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Role of the Exit-Channel Distorting Potential in Heavy-Ion–Induced Inelastic Scattering

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The application of the distorted-wave Born approximation (DWBA) to inelastic scattering is examined. Particular attention is given to the $^{16}O+^{40}Ca$ system. The one-channel optical-potential wave function is found to be an adequate representation of the entrance-channel relative motion. The choice of exit-channel distorting potential DWBA is found to be crucial and a procedure to choose it correctly is discussed.

A recent Letter[1] emphasized the superiority of coupled-channels (CC) calculations to conventional distorted-wave Born-approximation (DWBA) calculations for the inelastic scattering to the lowest 2^+, 3^-, and 5^- states of ^{40}Ca by an ^{16}O projectile. The analysis of Ref. 1 showed that a conventional DWBA fails to describe the 5^- data, whereas a CC analysis succeeds, even though the coupling of the 5^- to either the 3^- or the ground state is weak. The success of the CC calculations was attributed to a more accurate treatment of the elastic-scattering wave function by the explicit coupling of the important inelastic channel (the 3^- of ^{40}Ca) to the ground state.

Such a feature, if general, would have profound consequences for heavy-ion–induced–transfer calculations since, even for a one-step transfer process involving nuclei with moderate inelastic collectivity, a CC calculation of inelastic scattering would be required in order to obtain an accurate description of the relative motion.

Fortunately, it is possible to view the results of Ref. 1 from a significantly different perspective, one in which the failure is *not* of the DWBA but only of what is called (in Ref. 1) the *conventional* DWBA (i.e., one in which the elastic-scattering optical potential is used as the exit-channel distorting potential). This conventional prescription, however, is clearly incorrect for many applications. We shall conclude from a direct comparison with the CC wave function that there is no serious problem associated with the adequacy of a one-channel optical-potential description of the elastic-scattering wave function. Rather, the error in the conventional DWBA is in the choice of the exit-channel distorting potential. In particular, this Letter will focus on the exit-channel distorting potential within the DWBA, and demonstrate that the distorted waves of the entrance and exit channels often must be calcu-

lated with different potentials. When the exit-channel distorting potential is chosen properly (not by *ad hoc* convention), the DWBA describes the data rather well.

The $^{16}O+^{40}Ca(5^-)$ inelastic scattering under discussion here is typical of a large class of reactions where one should expect the entrance- and exit-channel potentials to be different in a DWBA analysis. This difference stems from the presence of a state (the 3^- of ^{40}Ca) which couples strongly to the ground state but only weakly to the excited state of interest (the 5^- of ^{40}Ca). A one-channel optical-potential description of the elastic scattering therefore must reflect that coupling. On the other hand, a distorting potential governing the exit-channel relative motion should not. This asymmetry is automatically included in a CC calculation and will also exist for the DWBA, provided that the proper limit of the CC equations is taken.

To make these arguments more precise, consider a simplified treatment of the DWBA. The exact inelastic-scattering transition amplitude may be written in terms of an exit-channel distorting potential (U) as follows[2]:

$$T_{\alpha_0 J_0 \to \alpha J} = \langle \psi_{\alpha J}^{(-)} \varphi_{\alpha J} | (V - U_\alpha) | \Psi^{(+)} \rangle, \tag{1}$$

where $\Psi^{(+)}$ is the exact wave function for the system, $\varphi_{\alpha J}$ is the internal state $-\alpha J$, and V is the interaction which induces the scattering:

$$V = \sum_{\alpha J} \varphi_{\alpha J}^{\dagger} \varphi_{\alpha J} V_{\alpha J, \alpha J} + \sum_{\alpha J \neq \alpha' J'} \varphi_{\alpha J}^{\dagger} \varphi_{\alpha' J'} V_{\alpha J, \alpha' J'}. \tag{2}$$

U_α is used to calculate the exit-channel distorted wave $\psi_{\alpha J}^{(-)}$. The amplitude T is invariant to the choice of U_α as long as $\Psi^{(+)}$ is exact. The DWBA, however, assumes that elastic scattering is the dominant process and, furthermore, that the transition proceeds through a single inelastic excitation from the ground state (one-step process). Thus, the full wave function is truncated to include only the ground-state component ($\Psi^{(+)} \approx \varphi_{\alpha_0 J_0} \chi_{\alpha_0 J_0}$). In addition, the relative-motion wave function is calculated with a one-channel optical potential chosen to fit the elastic scattering ($\chi_{\alpha_0 J_0} \approx \psi_{\alpha_0 J_0}^{(+)}$) and hence must reflect the influence of all levels strongly coupled to the ground state. To minimize the error introduced by these approximations, it makes sense to choose U_α so that the ground-state component of $\Psi^{(+)}$ makes the major contribution to the calculation of T. For example, $\Psi^{(+)}$ has the component $\varphi_{\alpha J} \chi_{\alpha J}$ and, because of the large nuclear overlap, the contribution to T from the omitted term

$$\langle \psi_{\alpha J}^{(-)} \varphi_{\alpha J} | (V - U_\alpha) | \varphi_{\alpha J} \chi_{\alpha J} \rangle, \tag{3}$$

can be significant. The choice $U_\alpha = V_{\alpha J, \alpha J}$ renders this term zero and thus reduces the error introduced through the truncation. Note that this choice of U_α may not correspond to "elastic scattering" of the projectile with the nucleus in the state $-\alpha J$.

In the case of strong coupling between the ground state and the state $-\alpha J$ (e.g., the 3^-), $U_\alpha = V_{\alpha J, \alpha J}$ will certainly *not* correspond to elastic scattering from state $-\alpha J$. That process requires an operator that must explicitly include the propagation of the wave while the nucleus is de-excited to the ground state, propagated, and then re-excited. In the case of *weak coupling* (e.g., the 5^-), the choice $U_\alpha = V_{\alpha J, \alpha J}$ (provided that the state $-\alpha J$ is not coupled to any other level) is closely related to elastic scattering from state $-\alpha J$.

To be more explicit about V, and to obtain the DWBA limit discussed above, consider the CC system of equations. In this approach, one exactly solves a system of equations corresponding to a small number of open channels (αJ) and includes the influence of the omitted channels through a complex, energy-dependent, effective interaction. Operators $P = \sum_{\alpha J} \varphi_{\alpha J}^{\dagger} \varphi_{\alpha J}$ and $Q = 1 - P$ project, from $\Psi^{(+)}$, the channel space of interest ($P\Psi^{(+)} = \sum_{\alpha J} \varphi_{\alpha J} \chi_{\alpha J}$) and the excluded component ($Q\Psi^{(+)}$), respectively. Within the truncated space, the effective interaction take the familiar form[3]

$$V_{\text{eff}} = V + V \frac{Q}{E - H + i\epsilon} V, \tag{4}$$

where H is the full Hamiltonian for the projectile-target system. The propagator expresses the open-channel boundary condition. Diagonal matrix elements of (4) for any channel (αJ) within the P space are

$$V_{\alpha J, \alpha J} = \langle \varphi_{\alpha J} | V | \varphi_{\alpha J} \rangle + \sum_{\alpha_q J_q} \frac{\langle \varphi_{\alpha J} | V | \Psi_{\alpha_q J_q} \rangle \langle \Psi_{\alpha_q J_q} | V | \varphi_{\alpha J} \rangle}{E - E_q + i\epsilon}. \tag{5}$$

These we refer to as *bare optical potentials*. If there are no remaining channels in the Q space that couple strongly to any in the P space, the imaginary part of the bare optical potential will be similar for all channels since it is determined mainly by the high-excitation part of the energy spectrum where the level density is large. Any channel dependence, therefore, is reflected

through the real-component $\langle\varphi_{\alpha J}|V|\varphi_{\alpha J}\rangle$.

If a transition to a given state $-\alpha J$ in the P space proceeds mainly by a single excitation from the *ground state* and, in addition, is weak, the CC equations reduce to the separated form

$$(T_{\alpha_0 J_0}+V_{\alpha_0 J_0,\alpha_0 J_0}-E_{\alpha_0 J_0})\chi_{\alpha_0 J_0} = -\sum_{\alpha' J'\neq\alpha_0 J_0,\alpha J} V_{\alpha_0 J_0,\alpha' J'}\chi_{\alpha' J'}, \quad (6)$$

$$(T_{\alpha J}+V_{\alpha J,\alpha J}-E_{\alpha J})\chi_{\alpha J} = -V_{\alpha J,\alpha_0 J_0}\chi_{\alpha_0 J_0}. \quad (7)$$

The scattering to state $-\alpha J$ may be described by the DWBA, provided that the influence of the states $-\alpha' J'$ can be incorporated into a one-channel optical-potential description of the elastic scattering ($\chi_{\alpha_0 J_0}\approx\psi_{\alpha_0 J_0}^{(+)}$). The DWBA limit thus becomes

$$T_{\alpha_0 J_0\to\alpha J}=\langle\chi_{\alpha J}^{(-)}|V_{\alpha J,\alpha_0 J_0}|\psi_{\alpha_0 J_0}^{(+)}\rangle, \quad (8)$$

where $\chi_{\alpha J}^{(-)}$ is the exit-channel distorted wave, calculated from the *bare optical potential* $V_{\alpha J,\alpha J}$; whereas $\psi_{\alpha_0 J_0}^{(+)}$ corresponds to an optical potential that reflects the influence of all states $-\alpha' J'$ strongly coupled to the ground state. The bare optical potential can be approximated by the underlying potential used in a CC calculation of the elastic scattering.

Figure 1 shows the success of the DWBA for the reaction $^{16}O+^{40}Ca$ of Ref. 1 with the CC potential used as the bare optical potential for the

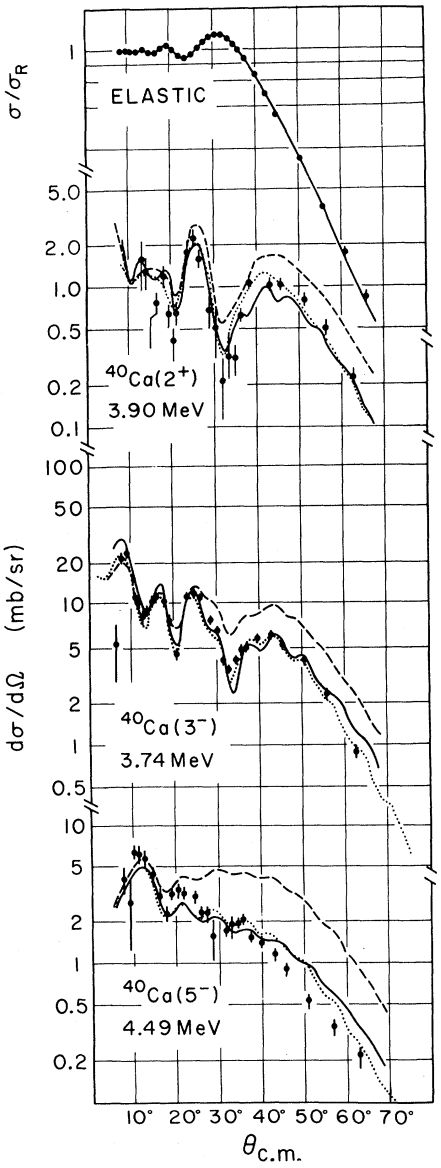

FIG. 1. Inelastic-scattering angular distributions for $^{16}O+^{40}Ca$ at a bombarding energy of 60 MeV. Solid lines are CC calculations based on the parameters of Ref. 1. Dashed lines are the conventional DWBA calculations of Ref. 1 with their suggested normalization to the data at forward angles by taking δ_N as a free parameter. [This normalization when consistently applied gives the 2^+ fit shown ($\delta_N=0.56$).] Dotted lines are DWBA calculations using the CC potential as the exit-channel distorting potential.

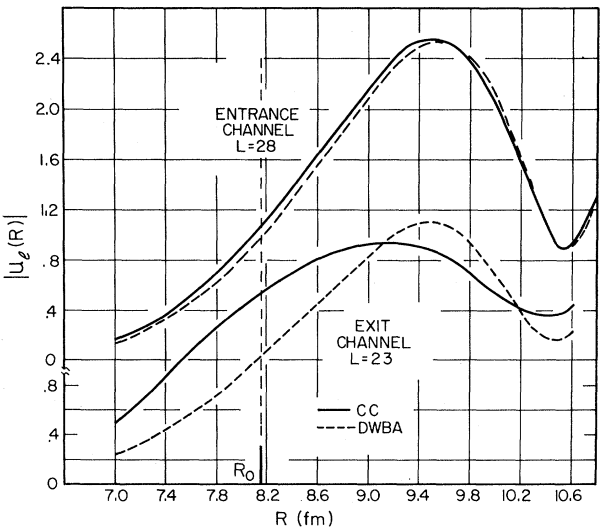

FIG. 2. Comparison of relative-motion radial wave functions for the $^{16}O+^{40}Ca$ reaction. For the entrance channel, the solid line corresponds to the CC calculation and the dashed line to a one-channel optical-potential calculation based on the DWBA parameters discussed in text. For the exit channel, the solid line corresponds to using the bare optical potential and the dashed line to using the conventional DWBA choice of Ref. 1.

TABLE I. Optical-potential parameter used in the DWBA and CC calculations as discussed in the text and Ref. 1.

Potential	V (MeV)	r_R (fm)	a_R (fm)	W (MeV)	r_I (fm)	a_I (fm)
DWBA[a]	14.6	1.373	0.515	7.3	1.381	0.441
CC (bare optical potential)	14.6	1.376	0.515	6.35	1.267	0.674
DWBA[b]	14.6	1.373	0.515	5.563	1.373	0.552

[a]Reference 1.
[b]DWBA parameters of the present work, in which the imaginary part of the optical potential was modified so as to reproduce the elastic scattering of the CC calculation.

exit channel, and the one-channel optical potential of Ref. 1 used to describe the entrance channel. When the incorrect exit-channel distorting potential is used (i.e., the conventional DWBA choice) the fits for all transitions are poor, even though the 2^+ state has a significant contribution from Coulomb excitation and the 3^- and 5^- states do not.

We consider next the DWBA as a limiting case of the CC equations. This allows us to address the specific issue raised in Ref. 1, namely, the validity of the DWBA and the accuracy of a one-channel optical-potential relative-motion wave function. We show, therefore, a detailed test for the 5^- transition. Figure 2 compares, for the grazing partial wave $l_{in} = 28$, the entrance-channel radial wave function obtained from a CC calculation with the corresponding wave function from a one-channel optical-potential calculation that produces the same elastic scattering as the CC in the angular range 0°–70°. [The potential of Ref. 1 does not produce the *same* elastic scattering as their CC calculation and, for the theoretical discussion presented here, a more meaningful comparison between the DWBA and the CC calculations is obtained in the limit that the two approaches yield the same elastic scattering. The changed parameters are $W = 5.563$, $r_I = 1.373$, and $a_I = 0.552$ (see Table I).] The entrance-channel wave functions as shown in Fig. 2 are nearly identical even at the half-radius (8.156 fm). This agreement is in contrast to the dramatic difference in the exit-channel wave functions (e.g., $l_{out} = 23$) resulting from the conventional DWBA choice for the exit-channel distorting potential. Figure 2 suggests that the relative-motion wave function in the DWBA is an adequate representation of the entrance channel and, furthermore, that the failure to use the correct exit-channel distorting potential leads to the inferior results obtained with the conventional DWBA (see Table II). A CC calculation for the 5^- transition, when compared to the DWBA limit of the CC equations [Eq. (8)], shows the two results are identical beyond 40° and agree to within 25% elsewhere.

Other areas exist where these considerations will be important. For example, knowledge of the role of the exit-channel distorting potential is particularly important in view of the current attempts to establish the microscopic structure

TABLE II. Coulomb and nuclear deformation lengths for states of ^{40}Ca.

J_i^π	J_f^π	Transition energy (MeV)	δ_C (fm)	δ_N(DWBA)[a] (fm)	δ_N(CC)[b] (fm)	δ_N(DWBA)[c] (fm)
0^+	3^-	3.737	1.79	1.27[d]	1.08	1.14
0^+	2^+	3.904	0.47	0.56[e]	0.41	0.42
0^+	5^-	4.492	1.22	0.90[d]	0.54	0.68

[a]Conventional DWBA calculation (see text) in the manner of Ref. 1.
[b]CC calculation of Ref. 1.
[c]Present work.
[d]Parameters taken directly from Table II of Ref. 1.
[e]Cf. caption of Fig. 1.

of the nuclear inelastic transition density by fitting angular distributions.[4] Secondly, a similar failure of the conventional DWBA has been observed for transfer reactions leading to excited states of the residual nucleus.[5] Using the prescription suggested in this Letter, we have performed transfer calculations and found large increases in the cross sections at forward angles, as for the inelastic scattering.

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