in order to investigate more precisely the cases in which the linear-polarization direction has been predicted to be randomly distributed. On the other hand, if other excitation processes such as two-photon absorption are used, then new polarization properties are to be expected: These experiments are now in progress.

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Evidence for Magnetic Fluctuations as the Heat-Loss Mechanism in the Alcator Tokamak

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It is argued that resonant magnetic fluctuations are the mechanism of anomalous electron heat loss in tokamaks. The experimental and theoretical support for this assertion are given.

While anomalous electron heat loss is clearly one of the major physics problems in toroidal confinement, the process or processes responsible for it are still largely unknown. Traditionally, anomalous losses have been attributed to drift waves of various kinds,[1] but the experimental evidence for this assumption is ambiguous at best. More recent measurements[2] seem to rule out drift waves, for densities $> 10^{14}$, at least in the usual quasilinear sense. However, even at fluctuation levels below those required to produce significant quasilinear transport, the inherently nonlinear effect of magnetic-surface destruction can lead to sizable transport. These points were emphasized recently by Callen.[3] There are basically two forms this magnetic-surface destruction can take. In the first,[3] the so-called "magnetic-flutter" model, one has essentially coherent island structures most of the time. These grow up out of noise to an amplitude at which islands overlap. Stochasticity of the field lines ensues and presumably damps the underlying drift waves, allowing the original equilibrium to reform, wherein the process repeats itself. Alternatively one can have some quasisteady saturated turbulence level in which "stochasticity" prevails.[4] Either form can lead to substantial enhancements of thermal transport.

In this Letter, we make a connection between these magnetic fluctuations and an anomaly in the soft-x-ray spectrum that has been found persistently in the Alcator tokamak.[5] This anomaly cannot be explained by classical processes. We show that magnetic fluctuations give rise to an enhanced suprathermal tail in the electron distribution function which is related to the thermal flux. The tail and energy flux are both gauged by the same parameter, τ_E, the energy confinement time. By fitting the x-ray spectrum we determine a τ_E^*. The τ_E^* so obtained agrees with the bulk energy confinement measurements both in absolute magnitude and scaling with the plasma density.

Soft-x-ray spectra between 1 and 7 keV have been collected from the Alcator tokamak using a Si(Li) crystal and pulse-height-analysis system, with a collimated detector viewing along chords of the minor cross section. Figure 1 shows a typical nonthermal spectrum taken during the

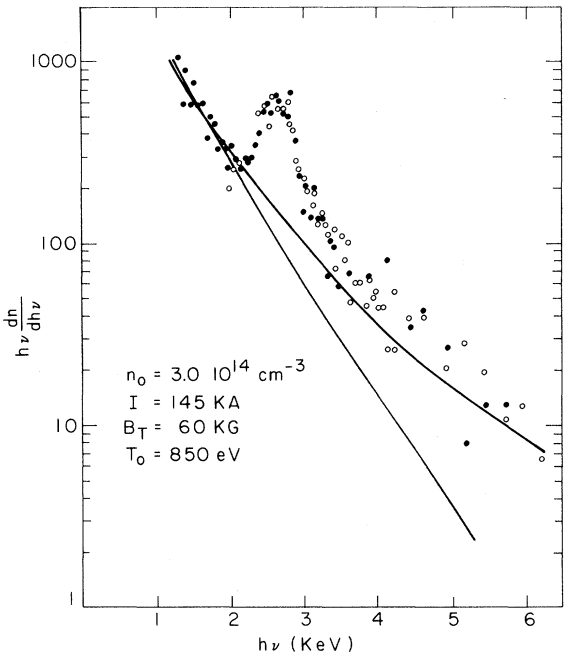

FIG. 1. An x-ray spectrum for a typical nonthermal Alcator discharge. The lower curve is from an 850-eV Maxwellian distribution; the upper curve includes the perturbation induced by magnetic fluctuations.

steady-state portion of a 145-kA discharge with a peak density of 3×10^{14} cm^{-3}, a toroidal magnetic field of 60 kG, and $Z_{\text{eff}} \sim 1$. The feature at 2.7 keV is due to L-line radiation from the molybdenum introduced into the plasma by interaction with the limiter. The open circles were taken through a 0.005-cm beryllium filter to enhance the high-energy region. A temperature of 850 eV is deduced by fitting the points between 1.3 and 2.1 keV and accounting for density and temperature profiles. The portion of the spectrum between 3.5 and 6.25 keV is best described by a temperature of 1500 eV. At the highest densities the discharges are "thermal" with only a single temperature. The lower curve in Fig. 1 represents the spectrum from a temperature of 850 eV. From chordal scans, this nonthermal behavior is seen to occur at all radii in the plasma.

This high-energy component cannot be due to pulse pileup because it persists at low counting rates (< 10 kHz) with a short ($\frac{1}{2}$ μs) shaping time constant in the amplifier. A Be filter is used to attenuate the low-energy events which at high counting rates would combine to form false high-energy counts. The shape of this spectrum is not characteristic of a piled-up spectrum which would have only one "temperature" and not a tail. Furthermore, this tail is seen to disappear as the density and subsequent counting rate are increased.

The classical modifications to the Maxwellian (including Spitzer-Härm,[6] neoclassical,[7] Kruskal-Bernstein,[8] and runaway effects) fail to account for the spectrum by several orders of magnitude, primarily because the dominant perturbation is odd in pitch angle and makes no contribution to the spectrum.

The quasilinear effects of drift waves cannot account for the spectrum for two reasons. First, drift waves are resonant with slow electrons, $\omega/k_{\parallel}v_e \ll 1$ (in the sheared-field case, only a small portion of the eigenmode has $\omega/k_{\parallel}v_e > 1$). Second, if they were resonant at high phase velocities, the density fluctuations in the center on the Alcator[2] ($\tilde{n}/n < 10^{-2}$) are known to be too small to account for the energy losses.[3] It follows from what we are about to show that they then cannot explain the x rays either.

In the following, for definiteness, we consider the consequences of the "stochasticity" limit of magnetic fluctuations. It will then become apparent which aspects of the model are necessary, allowing us to generalize the conclusions and make a comparison with "magnetic flutter" or other possible models.

Several authors[4,9] have considered the diffusion of test particles in a stochastic magnetic field. Estimates of heat transport are made by equating the diffusion coefficient to the thermal conductivity. Of course, to be more accurate one must use the appropriate electron kinetic equation. Its solution gives distortions (from Maxwellian) associated with the energy flow.

From the test-particle diffusion picture one is led to add a term, $(\partial/\partial x)[D\partial f_e/\partial x]$, to the electron kinetic equation (here, x is the radial variable). However, such a procedure omits the possible ambipolar potential and gives a kinetic equation which does not conserve particles locally (except in the trivial case $\partial f_e/\partial x = 0$). The radial ambipolar field, E^A, will cause electrons to lose energy as they step out radially (and vice versa). Thus the random walk occurs not in space but along paths in the energy-radius (w,x) plane given by $dx/dl = 1$ and $dw/dl = -eE^A/m$. More generally, then, stochastic diffusion gives rise to a term $\mathcal{L}D\mathcal{L}f_e$, where $\mathcal{L} = \partial/\partial x - (eE^A/m)\partial/\partial w$, and the appropriate kinetic equation is therefore

$$\frac{\partial f_e}{\partial t} = \frac{e}{m}\vec{E}^T \cdot \frac{\partial f_e}{\partial \vec{v}} + C(f_e) + \mathcal{L}D\mathcal{L}f_e, \qquad (1)$$

where $\vec{E}^T$ is the applied electric field, C is the collision operator, and $w = v^2$ has the dimensions of velocity squared. In the limit most appropriate to tokamaks, the collisional mean free path is longer than the stochastic mixing length, $L_c = \pi R/\ln(\pi S/2)$ (less than πR, when the stochasticity parameter, S, is large) and the diffusion coefficient is given by[4]

$$D = \pi R |v_\parallel| \sum_{m,n} |\tilde{B}_{mn}/B|^2 \delta(n - m/q). \quad (2)$$

For an explanation of the notation and some of the background, we refer the reader to Ref. 4.

A more rigorous derivation of Eq. (1) can be given by solving the drift kinetic equation directly for the slow evolution of the smooth distribution function.[10] The derivation shows that (a) stochasticity of the lines is necessary to obtain Eqs. (1) and (2), and (b) the δ function in (2) is actually a resonance function of sufficient width to make several modes contribute to D at each spatial point [D is then smooth as it would not be from a strict application of Eq. (2)].

We thus seek a solution to (1) in the steady state by an expansion in powers of E/E_R, $f = f^{(0)} + f^{(1)} + f^{(2)} + \cdots$, regarding D as second order. The ordering is suggested by both the size of D and the fact that the diffusive loss process must balance Ohmic heating, which is second order in E/E_R.

To the lowest order we have $0 = C(f_e)$, which says that f_e is a local Maxwellian but gives no information on the density and temperature. The first-order equation is

$$0 = C(f^{(1)}) - \frac{e}{m} \vec{E}^T \cdot \frac{\partial f^{(0)}}{\partial \vec{v}}, \quad (3)$$

or the Spitzer-Härm problem. The solubility conditions on (3) permit the inversion of the C operator to obtain $f^{(1)}$. These are obtained by applying annihilators of C to Eq. (3). In the present case, these are the particle and energy moments. (C allows momentum transfer to the ions.) This order gives no information on n and T_e, since both terms in (3) annihilate, but provides $f^{(1)} = f_{SH}$ and the electrical conductivity.

At second order,

$$0 = C(f^{(2)}) + \mathfrak{L} D \mathfrak{L} f^{(0)} - \frac{e}{m} \vec{E}^T \cdot \frac{\partial f^{(1)}}{\partial \vec{v}}. \quad (4)$$

The particle moment of (4) requires no particle fluxes on the electron time scale, or an ambipolarity constraint,

$$0 = \frac{\partial^2}{\partial x^2} n v_e + \frac{eE^A}{2T_e} v_e^2 \frac{\partial}{\partial x} \frac{n}{v_e} + \frac{\partial}{\partial x} \frac{eE^A}{2T_e} n v_e + n v_e \left(\frac{eE^A}{2T_e}\right)^2, \quad (5)$$

which will be recognized as a Riccati equation for E^A. From the energy moment, we find, using (5), the heat transport equation

$$0 = \vec{J} \cdot \vec{E} + \frac{2}{\sqrt{\pi}} D_0 \left[n \frac{\partial^2 T_e}{\partial x^2} + \frac{2}{v_e} \left(\frac{\partial n v_e}{\partial x} + \frac{eE^A}{2T_e} n v_e \right) \frac{\partial T_e}{\partial x} \right], \quad (6)$$

where $D_0 \equiv D(v_e)$. For simplicity in writing (6) we have neglected heat transfer to the ions (a process involving the low-energy part of f_e, which is not of interest here).

The distribution-function distortion associated with heat transport is obtained by solving Eq. (4) for $f^{(2)}$ once the solubility conditions (5) and (6) are satisfied. The dominant contribution for $u \equiv v/v_e \gg 1$, comes from the spatial diffusion term, $f^{(2)} = -C^{-1}[(\partial/\partial x)(D \partial f^{(0)}/\partial x)]$, where C^{-1} denotes the inverse of the collision operator. For $u^2 \gg 1$, the collision operator is linear, asymptotically, and can be easily inverted, giving to order u^{-2},

$$f^{(2)} = \frac{D_0}{\nu} \left| \frac{d \ln T_e}{dx} \right|^2 f_{max} \sum_m P_m(\mu) \frac{a_m}{4[8 + m(m+1)]} \left(\frac{v}{v_e}\right)^8, \quad (7)$$

where $a_m = [(2m+1)/(m+2)][P_{m-2}(0) + P_m(0)]$, and the P_m's are Legendre polynomials. Defining the local energy confinement time, $\tau_E^* = 1/D_0 |d \ln T_e/dx|^2$, (7) can be written schematically

$$f^{(2)} \sim \frac{1}{\nu \tau_E^*} \left(\frac{v}{v_e}\right)^8 f_{max}, \quad (8)$$

so that the perturbation to the Maxwell measures τ_E^*.

The x-ray spectrum can now be found using this distribution function. The fitting procedure was to find first the bulk temperature by a fit to the low-energy ($h\nu < 2.1$ keV) points. This gives the lower curve in Fig. 1. The parameter τ_E^* is then varied to fit the tail, with the overall spectra

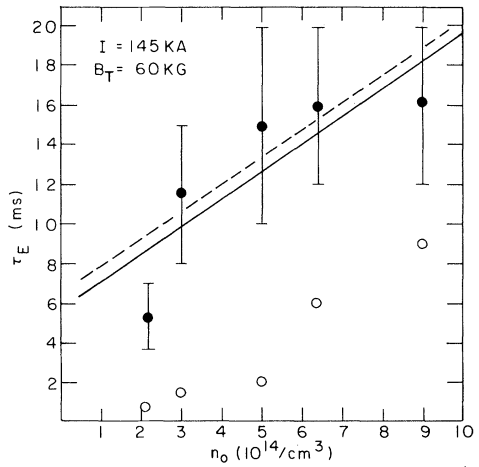

FIG. 2. τ_E vs η_0. The open circles are τ_E* deduced from fitting the tail of the x-ray spectrum; uncertainties are about a factor of 2. Solid points are τ_E determined from energy content divided by the input power. The dashed line is from Ref. 11. The solid line is the best linear fit to the points.

shown by the upper curve in Fig. 1. Although, as a result, $f^{(2)}$ is of order $f^{(0)}$ at high velocities and perturbation theory is not *a priori* valid there, the series it generates is correct, as can be shown by a proper asymptotic procedure paralleling the region II expansion of Ref. 9. (The spectra of Fig. 1 includes the next term of this series.)

A sequence of discharges of varying density but constant current and toroidal field (145 KA and 60 kG) were similarly analyzed. The values for the confinement times are shown as open circles in Fig. 2. Shown for comparison are confinement times, τ_E, obtained by the standard method.[11]

A comparison such as Fig. 2 has some obvious limitations, notably the discrepancy between the local values of $\tau_E^* = 1/D_0/d \ln T_e/dx|^2$ and the global parameter, τ_E. In fact, we have treated τ_E* as a constant, while retaining the radial dependence of the remaining parameters in f_e. For a more quantitative comparison one needs profiles of spectra (which are obtainable in practice) to determine D_0 as a function of radius and then explicit transport-code calculations of τ_E. In view of these limitations the agreement between the two τ_E determinations is rather remarkable, particularly since the slopes of the two curves are virtually identical. Without including correct profiles better agreement in magnitude is not meaningful.

At this point what we have shown is that when a kinetic equation of the form (2) is valid, the amount of D required to account for the soft x rays also accounts for the observed anomalous losses, and vice versa. The evidence, then, suggests that Eq. (2) is appropriate for tokamak plasmas. Magnetic fluctuations are the only known process that can lead to an equation of the form (2), consistent with other observations. Thus, while it is conceivable that some very unusual drift wave could be resonant with high-energy electrons and give an equation like (2) in quasilinear theory, the D implied by observed density fluctuations (2) is too small to explain either the x rays or the energy transport. Our tentative conclusion is that resonant magnetic fluctuations are responsible for anomalous electron heat loss in the Alcator and presumably similar tokamaks as well.

A definitive conclusion should be possible through measurement of the profiles, establishing a consistent fitting procedure, and possibly extending the spectra to higher energy. The latter measurement also has a bearing on which of the two competing models, "magnetic flutter" or "stochasticity," is correct. It has been suggested[12] that magnetic flutter will also give an equation like (2), since it is essentially a diffusion process in which the step size and time are the average island width and lifetime. The diffusion coefficient is then independent of velocity and leads to a perturbation of the form $f^{(2)} \sim (1/\nu\tau_E^*) \times (v/v_e)^7 f_{max}$, as compared to Eq. (11) for stochasticity. These differences are indistinguishable in the present analysis but ultimately lead to different asymptotic behavior at high v/v_e which should be measurable.

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Empirical Criterion for the Glass Transition Region Based on Monte Carlo Simulations

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We propose a new empirical criterion using the pair distribution function for specifying the supercooled-liquid/amorphous phase boundary.

Modeling techniques and computer simulations as applied to noncrystalline solids have developed into a very active and rewarding area of research, and generally seek to reproduce the salient features of the experimentally determined "pair distribution function" (hereafter designated PDF) obtained from radiation diffraction techniques.[1-3] Tools for the theoretical analysis of the amorphous state include hard- and soft-sphere packing,[1] molecular dynamics,[2] and, very recently,[4] the Monte Carlo method[5] for a canonical ensemble (temperature T, volume V, and number of atoms N are fixed).

However, the onset of the glass transition region, as a function of *both* temperature and pressure, has heretofore not received significant attention. By employing the isothermal-isobaric Monte Carlo method of classical statistical mechanics,[1,2] we are presently investigating whether there exist unique, or distinctive, structural and thermodynamic features associated with the {supercooled-liquid $\rightarrow$ glass} transition region reached by abruptly cooling and/or compressing an equilibrated simple liquid.

The (N, P, T) Monte Carlo procedure used in this study has been described by McDonald.[6] The interatomic force law was arbitrarily taken to be Lennard-Jones 12-6 with (ϵ, σ) denoting the well-depth and size parameters, respectively. In order to simulate the bulk, the standard periodic boundary conditions were imposed with respect to translations parallel to the faces of the computational cube composed of 108 atoms. In each simulation "experiment," we started with the last fluid configuration of a well-equilibrated fluid state at reduced temperature $T^* = kT/\epsilon = 1.0$ and reduced pressure $P^* = P\sigma^2/\epsilon = 1.0$. Then we abruptly quench and/or compress the system to a new (P^*, T^*) by simply setting the pressure and temperature in the Monte Carlo procedure to the new desired values.[7] After "equilibrating" the system through 2×10^5 Monte Carlo moves with a 50% acceptance ratio, 6×10^5 further moves were performed to obtain the enthalpy H, the average density $\langle\rho\rangle$, the PDF $g(r)$, and other quantities. Experiments were simulated for constant $P^* = 1$ and various $0.1 \leq T^* < 1.0$, constant $T^* = 1.0$ and various $1 < P^* \leq 20$, and various combinations of $1 < P^* \leq 9$ and $0.6 \leq T^* < 1$.

As illustrated in Fig. 1, the PDF's generally show the features expected. In the supercooled-liquid state, two smooth peaks exist: a first prominent peak and a second smaller peak, corresponding to the first and second coordination shells of an atom in the liquid, respectively. As the instantaneous temperature quenches probe deeper into the metastable region, the first peak becomes more pronounced in magnitude and narrower in width, the first minimum decreases in magnitude, and the second peak gradually flattens in shape with an eventual "bimodal splitting" at very low temperatures ($P^* = 1.0$, $T^* \leq 0.4$). Clearly, the development of the split second peak indicates an amorphous atomic packing,[1-3] this being the principal structural feature in the experimental PDF of amorphous materials that previous theoretical models have attempted to describe. However, the second-peak flattening preceding the fully developed splitting may be the "signature" that the glass transition region has been reached.