1978 (unpublished); E. A. Treadwell, Ph.D. thesis, Cornell University, 1978 (unpublished).
[4]J. F. Martin *et al.*, Phys. Rev. Lett. 40, 283 (1978).
[5]C. T. Day, Ph.D. thesis, Cornell University, 1978 (unpublished).
[6]P. Joos *et al.*, Nucl. Phys. B113, 53 (1976).
[7]J. Ballam *et al.*, Phys. Rev. D 5, 545 (1972).
[8]L. A. Ahrens, Ph.D. thesis, Cornell University, 1978 (unpublished).
[9]J. Ballam *et al.*, Phys. Rev. D 10, 765 (1974).

Violations of SU(6) Selection Rules from Quark Hyperfine Interactions

Nathan Isgur
Department of Physics, University of Toronto, Toronto M5S 1A7, Canada

and

Gabriel Karl
Department of Physics, University of Guelph, Guelph N1G 2W1, Canada

and

Roman Koniuk
Department of Physics, University of Toronto, Toronto M5S 1A7, Canada
(Received 18 September 1978)

The decay amplitudes for $D_{15}\to p\gamma$, the amplitude for $D_{05}\to \bar{K}N$, and the charge radius of the neutron are zero in the SU(6) limit, but are observed to be nonzero. We show that all of these SU(6)-violating effects can be understood quantitatively in terms of the admixtures of excited-state configurations in the nucleon expected on the basis of color hyperfine interactions. In particular, the admixture of ^2S_M (i.e., $[\underline{70}, 0^+]$) with an amplitude of about $-\frac{1}{4}$ is central to understanding all three effects.

In the consideration of baryon decays[1] within the framework of the SU(6) quark model or $SU(6)_W$, two simple selection rules relevant to the negative-parity P-wave baryons emerge: Decays of the types

$$N\,{}^4P_M \to p\gamma \tag{1}$$

and

$$\Lambda\,{}^4P_M \to \bar{K}N \tag{2}$$

are forbidden[2] (our notation is $X\,{}^{2S+1}L_\sigma$, where $\sigma = S, M, A$ is the symmetry of the spatial wave function). These selection rules are most clearly tested in the specific decays $D_{15}(1670)\to p\gamma$ and $D_{05}(1830)\to\bar{K}N$ since in these cases mixing with other members of the $[\underline{70}, 1^-]$ multiplet is impossible. In both cases the selection rule is found to be approximately satisfied; for example, the branching ratio for $\Lambda(\frac{5}{2}^-; 1830)\to\bar{K}N$ is less than 10%. Nevertheless, in both cases violations of the selection rule are clear and well established. Columns 1 and 3 of Table I summarize the situation.

Recent work on quark models[8,9] has provided a considerable body of evidence in favor of the idea that violations of SU(6) symmetry of another kind —those responsible for the mass splittings and mixings of the SU(6) multiplets—can be explained in terms of color hyperfine interactions between quarks. These are interactions which can arise from one-gluon exchange which are analogous to

TABLE I. Violations of some SU(6) rules.

Quantity	SU(6) (Relative values)	This calculation (Relative values)	Experiment (Various units)
$A_{3/2}^n(D_{15}\to n\gamma)$	$-\alpha$	$-\alpha$	-60 ± 33 [a]
$A_{1/2}^n(D_{15}\to n\gamma)$	-0.71α	-0.71α	-33 ± 25 [a]
$A_{3/2}^p(D_{15}\to p\gamma)$	0	$+0.31\alpha$	$+20\pm13$ [a]
$A_{1/2}^p(D_{15}\to p\gamma)$	0	$+0.22\alpha$	$+19\pm14$ [a]
$A(D_{15}\to\bar{K}N)$	β	β	$+0.41\pm0.03$ [b]
$A(D_{05}\to\bar{K}N)$	0	-0.28β	-0.09 ± 0.04 [c]
$\langle\sum e_i r_i^2\rangle_p$	γ	γ	$+0.82\pm0.02$ [d]
$\langle\sum e_i r_i^2\rangle_n$	0	-0.16γ	-0.12 ± 0.01 [e]

[a]Ref. 3.
[b]Refs. 3 and 4.
[c]Refs. 4 and 5.
[d]Ref. 6.
[e]Ref. 7.

ordinary magnetic-dipole–magnetic-dipole forces of electromagnetism:

$$H_{\mathrm{hyp}}{}^{ij} = A_{ij}\left[\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}{}^3}\left(\frac{3(\vec{S}_i\cdot\vec{r}_{ij})(\vec{S}_j\cdot\vec{r}_{ij})}{r_{ij}{}^2} - \vec{S}_i\cdot\vec{S}_j\right)\right], \tag{3}$$

where in quantum chromodynamics in lowest order $A_{ij} = 2\alpha_s/3m_im_j$ for two quarks in a baryon. We will show that this same interaction can also account for the observed violations of the SU(6) selection rules forbidding the decays (1) and (2) via the mixing which it causes of non-$N\,{}^2S_S$-type configurations into the nucleon. In particular we are able to attribute these violations to transitions into an $N\,{}^2S_M$ component in the nucleon, the presence of which is independently indicated by the observed charge radius of the neutron.

As usual we frame our discussion in terms of the harmonic-oscillator model. It will, however, be clear that our conclusions are for the most part independent of that language. The interaction (3) can mix into the ground-state configuration $N\,{}^2S_S$ ($[\underline{56}, 0^+]$) of the (spin-independent) confinement-potential problem a variety of other configurations, including $N\,{}^2S_S{}'$ ($[\underline{56}', 0^+]$), $N\,{}^2S_M$ ($[\underline{70}, 0^+]$), and $N\,{}^2D_M$ ($[\underline{70}, 2^+]$). Using harmonic-oscillator wave functions and an analysis of the positive-parity excited baryons based on the interaction (3),[8] one can conclude that the physical nucleon is of the form

$$|N\rangle \simeq 0.90|N\,{}^2S_S\rangle - 0.34|N\,{}^2S_S{}'\rangle - 0.27|N\,{}^2S_M\rangle - 0.06|N\,{}^2D_M\rangle. \tag{4}$$

This composition follows from using perturbation theory to calculate mixing matrix elements[10] in the relevant oscillator wave functions and then diagonalizing a 4×4 matrix with diagonal entries chosen to give eigenvalues corresponding to the observed $N(940)$, $N(1410)$, and $N(1710)$ and to a presumed $N\,{}^2D_M$ at around 1900 MeV. The resulting admixture of D waves into the nucleon is quite small, in accord with the well-established smallness of the $E2$ moment in the decay $\Delta\to N\gamma$; its effects here are also quite small and we shall henceforth neglect this component. If we define

$$|N_S\rangle \equiv \tfrac{1}{2}\sqrt{2}\,(\chi_+{}^\rho\varphi_N{}^\rho + \chi_+{}^\lambda\varphi_N{}^\lambda)(\psi_{00}{}^S\cos\theta + \psi_{00}{}^{S'}\sin\theta), \tag{5}$$

$$|N_M\rangle \equiv |N\,{}^2S_M\rangle \equiv \tfrac{1}{2}(\chi_+{}^\rho\varphi_N{}^\rho\psi_{00}{}^{M\lambda} + \chi_+{}^\rho\varphi_N{}^\lambda\psi_{00}{}^{M\rho} + \chi_+{}^\lambda\varphi_N{}^\rho\psi_{00}{}^{M\rho} - \chi_+{}^\lambda\varphi_N{}^\lambda\psi_{00}{}^{M\lambda}), \tag{6}$$

where χ and φ are the usual spin-$\frac{1}{2}$ and isospin-$\frac{1}{2}$ wave functions[8,9] and

$$\psi_{00}{}^S = (\alpha^3/\pi^{3/2})\exp[-\tfrac{1}{2}\alpha^2(\rho^2+\lambda^2)], \tag{7}$$

$$\psi_{00}{}^{S'} = \tfrac{1}{3}\sqrt{3}(\alpha^5/\pi^{3/2})(\rho^2+\lambda^2-3\alpha^{-2})\exp[-\tfrac{1}{2}\alpha^2(\rho^2+\lambda^2)], \tag{8}$$

$$\psi_{00}{}^{M\lambda} = \tfrac{1}{3}\sqrt{3}(\alpha^5/\pi^{3/2})(\rho^2-\lambda^2)\exp[-\tfrac{1}{2}\alpha^2(\rho^2+\lambda^2)], \tag{9}$$

$$\psi_{00}{}^{M\rho} = \tfrac{2}{3}\sqrt{3}(\alpha^5/\pi^{3/2})\vec{\rho}\cdot\vec{\lambda}\exp[-\tfrac{1}{2}\alpha^2(\rho^2+\lambda^2)], \tag{10}$$

with $\sin\theta \simeq -0.35$, we can then simply write

$$|N\rangle \simeq |N_S\rangle\cos\varphi + |N_M\rangle\sin\varphi, \tag{11}$$

where $\sin\varphi \simeq -0.27$.

This structure for the nucleon has a simple physical interpretation. The first term in (3), called the contact term, is responsible for the Δ-N mass difference and so is attractive for antiparallel and repulsive for parallel spins. The net effect of this term is both to lower the energy of the nucleon and to reduce its size. All three quarks are not, however, equally affected by this force: The two identical quarks (uu in p and dd in n) are necessarily in a spin-1 state and so certainly repel each other. The resulting distortion of the nucleon therefore has two components: a symmetric part measured by θ and a part with mixed symmetry measured by φ.

The mixed piece has one consequence that can be immediately confronted with experiment: The two d quarks in the neutron will repel each other leaving the neutron with a positive center. More precisely, one finds that

$$\alpha^2\langle\textstyle\sum e_ir_i{}^2\rangle_n = \frac{2\sin\varphi\cos\varphi}{\sqrt{6}}\left(\cos\theta + \frac{2}{\sqrt{3}}\sin\theta\right), \tag{12}$$

$$\alpha^2\langle\textstyle\sum e_ir_i{}^2\rangle_p = \cos^2\varphi\left(\cos^2\theta + \frac{2}{\sqrt{3}}\cos\theta\sin\theta + \frac{1}{6}\sin^2\theta\right) - \frac{2\sin\varphi\cos\varphi}{\sqrt{6}}\left(\cos\theta + \frac{2}{\sqrt{3}}\sin\theta\right) + \frac{5}{3}\sin^2\varphi, \tag{13}$$

which gives

$$\frac{\langle\sum e_i r_i^2\rangle_n}{\langle\sum e_i r_i^2\rangle_p} \simeq -0.16 \tag{14}$$

in good agreement[11] with the observed value[6,7] of this ratio which is -0.15 ± 0.01. With this confirmation of the wave function (11), we turn to consider the effects of such a nucleon on the decays (1) and (2).

We begin by discussing the radiative decay $D_{15} \to p\gamma$. There are two amplitudes $A_{3/2}^p$ and $A_{1/2}^p$ for this decay corresponding to the decay of the $J_z = \frac{3}{2}$ and $\frac{1}{2}$ components of the resonance into a photon moving along the z axis with positive helicity.[12] Using the usual interaction obtained by nonrelativistic reduction of the γ^μ quark-photon interaction,

$$\langle B'(p's')\gamma(q\lambda)|T|B(ps)\rangle = -\frac{3ie}{(2\pi)^{3/2}}\left\langle B'\left|\frac{e_3}{e}\left(\frac{\vec{\sigma}\cdot\vec{q}\times\vec{\epsilon}^*}{2m} + i\,\frac{\vec{\epsilon}^*\cdot\vec{p}_3'}{m}\right)\exp\left[i\left(\frac{2}{3}\right)^{1/2}\vec{q}\cdot\vec{\lambda}\right]\right|B\right\rangle, \tag{15}$$

where the photon polarization vector is $\vec{\epsilon}(q\lambda)$ and where e_3 and p_3' are the charge and momentum of the third quark in B', we obtain

$$\frac{A_{3/2}^p}{A_{3/2}^n} = \left(\frac{2}{3}\right)^{1/2}\frac{\tan\varphi}{\cos\theta + \frac{1}{3}\sqrt{3}\sin\theta} \simeq -0.31, \tag{16}$$

$$A_{1/2}^p / A_{3/2}^p = \tfrac{1}{2}\sqrt{2}, \tag{17}$$

which compare very favorably in *sign and magnitude* with the experimental values of Table I.

We next consider the decay $D_{05}(1830) \to \bar{K}N$. The interaction responsible for this decay may be written as[1]

$$\langle B'(p's')K^-(q)|T|B(ps)\rangle = -[3\sqrt{2}i/(2\pi)^{3/2}]\langle B'|(g\vec{q}\cdot\vec{\sigma}_3 + h\vec{\sigma}_3\cdot\vec{p}_3')\exp[i(\tfrac{2}{3})^{1/2}\vec{q}\cdot\vec{\lambda}](v_3)_+|B\rangle, \tag{18}$$

where g and h are two phenomenological parameters describing the effective interaction $q \to qM$, where $M = \pi, K, \ldots$. In terms of this interaction one can show that

$$\frac{A(D_{05} \to \bar{K}N\,^2S_M)}{A(D_{15} \to \bar{K}N\,^2S_S)} = \frac{1}{\sqrt{2}}. \tag{19}$$

The nucleon structure (11) also affects the allowed decay of $D_{15}(1765)$; taking this into account reduces its $\bar{K}N$ amplitude to about two-thirds of its SU(6) value, giving the result

$$\frac{A(D_{05} \to \bar{K}N)}{A(D_{15} \to \bar{K}N)} \simeq -0.28, \tag{20}$$

which compares favorably to the experimental value shown in the table[4]:

$$\left|\frac{A(D_{05} \to \bar{K}N)}{A(D_{15} \to \bar{K}N)}\right|_{\text{exp}} = 0.22 \pm 0.09. \tag{21}$$

The sign of the SU(6)-violating amplitude (20) (which is convention dependent) may be checked by comparing the amplitudes for the processes $\bar{K}N \to D_{05}(1830) \to \Sigma\pi$ and $\bar{K}N \to D_{15}(1765) \to \Sigma\pi$; we find that these amplitudes are of opposite sign as found experimentally. These latter two amplitudes can also be compared directly, though in this case the comparison is more model dependent than the parameter-free relation (19): In addition to involving g and h of Eq. (18) separately, it also involves the coefficients of an expansion analogous to (4) for the $\Sigma(1195)$. These coefficients may, of course, be calculated using (3), and the parameters g and h may be determined from other decays; doing this we find

$$\frac{A(KN \to D_{05}(1830) \to \Sigma\pi)}{A(KN \to D_{15}(1765) \to \Sigma\pi)} \simeq -1.0 \tag{22}$$

compared to the experimental value[3,5] of -1.1 ± 0.3.[4]

The internal composition (4) of the nucleon predicted by the hyperfine interactions (3) thus seems capable of explaining in both sign and magnitude the violations of SU(6) observed in the nonzero values for the neutron charge radius, the amplitudes for $D_{15} \to p\gamma$, and the amplitude for $D_{05} \to \bar{K}N$. Our conclusions may also be stated more phenomenologically: We find that all three types of violations have a common source in the admixture of ^2S_M in the nucleon with an amplitude of approximately $-\frac{1}{4}$.

The presence of representations other than the $[\underline{56}, 0^+]$ in the nucleon and analogous mixings in other particles will not only allow once-forbidden processes but, as already mentioned above, may also cause significant changes in the SU(6) predictions for allowed decays.[13] This leads to a variety of effects, some of which may be associated with the Melosh transformation. The Melosh transformation corresponds to taking a specific *Ansatz* for the single-quark transition operators but leaving the hadron states untransformed. Here we mix the states in a very specific way which is dictated by the hyperfine interactions (3), but use the elementary transition amplitudes (15) and (18). As normally applied, the two methods are not equivalent: The transitions (1) and (2) remain forbidden in the Melosh approach and the incorporation of such effects will presumably require some elaboration of the original simple *Ansatz*.

Finally, we mention that configuration mixing does not seem capable of resolving all problems of SU(6) breaking. In particular we are unable to account for the factor of $\frac{3}{4}$ required to bring the SU(6) value of $\frac{5}{3}$ for G_A/G_V into agreement with experiment; the mixing in (4), though it reduces the prediction, does so only to the extent of about 6%. We tentatively associate the remaining discrepancy with our neglect of relativistic effects which are known[14] to suppress G_A/G_V.

Two of us (N.I. and R.K.) wish to thank the Department of Theoretical Physics at Oxford for its hospitality during the course of this work; the other (G.K.) similarly thanks the Theory Division of the Rutherford Laboratory. We also express our appreciation to Dr. S. L. Glashow, Dr. G. P. Gopal, Dr. P. J. Litchfield, and Dr. R. G. Moorhouse for discussions. This work was supported in part by the National Research Council of Canada.

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[1]See, for example, H. J. Lipkin, Phys. Rep. 8C, 173 (1973); J. L. Rosner, Phys. Rep. 11C, 189 (1974); David Faiman and Archibald W. Hendry, Phys. Rev. 173, 1720 (1968); R. Horgan, in *Proceedings of the Topical Conference on Baryons, Oxford, 1976*, edited by R. T. Ross and D. H. Saxon (Rutherford High Energy Laboratory, Chilton, Didcot, Oxon, England, 1976).

[2]The selection rule for photoproduction is due to R. G. Moorhouse, Phys. Rev. Lett. 16, 771 (1966); a discussion of the $\bar{K}N$ selection rule may be found in D. Faiman and D. E. Plane [Nucl. Phys. B50, 379 (1972)] and in A. J. G. Hey, P. J. Litchfield, and R. J. Cashmore, [Nucl. Phys. B95, 516 (1975)].

[3]C. Bricman *et al.*, Phys. Lett. 75B, 1 (1978).

[4]We have used a simple D-wave phase-space factor of q^5 to extract amplitudes from the experimentally measured widths.

[5]G. P. Gopal *et al.*, Nucl. Phys. B119, 362 (1977).

[6]F. Borkowski *et al.*, Nucl. Phys. A222, 269 (1974).

[7]V. E. Krohn and G. R. Ringo, Phys. Rev. D 8, 1305 (1973); R. W. Berard *et al.*, Phys. Lett. 47B, 355 (1973).

[8]Nathan Isgur and Gabriel Karl, Phys. Lett. 72B, 109 (1977), and Phys. Lett. 74B, 353 (1978), and Phys. Rev. D (to be published), and to be published. For other related work on baryons see, for example, D. Gromes, Nucl. Phys. B130, 18 (1977); L. J. Reinders, to be published; U. Elwanger, University of Heidelberg Report No. HD-THEP-78-1, 1978 (to be published). For related work on mesons see, for example, A. B. Henriques, B. H. Kellet, and R. G. Moorhouse, Phys. Lett. 64B, 85 (1976); H. Schnitzer, Phys. Lett. 65B, 239 (1976), and 69B, 477 (1977), and to be published; R. H. Graham and P. J. O'Donnell, Phys. Rev. D (to be published).

[9]For recent reviews of applications of quantum-chromodynamic ideas to low-energy hadron structure see Nathan Isgur, Oxford University Report No. 67/78, 1978 (unpublished) and G. Karl, in Proceedings of the Nineteenth International Conference on High Energy Physics, Tokyo, Japan, 23-30 August 1978 (to be published).

[10]Strictly speaking, the interaction (3) should not be used in higher than first order because it is an illegal operator in the Schrödinger equation. However, multiple gluon exchanges can be expected to tame the otherwise singular behavior one would get by smearing out, for example, the δ function over a region of size m_q^{-1}. We have implicitly assumed that such a smearing has occurred by truncating the mixing series with the lowest-lying excited states.

[11]R. Carlitz, S. D. Ellis, and R. Savit, Phys. Lett. 64B, 85 (1976); Nathan Isgur, Acta Phys. Pol. B8, 1081 (1977).

[12]See, for example, L. A. Copley, G. Karl, and E. Obryk, Nucl. Phys. B13, 303 (1969).

[13]Nathan Isgur, Gabriel Karl, and Roman Koniuk, unpublished.

[14]P. N. Bogoliubov, Ann. Inst. Henri Poincaré 8, 163 (1968); A. Le Yaouanc *et al.*, Phys. Rev. D 15, 844 (1977), and references therein. Le Yaouanc *et al.* have also emphasized the importance of $[70, 0^+]$ components in the nucleon as a source of SU(6) breaking.