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^{12}C-Pion Monopole Scattering

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Inelastic pion excitation of the 0^+ (7.65-MeV) level in ^{12}C is studied using a Kisslinger optical potential in both a distorted-wave impulse approximation and a coupled-channels formalism. In clusion of coupling through the first excited 2^+ state in ^{12}C causes a dramatic suppression of this cross section for pion energies below 75 MeV. This explains the unexpectedly small 0_1^+ cross sections recently observed, and may permit detailed testing of pion-nucleus reaction theories.

Comparison of distorted-wave impulse-approximation (DWIA) calculations of the monopole cross section for the reaction ^{12}C(g.s.)$(\pi^+, \pi^{+\prime})^{12}C^*(0^+$, 7.65 MeV) and the newly obtained inelastic-pion-scattering data[1,2] at 50 MeV demonstrates major inadequacies in the reaction-theory formalism for low pion scattering energies. This glaring disagreement derives from the failure to include contributions to the cross section arising from two-step processes. Interference between the direct monopole and two-step contributions results in a reduction of the cross section by an order of magnitude at 50 MeV and brings the theoretical results into accord with the experimental data.

This failure to explain the 50-MeV monopole scattering data is a somewhat unique exception to the general success of the DWIA approach for inelastic pion scattering. For example, in ^{12}C, which is the most extensively studied nucleus experimentally and theoretically, the 50-MeV inelastic pion scattering to the collective states 2^+(4.44 MeV) and 3^-(9.63 MeV) has been well described by several workers.[2-4]

All of the calculations presented here are obtained using a Kisslinger optical-potential form with an angle transformation of the type used by diGiacomo *et al.*,[5] but without any other corrections. This approach is known to give a good qualitative description of the scattering to collective states of ^{12}C from about 50 to 200 MeV. For the monopole excitation, which is not amenable to a collective-model description, it was necessary to construct a model for ^{12}C based on the measured properties. Transition-charge densities for transitions involving the ground state were obtained from fits to the inelastic electron scattering form factors.[6] The form factor for the transition between excited states was taken to have the same shape as the g.s.-to-2^+ transition but was normalized to reproduce the γ-decay width. The transition densities have the form

$$\rho_{\rm tr} = \pi^{-3/2} b^{-3}(A + Br^2/b^2 + Cr^4/b^4)e^{-r^2/b^2} \qquad (1)$$

with parameters given in Table I.

The breakdown of the DWIA formalism for monopole excitations is obvious only at lower pion energies. From Fig. 1 it can be seen that a DWIA calculation of the monopole excitation in ^{12}C does give a rough description of the data for 150-MeV pions.[7] However, a similar calculation of the direct contribtuion at 50 MeV, shown as the solid line in Fig. 2, predicts a cross section on the order of 1 mb/sr while the observed cross

TABLE I. Parameters of the transition densities.

Transition	A	B	c	b
Diagonal	0.333	0.444	0	1.637
$0_1^+ \leftrightarrow 2_1^+$	0	0.0096	0.1344	1.414
$0_2^+ \leftrightarrow 2_1^+$	0	0.0056	0.0788	1.414
$0_1^+ \leftrightarrow 0_2^+$	−0.0579	−0.1652	0.0815	1.625

section is only 50 μb/sr or smaller. A coupled-channels impulse-approximation (CCIA) calculation which includes the complete coupling between these three states is shown as the dashed curve in Fig. 2. The CCIA result is an order of magnitude smaller than the DWIA result primarily because of the interference between the direct and two-step contributions. The reason for this large change is twofold. First, the ^{12}C nucleus is relatively transparent, not opaque, to 50-MeV pions. Secondly, and more important, although the monopole matrix element connecting the 0^+ states is large causing a strong direct transition, both the ground and excited 0^+ states have large $E2$ matrix elements to the 2^+ state at 4.44 MeV.

The two-step calculation is not straightforward in that there is one relative phase which must be determined. By making a model of ^{12}C as two mixed deformed bands one obtains two mixed 0^+ and two mixed 2^+ states in the physical spectrum. By neglecting excitation energies and using closure to estimate the total two-step contribution from the ground state to the excited 0^+ state through *both* 2^+ states, one can relate this relative phase to the rms radii and the magnitudes of the quadrupole moments of the two assumed intrinsic states. If the state with a larger quadrupole moment squared has a larger rms radius, the phase leads to cancellation in the pion scattering, as assumed here. If the larger-radius state had a smaller quadrupole moment, the coupled-channels result would be increased over the

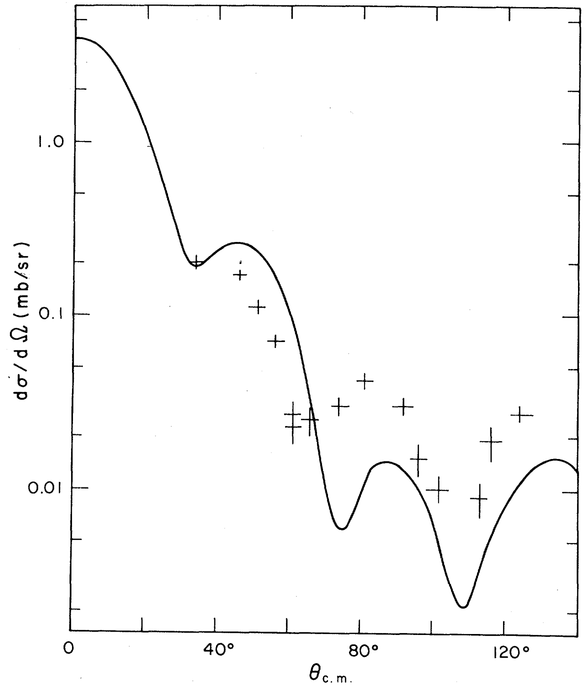

FIG. 1. Comparison of DWIA calculations with the data of Ref. 6 for the monopole excitation in ^{12}C at T_π =150 MeV.

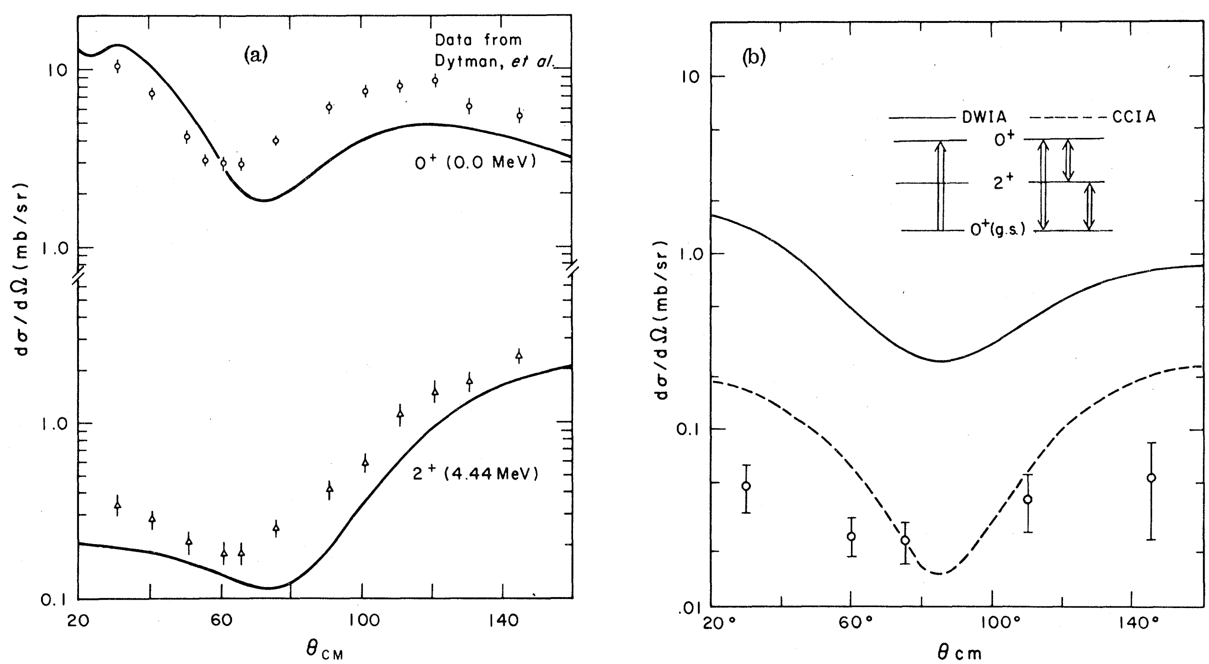

FIG. 2. (a) Elastic scattering and quadrupole excitation at T_π=50 MeV. (b) The effect of channel coupling on the monopole excitation.

DWIA result by about a factor of 2 rather than decreased by a factor of 10.

Given this specification of the phase, the strong cancellation which is obtained here is relatively insensitive to other variations in the calculations. One check was to use the optical potentials of Refs. 4 and 5 with the higher-order corrections, particularly s-wave absorption which may tend to suppress the two-step mechanism. The result is a somewhat smaller DWIA amplitude, and also a less dramatic reduction (a factor of 5 rather than 10 at 20°) as a result of the considerably smaller two-step contribution. This leads to almost exactly the same CCIA prediction for the angular distribution.

The contribution of double scattering through the 3^- state was also estimated. This would certainly be small compared with the DWIA result, but perhaps not with the suppressed cross section. There is no measured transition from the ^{12}C 3^- state to the excited 0_1^+, and so for the purposes of an estimate we assumed that the intrinsic $E3$ strength is the same as that to the ground state. Even this very generous estimate of this transition strength led to a very modest calculated contribution.

One very natural way in which to test these calculations of double quadrupole contributions to the scattering would be to compare with data on the excitation of the 4^+ state at 14.08 MeV. In terms of a shell model an $L=4$ excitation of p-shell nucleons is impossible in a single step. This view is confirmed by the exceedingly small measured electron-scattering form factor.[8] Two-step calculations of an excitation which proceeds through the 2^+(4.44 MeV) predict a slowly varying, *forward-peaked* cross section for the 4^+, rather than a characteristic $L=4$ angular distribution. Unfortunately, the predicted cross sections are all below 10 μb/sr, and so the prospects of obtaining a measured angular distribution with which to compare are remote.

There have been successful descriptions of the elastic scattering and 2^+ excitation which neglect the effects of multiple excitation and deexcitation included here, and it is important to note that these calculations are valid. There are slight changes in these differential cross sections, generally less than 20%, due to the multistep processes. These are modest effects when compared to current differences between different sets of data. The explicit inclusion of the coupling to and from the 2^+ state does, however, have substantial effect on the reaction cross section at the low energies, increasing it from 160 to 220 mb. This derives from constructive interference at forward angles between the first-order distorted

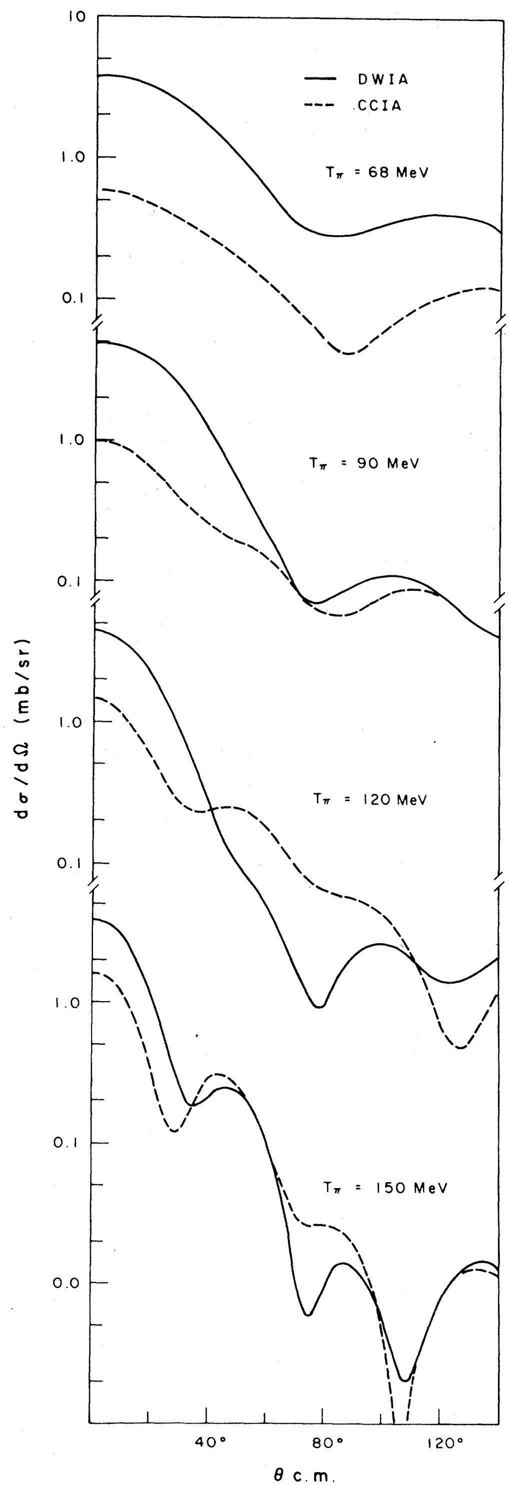

FIG. 3. Energy dependence of the DWIA and CCIA predictions for the monopole excitation.

wave and the second-order contribution proceeding through the 2^+. This is masked in the differential cross section by the Coulomb scattering. At energies above 70 MeV, the effect on the reaction cross section rapidly decreases, absolutely as well as relatively. Physically this may be understood by recalling that the first-order optical potential includes only nucleon knockout contributions to the reaction cross section.[9] These calculations show that at 50 MeV where the total π-N cross section is small this is not a good approximation and the large quadrupole deformation has an appreciable effect. As the total π-N cross section increases, nucleon knockout becomes dominant, making the two-step contribution to the reaction cross section smaller, in magnitude as well as relative to the one-step part.

The relative importance of channel coupling for monopole cross sections is also energy dependent. In Fig. 3 we present a comparison of the DWIA and the CCIA results for pion energies of 68, 90, 120, and 150 MeV. The predicted suppression becomes less dramatic as the energy increases and is generally restricted to forward cones but is still a factor of 5 out to 50° for 90-MeV pions. The differences between the DWIA and CCIA cross sections are only slightly influenced by the inclusion of higher-order corrections such as true pion absorption, Pauli blocking, and the Lorentz-Lorenz effect.

There are several exciting consequences of these results. This is the first instance of an allowed reaction with a fundamental particle which is dramatically influenced by a competing two-step process. Previous calculations[10,11] of pion scattering have shown these effects to be small, although in ^{3}He- and ^{4}He-induced reactions there is some evidence[12] for competing two-step processes in monopole excitations. Further, the nature of the interference between the direct and two-step mechanism is readily understood, and readily related to measured quantities. Also, since there is destructive rather than constructive interference in the two contributing processes the possibility exists for very detailed testing of reaction theories through the study of pion-induced monopole excitations.

These results also have strong implications for studies of the second-order optical potential. An excitation has been identified which, through correlations arising from nuclear collective motion, substantially increases the reaction cross section and drastically reduces the inelastic excitation of the 0^+ state. Calculations are in progress for heavier nuclei, particularly ^{28}Si and ^{40}Ca, both of which have resolvable 0^+ excited states and more reliable theoretical wave functions than are available for ^{12}C.

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