Study of the Reactions $\nu p \to \mu^- \Delta^{++}$ at High Energies and Comparisons with Theory

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We present production and decay distributions for the reaction $\nu p \to \mu^- p \pi^+$ with $M_{\pi^+ p} < 1.4$ GeV. This reaction is primarily $\nu p \to \mu^- \Delta^{++}$, but we find evidence for the presence of non-Δ amplitudes for $Q^2 > 1$ GeV^2. The data are fitted by various theoretical models. For $Q^2 < 1$ GeV^2 the best fit yields the value $M_A = 1.25^{+0.15}_{-0.13}$ GeV, using a dipole-nucleon axial-vector form factor in Adler's model, although none of these models successfully reproduces the observed Δ-decay distributions.

Several theoretical models for weak single-pion production and, in particular, for the reaction $\nu p \to \mu^- \Delta^{++}$ have been published and fitted to data below 1.5 GeV.[1-3] In this paper we present new data on this reaction at energies from 5 to 100 GeV and fit our data by the same parametrized models used in Ref. 1, and also by a more recent version of the full Adler model.[2]

From a sample of 201 events with kinematic fits to the reaction $\nu p \to \mu^- p \pi^+$, we obtain 138 events with the $\pi^+ p$ invariant mass, $W < 1.4$ GeV, corresponding to an average cross section of $(0.55 \pm 0.08) \times 10^{-38}$ cm^2 between 15 and 40 GeV, as described in the preceding paper.[4] The Q^2 distribution $[Q^2 = -q^2 = -(p_\nu - p_\mu)^2]$ from these events, predominantly from the reaction $\nu p \to \mu^- \Delta^{++}$, is shown in Fig. 1.

The decay-angle distributions from the events with $W < 1.4$ GeV are shown in Fig. 2 and are summarized in Table I for various Q^2 regions. The decay angles θ and Φ are calculated for the π^+ in the $\pi^+ p$ rest frame with respect to a Gottfried-Jackson system of right-handed axes with the polar axis, z along the momentum-transfer

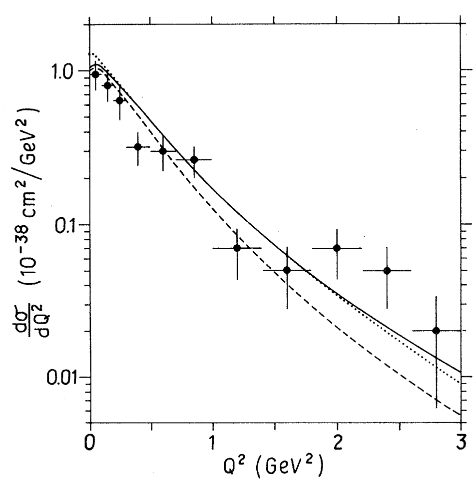

FIG. 1. The $d\sigma/dQ^2$ distribution. The solid and dashed curves are the projections of the fits by the full Adler model with $M_A = 1.43$ GeV and 1.25 GeV, respectively. The dotted curve shows where the fit by the parametrized Adler model with $M_A = 1.15$ GeV differs from the solid curve. In all cases the curves are absolute predictions.

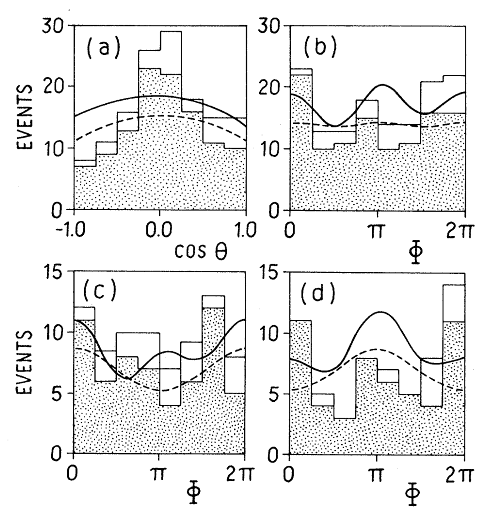

FIG. 2. Decay-angle distributions from events with $W < 1.4$ GeV: (a) $\cos\theta$, (b) Φ, (c) Φ for $\cos\theta > 0$, (d) Φ for $\cos\theta < 0$. The shaded areas correspond to events with $Q^2 < 1$ GeV^2. The solid curve is from the full Adler model with $M_A = 1.43$ GeV averaged over all Q^2, and the dashed curve is from the parametrized Adler model with $M_A = 1.0$ GeV for $Q^2 < 1$ GeV^2.

TABLE I. π^+p decay-angle distributions for $W < 1.4$ GeV.

Q^2 (GeV)^2	Number of events	cosθ (c) <0	>0	$\lvert c\rvert<0.5$	$\lvert c\rvert>0.5$	Φ $(0-2\pi)$ $<\pi$	$>\pi$	$>\pi/2$ or $>3\pi/2$	$\pi/2-3\pi/2$	$<\pi/2$	$>3\pi/2$
0–1	111	52	59	74	37	58	53	64	47	32	32
1–2	18	8	10	10	8	3	15	10	8	0	10
>2	9	1	8	5	4	6	3	5	4	4	1
0–0.3	60	24	36	40	20	32	28	34	26	17	17

direction, $\hat{z}=\vec{q}=\vec{p}_\nu-\vec{p}_\mu$, and with the y axis along the production plane normal, $\hat{y}=\vec{p}_\nu\times\vec{p}_\mu$. For $Q^2 < 1$ GeV^2 both the cosθ and Φ distributions are symmetric, as required for pure Δ^{++} production, but for $Q^2>2$ GeV^2 there is an excess of events with forward π^+ (as observed in the data with $W > 1.4$ GeV), and for $1<Q^2<2$ GeV^2 there is an asymmetry which is evidence for a parity-nonconserving production process. These results suggest that background amplitudes may be comparable with the Δ^{++} amplitudes for $Q^2>1$ GeV^2. The decay-angle distributions for $Q^2<1$ GeV^2 and $W<1.4$ GeV are well described with the spherical harmonic expansion up to $l=2$. The coefficients calculated by the moments method are given in Table II. The related density matrix elements (also for $Q^2<1$ GeV^2), assuming a pure spin-$\frac{3}{2}$ state, are $\tilde{\rho}_{33}=0.90\pm0.12$, $\tilde{\rho}_{31}=0.01\pm0.05$, $\tilde{\rho}_{3-1}=-0.07\pm0.07$, where the decay-angle distribution is given by

$$\frac{d\sigma}{d\cos\theta\, d\Phi}=\frac{\sigma}{\sqrt{4\pi}}\left[Y_0{}^0-\frac{2}{\sqrt{5}}(\tilde{\rho}_{33}-\tfrac{1}{2})Y_2{}^0-\frac{4}{\sqrt{10}}\tilde{\rho}_{3-1}\,\mathrm{Re}Y_2{}^2+\frac{4}{\sqrt{10}}\tilde{\rho}_{31}\,\mathrm{Re}Y_2{}^1\right].$$

We have fitted our data with various parametrized models[1] listed in Table III and also using the full Adler model.[2,9] Since we have evidence that background amplitudes, not included in the parametrized models, are present at high Q^2, we first study the fits to events with $Q^2<1$ GeV^2 before attempting to fit the full Q^2 range. In the parametrized models we use the Rarita-Schwinger formalism for pure Δ^{++} production, using the notation described in Ref. 1.

For the vector form factors[10] we take

$$C_6{}^V=0,\quad C_4{}^V=(-M_p/W)C_3{}^V(Q^2),\quad C_5{}^V(Q^2)=0,$$

$$|C_3{}^V|^2=(2.05)^2(1+9\sqrt{Q^2})\exp(-6.3\sqrt{Q^2}).$$

For the axial vector form factors[11] we take

$$C_i{}^A(Q^2)=C_i{}^A(0)\exp[aQ^2/(1+bQ^2)]/D,$$

where $i=3$, 4, or 5; $D=(1+Q^2/M_A{}^2)^2$; and $C_i{}^A(0)$, a, and b are given in Table III for each theory for $i=4$ and 5. For $i=3$, $C_3{}^A(Q)^2=0$ except for Zucker's model in which $C_3{}^A(0)=1.8$, $a=-1.76$, and $b=0.62$. For all models

$$C_6{}^A(Q^2)=\frac{M_p g_\Delta f_\pi}{2\sqrt{3}\,(M_\pi{}^2+Q^2)D}=\frac{-1.07}{0.019\pm Q^2}\,\frac{1}{D},$$

where g_Δ is the Δ-$p\pi^+$ coupling constant and f_π is the pion decay constant. For each model the value of M_A was found using the maximum-likelihood method. The fit includes the values of Q^2, θ, Φ, W, and E_ν for each event, as well as the measured average cross section for the process. The results are given in Table IV with the pre-

TABLE II. $Y_l{}^m$ coefficients for the decay-angle distribution for events with $W<1.4$ GeV and $Q^2<1$ GeV^2.

	Re	Im
a_{10}	0.08 ± 0.08	...
a_{11}	-0.12 ± 0.07	-0.04 ± 0.07
a_{20}	-0.36 ± 0.08	...
a_{21}	-0.01 ± 0.06	0.01 ± 0.06
a_{22}	0.09 ± 0.07	-0.04 ± 0.08

TABLE III. Form-factor coefficients following Ref. 1.

Model	$C_4{}^A(0)$	a	b	$C_5{}^A(0)$	a	b
Salin[a]	2.7	0		0		
Adler[b]	−0.3	−0.61	0.19	1.2	−0.61	0.19
Bijtebier[c]	3	−0.61	0.19	1.2	−0.61	0.19
Zucker[d]	−1.8	−1.36	0.57	1.9	−0.84	0.32

[a]Ref. 5. [c]Ref. 7.
[b]Ref. 6. [d]Ref. 8.

TABLE IV. Results of fits using the dipole form of the nucleon axial-vector form factor for $Q^2 < 1$ GeV^2 and for all Q^2. Adler 75 refers to the full Adler model (Refs. 2 and 9). All the other models are in the parametrized form as described in the text.

Model	M_A (GeV)	Cross section[a] predicted from fit $\times 10^{-38}$ cm^2	χ^2/d.f. for $d\sigma/dQ^2$
		$Q^2 < 1$ GeV^2	
Adler	$1.0^{+0.14}_{-0.11}$	$0.50^{+0.04}_{-0.06}$	10.1/10
Zucker	$0.62^{+0.09}_{-0.04}$	$0.55^{+0.10}_{-0.04}$	35.7/10
Bijtebier	0.50 ± 0.04	$0.40^{+0.05}_{-0.04}$	12.3/10
Salin	0.42 ± 0.03	$0.24^{+0.07}_{-0.10}$	21.3/10
Adler 75	$1.25^{+0.15}_{-0.13}$	0.45 ± 0.05	10.9/10
		All Q^2	
Adler	1.15 ± 0.10	$0.67^{+0.04}_{-0.06}$	20.9/13
Adler 75	$1.43^{+0.07}_{-0.09}$	$0.66^{+0.04}_{-0.03}$	19.1/13

[a]For $W < 1.4$ GeV and over Q^2 range fitted. For $Q^2 < 1$ GeV^2 the measured cross section is $(0.44 \pm 0.07) \times 10^{-38}$ cm^2.

dicted cross section and, as an indication of the fit quality, the χ^2 of the projection of the fit to the Q^2 distribution. For the fits restricted to $Q^2 < 1$ GeV^2 the models of Adler[6] and Bijtebier[7] give good fits to the Q^2 distribution,[12] but the structure in the decay-angle distributions is not well reproduced in any of the parametrized models, as shown in Fig. 2 for Adler's model. In terms of the average density-matrix element $\tilde{\rho}_{33}$, Adler's and Bijtebier's models predict 0.59 and 0.62, respectively, compared with our value of 0.90 ± 0.12. Comparing the values of M_A obtained, only that from Adler's model, $M_A = 1.00^{+0.14}_{-0.11}$ GeV, is consistent with the value 0.95 ± 0.08 GeV obtained from the reaction $\nu n \to \mu^- p$[13] and therefore we have only considered this model for the fits over the full Q^2 region.

The parametrized Adler model with $M_A = 1.15 \pm 0.10$ GeV yields an acceptable fit to the full Q^2 distribution as shown in Fig. 1. It should be pointed out, however, that both for the $Q^2 < 1$ GeV^2 fits and for the full Q^2 fits the value of M_A obtained is strongly influenced by the cross-section term in the likelihood function. If the measured cross section is not included in the likelihood, the best fit over the full Q^2 range using the parametrized Adler model gives $M_A = 1.45 \pm 0.20$ GeV and predicts $\sigma = (0.88 \pm 0.11) \times 10^{-38}$ cm^{-2}. As in the case of the restricted-Q^2 fits, the decay-angle distributions are poorly fitted.

In an attempt to obtain better fits we have also used the full Adler model[2] (Adler 75) which includes non-Δ amplitudes. The best fits with this model give $M_A = 1.25^{+0.15}_{-0.13}$ GeV for $Q^2 < 1$ GeV^2, and $M_A = 1.43^{+0.07}_{-0.09}$ GeV over the full Q^2 range, as summarized in Table IV and shown in Fig. 1. The quality of these fits is very similar to that of the parametrized model. In particular there is no improvement in the fits to the decay distribution, although the model introduces asymmetries in the integrated $\cos\theta$ and Φ distributions as shown in Fig. 2.

To summarize, we have obtained new data on the reaction $\nu p \to \mu^- \Delta^{++}$ at higher energies and hence higher-Q^2 values than previously studied. The decay distributions show evidence that non-Δ^{++} amplitudes may be significant at higher Q^2. The full Adler model[2] using a dipole form[14] for the nucleon axial-vector form factor provides a good fit to the Q^2 distribution below $Q^2 = 1$ GeV^2 with $M_A = 1.25^{+0.15}_{-0.13}$ and adequately describes the full Q^2 region with $M_A = 1.43^{+0.07}_{-0.09}$ GeV. Fitting with the parametrized Adler model yields lower values of M_A, but gives very similar quality fits. None of the models considered reproduces successfully the observed decay distributions.

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[11]To extend the parametrization of Ref. 1 to higher Q^2, their factor $\{1 + cQ^2/(d + Q^2)\}$ has been replaced by $\{\exp[aQ^2/(1 + bQ^2)]\}$ with the constaints a and b chosen such that the factors have the same value at $Q^2 = 0$, and value at $Q^2 = 0.4$ GeV^2.

[12]The poor fit with Zucker's model is due to the cross-

section term forcing M_A much lower than that required by the shape of the Q^2 distribution.

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[14]Although we have only presented results for the dipole form of the axial-vector form factor it should be noted that similar fits are obtained using a monopole form and these data cannot distinguish between the two forms.