3. At high energies b is about 8, because of the greater absorption.

Figure 4 shows our measurements plotted with data at the two nearest energies,[3,4] for qualitative comparison. The optical theorem gives the value of the imaginary part of the square of the forward-scattering amplitude to be 8.68 ± 0.05 mb/sr.[5] Using a simple exponential model for extrapolation[6] we have

$$d\sigma/d\Omega^* = |f_{\rm opt}|^2(1 + \alpha_n^2 + \beta_n^2)e^{bt},$$

where $f_{\rm opt} = k\sigma_{\rm tot}/4\pi$ is the mean value of the imaginary part of the spin-dependent amplitudes, α_n is the rms value of the real part of the spin-dependent amplitudes, and β_n is the rms deviation of the imaginary part of the amplitudes from $f_{\rm opt}$. From this we obtain

$$(\alpha_n^2 + \beta_n^2)^{1/2} = 0.41 \pm 0.04, \quad \alpha_n \geq -0.43 \pm 0.04,$$

where we have assumed that the sign is negative as determined by other methods.[6,7]

Using a small-angle p-d scattering technique, Dutton and van der Raay[8] obtained α_n and β_n at four-momenta from 1.29 to 1.69 GeV/c. We have averaged their results to obtain $(\alpha_n^2+\beta_n^2)^{1/2} = 0.56 \pm 0.18$, $\alpha_n = -0.48 \pm 0.16$, and $\beta_n = 0.26 \pm 0.26$. Using dispersion techniques Bugg and Carter[7] obtained $\alpha_n = -0.32 \pm 0.20$. Our result is consistent with these values, but because of the large error bars on α_n an improved value of β_n cannot be obtained. Since backward-angle n-p scattering[9] shows effects due to π^+ exchange for $-u < 0.01$ (GeV/c)^2 the possibility exists[2] that π^0 exchange at $-t < 0.01$ (GeV/c)^2 would change the slope and intercept from our values. This effect has been ignored in past publications and will be the subject of further investigation by us.

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Vacuum Polarization at Long Distances and the Heavy-Quark–Antiquark Potential

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Vacuum polarization at long distances for confined heavy-quark–antiquark ($Q\bar{Q}$) pairs is considered. The vacuum-polarization–corrected static potential is shown to have a radial dependence which should allow interpolation between charmonium, upsilon states, and other heavy $Q\bar{Q}$ systems. It is argued that the static, confining potential cannot grow faster than a linear potential at large distances, within the framework of this analysis.

Heavy-quark–antiquark ($Q\bar{Q}$) spectroscopy is generally regarded as being well described phenomenologically by a nonrelativistic potential model, with a static, quark-confining potential.[1] Along with the model, one has the understanding that there are (at least) two importnat regions of coordinate space, roughly described by $r \lesssim (\alpha_s m)^{-1} \simeq a_0$, where quantum-chromodynamic (QCD) perturbative effects are dominant, and $r \gg a_0$, where confinement is most significant and conventional perturbation theory is inapplicable. Here M is the heavy-quark mass, α_s and a_0 are the QCD fine-structure constant and Bohr radius, respectively, and r is the $Q\overline{Q}$ separation.

A further refinement of the potential model provides for the coupling of the $Q\overline{Q}$ states to real and virtual decay channels.[1,2] This coupling can be described qualitatively in several closely related ways. For example, in the string model, at large distances it is energetically favorable for the string to split, rather than increase in length indefinitely. In field-theoretical language this means that at sufficiently large $Q\overline{Q}$ separation, vacuum polarization will produce $q\overline{q}$ pairs, which gives partial screening of the $Q\overline{Q}$ potential, and allows the coupling $Q\overline{Q} \to Q\overline{q} + \overline{Q}q$, where q represents a light quark. Therefore, for a satisfactory description of $Q\overline{Q}$ spectroscopy, one must also consider the physics of these couplings, which then implies three important regions in the problem:

$$r < (\alpha_s M)^{-1}, \tag{1a}$$

$$(\alpha_s M)^{-1} < r < (2m)^{-1}, \tag{1b}$$

$$r > (2m)^{-1}, \tag{1c}$$

where m is the light-quark constituent mass. It is the purpose of this paper to present a discussion of the $Q\overline{Q}$ potential in regions (1b) and (1c) based on the effects of vacuum polarization at *long distances,* motivated by field-theoretic ideas. As a result we will exhibit a confining potential which should allow one to interpolate from charmonium to heavier $Q\overline{Q}$ systems, and which is considerably better motivated than those previously suggested.

Substantial circumstantial evidence has accumulated to suggest that the long-distance confining potential transforms as a Lorentz scalar.[3] Accordingly we consider the effective Lagrangian for color-singlet meson states to be

$$\int d^4x\, \mathcal{L}_{\text{eff}} = \int d^4x\, \mathcal{L}_{\text{quark}} - \frac{1}{2}\int \sum_{i,j} \overline{\psi}_i \psi_i(x) V_c(x-y) \overline{\psi}_j \psi_j(y)\, d^4x\, d^4y, \tag{2}$$

where $\psi_i(x)$ is a quark with flavor i, and

$$V_c(x) = 2\pi\delta(x_0) V_c(r). \tag{3}$$

Our results and qualitative conclusions, however, are independent of whether a vector or scalar confining force is chosen. In this approximation we consider all quark-antiquark pairs to interact by means of an instantaneous static potential. (This same assumption is made by the Cornell group in their analyses.[2]) We regard $V_c(r)$ as representing all orders in gluonic self-interactions, but with all fermion closed loops omitted. One can then envision a fermion loopwise expansion of the $Q\overline{Q}$ potential, with $V_c(r)$ the zeroth-order term in the expansion. In this vein, we have calculated the one-particle irreducible vacuum-polarization contribution to $V_c(r)$, given diagramatically by an infinite chain of bubbles of light quarks, which should represent dominant features of the coupling of the $Q\overline{Q}$ to decay channels at long distances.

The one-loop quark vacuum-polarization correction to the $Q\overline{Q}$ potential is given in momentum space by

$$\delta V_1(p^2) = -iV_c(p^2)\pi_1(p^2)V_c(p^2), \tag{4}$$

where $V_c(p^2)$ is the Fourier transform of $V_c(r)$. For a scalar potential the vacuum polarization $\pi_1(p^2)$ is quadratically divergent, so that two subtractions are required to obtain the finite result

$$\pi_R(p^2) = C_1 + C_2 p^2 + p^4 \tilde{\pi}_1(p^2), \tag{5}$$

where π_R is the renormalized version of π_1, and C_1 and C_2 are finite constants. Rather than choose a specific model for the vacuum polarization, we use the spectral representation

$$\tilde{\pi}_1(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{ds\, \mathrm{Im}\pi_R(s)}{s^2(s - p^2 + i\epsilon)}, \tag{6}$$

with $\mathrm{Im}\pi_R \geqslant 0$ as a result of the locality of the interaction in (2). The dressed one-loop vacuum-polarization correction can be iterated to form the *one-particle irreducible* vacuum-polarization–corrected potential

$$V(p^2) = V_c(p^2)\{1 + i[C_1 + C_2 p^2 + p^4 \tilde{\pi}_1(p^2)]V_c(p^2)\}^{-1}. \tag{7a}$$

We assume that *both* $V(p^2)$ and $V_c(p^2)$ are confining potentials. Then

$$C_1 = C_2 = 0 \tag{7b}$$

is required. Since $\tilde{\pi}_1(0) \neq 0$ from the positivity of $\mathrm{Im}\,\pi_R$, one also deduces that $V(p^2)$ cannot grow faster than $(\vec{p}^2)^{-2}$ as $\vec{p}^2 \to 0$, which means that *$V(r)$ cannot grow faster than a linear potential for large r, within the framework of this analysis.*

For definiteness, assume that

$$V_c(r) = ar + b \tag{8}$$

for all r. Then in coordinate space the one-loop correction to $V_c(r)$ is given by

$$\delta V_1(r) = \int d^3r'\, V_c(\vec{r} - \vec{r}\,')D(\vec{r}\,') \equiv (a/\pi^2)K(\vec{r}; m^2), \tag{9}$$

where $D(\vec{r})$ is the Fourier transform of $\tilde{\pi}_1(-\vec{p}^2)$. With the use of methods suggested by Schwinger,[4] it is easy to show that an excellent approximation is given by

$$K(\vec{r}; m^2) \simeq 3\lambda(\pi/m^2)[(ar + b) + \tfrac{1}{2}(a/m^2)(1 - e^{-2mr})r^{-1}], \tag{10a}$$

where

$$\lambda = \tfrac{8}{3}\pi m^2 \int_{4m^2}^{\infty} (ds/s^3)\,\mathrm{Im}\,\pi_R(s) > 0. \tag{10b}$$

One proves by induction that the iteration of this one-loop correction is

$$\delta V(\vec{r}) = -(a/\pi^2)\exp[3\lambda(a/\pi)\,\partial/\partial m^2]K(\vec{r}, m^2) \tag{11}$$

for the physically interesting case of three light quarks of approximately equal mass m. One then obtains the potential

$$V(r) = V_c(r) + \delta V(r) = Ar + B - (3\lambda/2\pi)(A/m^2)^2\{1 - \exp[2mr(a/A)^{1/2}]\}r^{-1}, \tag{12}$$

where

$$A = a[1 + \lambda(3/\pi)a/m^2]^{-1} < a,$$
$$B = b[1 + \lambda(3/\pi)a/m^2]^{-1} < b. \tag{13}$$

There are two interesting limiting cases, i.e.,

$$V(r) \simeq Ar + B \quad \text{for } r \gg [2m(a/A)^{1/2}]^{-1} \tag{14a}$$

and

$$V(r) \simeq ar + V(0) \quad \text{for } r \ll [2m(a/A)^{1/2}]^{-1}, \tag{14b}$$

where

$$V(0) = B - \frac{3\lambda}{\pi}\left(\frac{A}{m^2}\right)^2\left(\frac{a}{A}\right)^{1/2} m. \tag{15}$$

The screening in region (14a), and the absence of the renormalization of the slope of the potential in region (14b) is evident.

It is obvious that the potential (12) has the *correct qualitative behavior* to permit a satisfactory interpolation between ψ and Υ spectroscopy, since previous analyses[5] suggest that charmonium is sensitive to $r \gg (2m)^{-1}$, while upsilon states probe $r < (2m)^{-1}$. Note that the assumption that the vacuum polarization is due to *free* quark loops is too restrictive to be useful phenomenologically, which is why we have not committed ourselves to a specific model for the vacuum polarization. Instead we have used the spectral representation (6) in conjunction with (9). However, as a result the free parameter λ, defined by (10b), appears so as to incorporate these unspecified features of $\tilde{\pi}_1(p^2)$. The strong interactions among the light quarks are included in two separate ways: (a) by the iteration of the one-loop (dressed) vacuum-polarization correction so as to sum up the infinite chain of bubbles to form the *one-particle irreducible* vacuum-polarization correction, and (b) by the use of the complete spectral representation (6) in our analysis. A reflection of this situation is found in (13), which relates A in a nonperturbative way to all orders in $\pi^{-1}(a/m^2)$, the natural dimensionless parameter of the problem, as well as to the parameter λ defined by (10b).

For $Q\bar{Q}$ phenomenology, we advocate the potential

$$V_{tot}(r) = -\tfrac{4}{3}\alpha_s/r + V(r), \tag{16}$$

where

$$V(r) = Ar + B - (3\lambda/2\pi)(A/m^2)^2\{1 - \exp[-2mr(a/A)^{1/2}]\}r^{-1}, \tag{17}$$

with the parameters λ, A, a, and b related by Eq. (13). The free parameters of the model are A, λ, and b, but m should be fixed at the light-quark constituent mass. Moreover, $\lambda \geq \frac{1}{10}$ is required by the positivity of the spectral representation for $\tilde{\pi}_1(p^2)$, with the free quark loop providing the lower bound for λ. We display the behavior of $V(r)$ in Fig. 1 for a plausible choice of parameters. If $b=0$,[6] then $V(0) \simeq -1$ GeV, which is compatible with the sign and magnitude of $V(0)$ obtained earlier,[1] but here it is related to other parameters of the model. One observes that $V(r)$ has the same qualitative features guessed by other workers[5] in order to interpolate between ψ and Υ states, although our potential is much better motivated theoretically. It should also be noted that $V(r)$ has the same Lorentz transformation properties as $V_c(r)$, which is important in extending the above considerations to the spin-dependent corrections of the $Q\bar{Q}$ spectrum.[3,7]

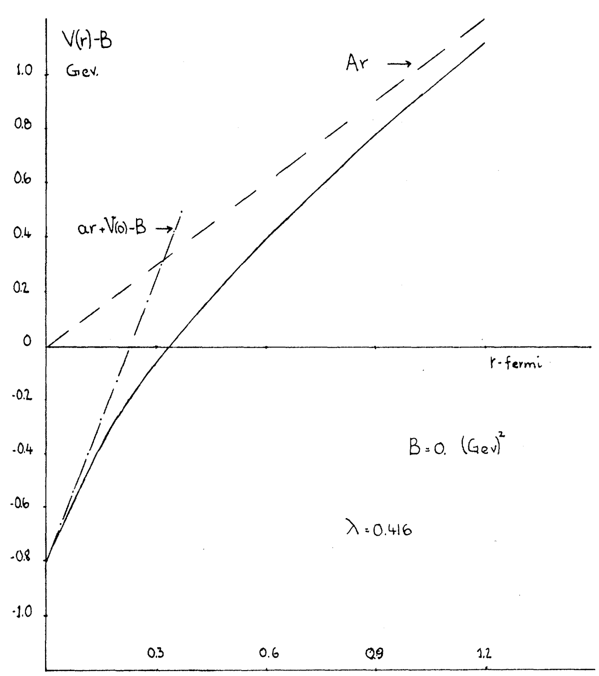

FIG. 1. A graph of $V(r)$, as given by Eq. (17), with $A=0.2$ GeV^2, $a/A=3.5$, $m=0.333$ GeV, and $b=0$. These parameters are *not* chosen to be best fits to data, but represent a plausible set of parameters based on earlier work (Refs. 1 and 5).

In summary, we have presented a treatment of the $Q\bar{Q}$ potential which is strongly motivated by field-theoretic ideas, an essential feature being the consideration of the *long-distance* vacuum-polarization effects, and which should interpolate between the various $Q\bar{Q}$ systems. Note that Υ is sensitive to $r<(2m)^{-1}$, where $V(r)\sim ar+V(0)$, as seen from Eq. (14b). Therefore if heavier $Q\bar{Q}$ states, such as $t\bar{t}$, exist, then $[E(n=2,{}^3S_1)-E(n=1,{}^3S_1)]_{t\bar{t}} < E(\Upsilon')-E(\Upsilon)$, since this energy difference should scale downward, roughly as $m_Q^{-1/3}$ from the $\Upsilon'-\Upsilon$ mass difference. On the other hand, the average size of the ψ states is *greater* than $(2m)^{-1}$, so that charmonium feels the *screened* potential, in contrast to the upsilon states, which lie *inside* the vacuum-polarization cloud of size $r=(2m)^{-1}$ due to the light quarks. This is the principal *physical* observation of this paper. We will report the details of the derivations in a separate communication.

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Precise Measurement of the Λ^0 Magnetic Moment

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The magnetic moment of the Λ^0 hyperon has been measured to be $\mu_\Lambda = (-0.6138 \pm 0.0047)\mu_N$.

Magnetic moments have played a major role in the development of our current understanding of the structure of matter. The Zeeman effect and the Stern-Gerlach experiments were crucial to modern ideas of angular momentum, spin, quantum mechanics, and atomic structure. Extraordinarily precise measurements of the magnetic moments of the electron and muon have supported the validity of quantum electrodynamics and established that these charged leptons behave as pointlike Dirac particles. The magnetic moments of the deuteron and other nuclei shed light on the structure of these composite systems. If the lessons of the past are any guide, precise measurements of baryon magnetic moments will provide us with strong constraints on models of hadronic structure, and important information about the nature of the constituents of hadrons.

The large anomalous moments for the neutron and proton have shown that these particles are not elementary. Their structure are related by unitary symmetry schemes which predict the ratio of their moments to 3% accuracy. Unitary symmetry also predicts the moments of the strange baryons. Previous measurements of the Λ^0 moment indicate that the symmetry is not exact, and that a symmetry-breaking parameter must be introduced into the theory.

In a simple s-wave quark model of the baryons, the nucleons contain only u and d quarks, and their moments can be used to calculate these quark moments. The magnetic moments of the other memebers of the baryon octet involve the strange quark. The lambda hyperon consists of u, d, and s quarks with the u and d quarks in a state with spin $J=0$. The spin and magnetic moment of the Λ^0 are identical to those of its s quark. Among the stable baryons, this property is unique. Thus, a precise measurement of the Λ^0 moment gives the s-quark moment directly. This, in turn, can be compared with the moment of the u quark to give the symmetry breaking. Further assumptions regarding the relationship between mass and magnetic moment allow calculations of quark masses which can be compared to those determined directly from hadron masses.

The observation that Λ^0's inclusively produced by 300-GeV protons are polarized has been reported.[1] This polarization offered an opportunity to measure the Λ^0 magnetic moment with unprecedented precision because of several advantages over earlier experiments. The large inclusive cross section and rapid data-acquisition techniques make it possible to obtain a large sample of polarized Λ^0's in a relatively short time. The high energy results in an average decay length of order 10 m. Conventional dc magnets over such distances give large precession angles. Finally, the Fermilab neutral hyperon spectrometer has high acceptance (greater than 70% averaged over momentum) which reduces systematic errors in measurements of the polarization vector. A measurement of the Λ^0 magnetic moment (to 9% uncertainty) was an intrinsic part of the original discovery of polarization. It was clear that a number of improvements could be made in a new experiment specifically designed to measure the moment.

The basic apparatus common to both measurements is illustrated in Fig. 1(a).[2] The coordinate system [Fig. 1(b)] has $\hat{Z}$ along the neutral beam direction. $\hat{Y}$ is vertical upwards, and $\hat{X} = \hat{Y} \times \hat{Z}$