amplitude. The collector current signal is sampled with a resolution of 5 μsec. In Fig. 4(a), one sees a pronounced increase of $T_{i\perp}$ as the decay waves grow, without much change in $T_{i\parallel}$. Although this ion heating can be partly due to ion-cyclotron damping of the IQM, the perpendicular interaction of the LHW[5] with the ions is likely to cause most of the $T_{i\perp}$ increase and the observed tail formation.

With the onset of the instability, one observes an enhanced absorption of the pump wave as shown in Fig. 4(b) where we have plotted the amplitudes of the pump, the LHW, and the IQM as functions of $V_{\rm mod}$. The associated increase in T_e, plotted in Fig. 4(c), appears to be due to electron Landau damping of the IQM for which $\zeta \simeq 1$. The increase in T_e is limited by the heat conduction loss to the end plate. Furthermore, for pump frequencies without parametric decay, the increases of T_i and T_e were small when compared with the heating when parametric instabilities were excited.

In summary, we have demonstrated that by modulating the density of an electron beam at or above $\omega_{\rm LH}$ the heating efficiency of both electrons and ions in a target plasma can be greatly increased. This increase is associated with the parametric excitation of lower-hybrid waves and ion-cyclotron quasimodes. Similar effects may be expected during heating of tokamaks with rf near $\omega_{\rm LH}$. Furthermore, similar heating techniques can be applied by modulating a beam at other plasma eigenmode frequencies.

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Diffuse-Boundary Rayleigh-Taylor Instability

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For density profiles, $N(y)$, making a smooth transition from $N(-\infty)=0$ to $N(+\infty)=$const with $d\ln N/dy$ decreasing monotonically with y, it is shown that the Rayleigh-Taylor instability exhibits essentially different behavior above and below a certain critical wave number, k_c. For $k > k_c$ the growth of the response to an initial perturbation is slower than exponential, $\sim t^{-1/2}\exp(\gamma_b t)$. For $k < k_c$ an unstable eigenmode (analogous to that in the sharp boundary case) exists, and purely exponential growth occurs.

Recently, as a result of developments in pellet fusion,[1] imploding liner fusion,[2] and ionospheric physics,[3,4] the Rayleigh-Taylor instability has been the subject of renewed interest. In this note we present results for the Rayleigh-Taylor instability with a diffuse density profile. For a fairly broad class of profiles we find that there exists a critical wave number (transverse to the density gradient) above which there are no eigenmode solutions, and the time-asymptotic response to an initial excitation exhibits slower than exponential growth. For smaller wave numbers eigenmode solutions occur, and the time-asymptotic response to an initial perturbation grows exponentially with time.

Taking gravity $\vec{g} = -g\vec{y}_0$ with equilibrium density gradients in y only ($\nabla N = \vec{y}_0 dN/dy$), and linearizing the fluid equations, we obtain for the y component of the fluid velocity

$$\frac{\partial}{\partial y} N \frac{\partial^3 \hat{v}}{\partial y \partial t^2} - k^2 N \left(\frac{\partial^2}{\partial t^2} - \frac{g}{N}\frac{dN}{dy}\right)\hat{v} = 0, \quad (1)$$

where we have neglected compressibility, equilibrium flow, and viscosity, and have assumed an x dependence $\exp(ikx)$. We consider the initial-value problem for (1) and introduce the Laplace transform:

$$N^{-1}\frac{d}{dy} N \frac{dv}{dy} - k^2\left[1 - \frac{g}{s^2}\frac{1}{N}\frac{dN}{dy}\right]v = -\Gamma(y), \quad (2)$$

where $\Gamma(y) = (k/s)^2[d\hat{v}/dt + s\hat{v}]_{t=0}$ and

$$\hat{v}(t,y) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} ds\, v(s,y)\exp(st),$$

with σ chosen so that v has no singularities in $\mathrm{Re}(s) \geq \sigma$. Equation (2) is to be solved subject to the condition that the energy density of the perturbation approaches zero as $|y| \to \infty$. Since the perturbation energy density is proportional to $N\hat{v}^2$, we have $N^{1/2}v \to 0$ as $|y| \to \infty$. (Note that v itself can actually become infinite if N approaches zero at infinity.) We set $q = N^{1/2}v$ and consider the Green's function associated with (2):

$$\frac{d^2 q}{dy^2} - k^2 Q(s,y) q = -K\delta(y + y_0), \quad (3)$$

where K is a constant, $q \to 0$ as $|y| \to \infty$, and

$$Q = 1 - \frac{g}{s^2}\frac{1}{N}\frac{dN}{dy} + \frac{1}{k^2 N^{1/2}}\frac{d^2 N^{1/2}}{dy^2}.$$

We first examine the following illustrative density profile for which $N(y)$ increases monotonically from zero to a constant value:

$$N(y) = N_0, \ y > 0; \quad N(y) = N_0 \exp(\beta y), \ y < 0. \quad (4)$$

Subsequently we shall show that certain results obtained for (4) apply more generally. Solving (3) and (4) for $y_0 > 0$ subject to the conditions at $|y| \to \infty$, the continuity of q at $y = 0$ and $y = -y_0$, and the jump conditions on dq/dy at $y = 0$ and $y = -y_0$, we have

$$q = \begin{cases} A(s)e^{\alpha y} & \text{for } y < -y_0, \\ B(s)e^{-\alpha y} + C(s)e^{\alpha y} & \text{for } -y_0 < y < 0, \\ D(s)e^{-ky} & \text{for } y > 0, \end{cases} \quad (5)$$

where

$$\alpha = [(\tfrac{1}{2}\beta)^2 + k^2(1 - \beta g/s^2)]^{1/2}, \quad A(s) = K\frac{\exp(\beta y_0)}{2\alpha}\left[\frac{\alpha - k + \beta/2}{\alpha + k - \beta/2}\exp(-\alpha y_0) + \exp(\alpha y_0)\right], \quad (6)$$

with similar expressions for the other coefficients. In accord with the condition at $y = -\infty$ the square root is defined so that

$$\mathrm{Re}(\alpha) > 0 \text{ for } \mathrm{Re}(s) > \sigma. \quad (7)$$

From (6) it is evident that the Green's function has branch points (associated with continuous spectra) at $s = \pm\gamma_b$ (i.e., $\alpha = 0$)

$$\gamma_b \equiv \left(\frac{\beta g}{1 + (\beta/2k)^2}\right)^{1/2}, \quad (8)$$

an essential singularity at $s = 0$, and possible poles (discrete spectra) at

$$\alpha = \tfrac{1}{2}\beta - k. \quad (9)$$

Equations (7) and (9) can only be satisfied for $2k < \beta$, and, under this condition, the poles are located at $s = \pm\gamma_p$, where

$$\gamma_p \equiv (kg)^{1/2}. \quad (10)$$

Remarkably, (10) coincides precisely with the sharp-boundary result. It is of interest to follow the migration of the pole as k is varied. For $\beta > 2k$, $\gamma_p > \gamma_b$, and as k is increased γ_p moves toward γ_b. At $\beta = 2k$, the pole and branch point coincide, $\gamma_p = \gamma_b$. For larger k, the pole drops on to the lower sheet of α [where the inequality (7) is reversed] and $\gamma_p > \gamma_b$ again. (Since for $\beta < 2k$, the pole is on the lower sheet, it will not be intercepted when the Laplace contour is deformed to the left.) For $t > 0$ we can deform the Laplace contour to the left. The long-time asymptotic behavior of $\hat{q}(t,y)$ will then be dominated by the intercepted singularity of $q(s,y)$ with the largest real part. For $\beta > 2k$ this singularity is the pole $s = \gamma_p$ and the resulting residue from the inverse Laplace transform yields

$$\hat{q} \sim \exp[(kg)^{1/2}t]. \quad (11)$$

On the other hand, for $\beta < 2k$, the dominant contribution is from the branch point at $s = \gamma_b$. The inverse Laplace transform can then be evaluated by expanding $q(s,y)$ about $s = \gamma_b$, and performing the integration asymptotically for large t. One then obtains slower than exponential growth[4,5] at any point:

$$\hat{q} \sim t^{-1/2}\exp\gamma_b t \exp[-b(y + y_0)^2/t] \quad (12)$$

for $b/t \lesssim y_0^2$, where $b = k^2 g\beta/2\gamma_b^3$. Thus, $\hat{q}$ is in the form of a spreading Gaussian whose area increases exponentially in time. For $t \gtrsim b(y+y_0)^2$, growth is slower than exponential, $\ddot{q} \sim t^{-1/2} \exp \gamma_b t$. Note that $\gamma_b \cong (\beta g)^{1/2}$ for $4k^2 \gg \beta^2$. These results show the transition from the growth rate $(\beta g)^{1/2}$, obtained from the local approximation [$k^2 \to \infty$ and d^2q/dy^2 neglected in (3)], to the sharp-boundary result (11) for long wavelength ($\gamma_p = \gamma_b$ at $\beta = 2k$). For the profile (4) we see that the local approximation corresponds to the continuous spectrum with slightly slower than exponential growth. {Note that the pole solution corresponds to an eigenfunction and can be simply obtained by considering the homogeneous version of (3). Applying the conditions at $y=0$ we obtain $\alpha = \beta/2 - k$, $q \sim \exp(-ky)$ in $y>0$, and $q \sim \exp[(\beta/2 - k)y]$ in $y<0$. From this we see that $q \to 0$ in $y \to -\infty$ only if $\beta > 2k$.}

We now consider how these results generalize to profiles other than (4). In order for eigenmode solutions to exist [i.e., homogeneous solutions of (3) satisfying $q \to 0$ at $|y| \to \infty$], Q must have the character of a potential well. In particular, Q must be positive for $|y| \to \infty$ (so that q decays exponentially at infinity), and must be sufficiently negative over some interval of y. [Note that on multiplying (3) by q and integrating over all y a quadratic form results which shows that any eigensolutions have s^2 real. Consequently Q is taken real in this discussion.] For short wavelength, $k^2 \to \infty$, the term $N^{-1/2}d^2N^{1/2}/dy^2$ in Q is negligible, and the y dependence of Q is determined by $N^{-1}dN/dy$. In many circumstances in which there is a transition from $N=0$ at $y=-\infty$ to $N=$ const at $y=\infty$, the term $N^{-1}dN/dy$ decreases monotonically with y. Thus under this fairly general condition Q does not have the character of a potential well and no eigenmode solutions (poles of the Green's function) exist for short wavelength. This agrees with our result for the profile (4). [For profile (4), $N^{-1}dN/dy$ decreases monotonically since we have $\beta > 0$ for $y<0$ and zero for $y>0$.] For k^2 small the term $N^{-1/2}d^2N^{1/2}/dy^2$ becomes important. For profiles with $N(-\infty) = 0$ and $N(+\infty) =$ const, this term must have the character of a potential well, and unstable eigenmodes can occur. In fact, for $k^2 \ll \beta^2(y) \equiv (N^{-1}dN/dy)^2$, the extent of the mode in y is much larger than β^{-1} and the sharp-boundary eigenmode is recovered. Thus, there will be some critical wave number k_c such that for $k<k_c$ ($k>k_c$) eigenmodes exist (do not exist). For the profile (4) we have determined k_c to be $\beta/2$. [Of ocurse for profiles where $\beta(y)$ assumes a local minimum somewhere ($y=y_m$, say), there *will* be eigensolutions for $k^2 \gg \beta^2$. In this case one can write $\beta(y) \cong \beta(y_m) + \beta''(y=y_m)(y-y_m)^2/2$ and obtain the harmonic-oscillator equation localized about $y=y_m$ with eigenvalue $s \cong \gamma_b$.] The result (12) applies to profile (4) which has $d\beta/dy = 0$ in $y>0$. However, (12) will still apply for times $t \lesssim t_0$, which are short enough that the width of the Gaussian in (12) is sufficiently small so that the perturbation does not see the variation of $\beta(y)$:

$$2\gamma_b t_0 \cong [2k/\beta(-y_0)]^{3/2} a^{3/2}(-y_0), \tag{13}$$

where $a(y) = \beta^2(d\beta/dy)^{-1} \sim 1$. Thus from (13), for short wavelengths, $k \gg \beta(-y_0)$, the instability can reach the nonlinear stage before the y dependence of β is felt.

Finally, in order to see how these results are modified if $N(-\infty) \neq 0$ we consider the profile $N(y) = N_0 \exp(-\beta y_1)$ for $y \leq -y_1$, $N(y) = N_0 \exp(\beta y)$ for $-y_1 \leq y \leq 0$, and $N(y) = N_0$ for $y>0$, which corresponds to (4) for $y_1 \to \infty$. Solution of the Green's-function problem, Eq. (3), for this case shows that no branch points are present since q turns out to be an even function of α. The poles of q are located at the solutions of

$$2k[(\beta/2)^2 - (k^2+\alpha^2)]^{-1} = \alpha^{-1} \tanh \alpha y_1. \tag{14a}$$

Since s^2 is real, α is either purely real or purely imaginary. First consider the case α real. Since the right-hand side of (14a) decreases with α while the left-hand side increases, there can at most be one root in α, and the condition for this root to exist is that the right-hand side of (14a) exceed the left-hand side at $\alpha = 0$, or

$$\beta > 2k[1 + 2(ky_1)^{-1}]^{1/2}. \tag{15}$$

This condition is analogous to the condition $\beta > 2k$ for profile (4). Also for $y_1 \to \infty$ the root of (14a) becomes $\alpha = \beta/2 - k$ or $s = \pm\gamma_p$. For $k \to \infty$ (14a) yields $s^2 = kg[N(\infty) - N(-\infty)]/[N(\infty)+N(-\infty)]$, which is the sharp-boundary result. Now consider the case α purely imaginary and define $\alpha = i\kappa$ so that (14a) becomes

$$2k[(\beta/2)^2 + \kappa^2 - k^2]^{-1} = \kappa^{-1} \tan \kappa y_1. \tag{14b}$$

It follows from the definition of α that any root for α imaginary satisfies $s^2 > \gamma_b^{\,2}$, while under condition (15) the pole for α real satisfies $s^2 > \gamma_b^{\,2}$. Thus when (15) applies the root with α real dominates. By sketching both sides of (14b) it is seen that one root of κ occurs in each of the intervals $3\pi/2 > \kappa y_1 > \pi/2$, $5\pi/2 > \kappa y_1 > 3\pi/2, \ldots$, and, if (15) is not satisfied, also in $\pi/2 > \kappa y_1 > 0$. Since $\kappa \to \infty$

corresponds to $s^2 \rightarrow 0$, there is a clustering of poles around $s=0$ (i.e., there is an infinite number of poles in any finite region around $s=0$). Also, for $y_1 \rightarrow \infty$ the density of poles in any region in $\gamma_b > s > -\gamma_b$ approaches infinity, and the continuum of profile (4) is recovered.

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Observation of Low-Frequency Excitations of Deuterium and Hydrogen in Niobium

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Measurements of the dispersion curves on single crystals of $NbD_{0.85}$ and $NbH_{0.82}$ reveal two new excitations at $\hbar\omega = 18.4$ and 10.8 meV. The positions of these excitations are independent of temperature and mass of the light atom, but the linewidth is broader in the α' solid-solution phase than in the β phase, and broader in the hydrogen system than in the deuterium system.

Hydrogen in metals has been extensively studied over the last two decades for reasons both basic and applied.[1] Since the diffusivity of hydrogen is very large in metals and since samples are readily made, the hydrogen-metal systems are ideal for studying the microscopic mechanisms of interstitial diffusion in solids by neutron scattering. The dynamics of hydrogen and deuterium in Pd, V, Nb, and Ta have been studied by inelastic neutron-scattering techniques.[2] These studies have concentrated on (i) the measurements of the diffusion constant by examining the width of the quasielastic incoherent scattering with energies generally less than 1.0 meV,[1] (ii) the lattice dynamics of the host lattice in the energy range of less than 30.0 meV,[2] and (iii) the study of the local vibration of H (or D) at energies frequently as high as 170 meV.[3] It has generally been accepted that the light atoms change the electronic properties of the metallic host, thereby affecting the electron-phonon interaction.[4] However, because of the large mass differences, the coupling of the light interstitial atoms to the heavy metal atoms is considered quite weak,[4] and no large change in the lattice dynamics of the host lattice is expected.

We report below measurements of the phonon dispersion curves of $NbD_{0.85}$ and $NbH_{0.82}$ in the α' solid solution and in the ordered β hydride phase. Observations of additional low-frequency excitations at $\hbar\omega = 18.4$ and $\hbar\omega = 10.8$ meV are reported.[5] These excitations are q independent, having energies well below the local-mode vibrational levels and within the range of the host lattice modes. The meaning of these new features is not understood but they undoubtedly will play an im-