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Reconciliation between Thermodynamics and Noise Measurements on Metal Film Resistors

Michael B. Weissman
Department of Chemistry, Harvard University, Cambridge, Massachusetts 02138
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The deviation of a resistor's I-V curve from linearity as a result of Joule heating and its temperature coefficient of resistivity are shown to be sufficient to predict the low-frequency limiting spectral density of resistance fluctuations caused by spontaneous enthalpy fluctuations. The prediction agrees with previous experiments. Previous predictions that substantially exceeded measured noise in metal film resistors are shown to rely on incorrectly normalized current-jump data.

A number of recent papers[1-7] have investigated the relation between the $1/f$ noise (noise with spectral density inversely proportional to frequency, f) observed in current-carrying metal film resistors and the resistance fluctuations which are expected to arise from spontaneous enthalpy (or "temperature") fluctuations predicted by thermodynamics.[8] The frequency spectrum of the expected spontaneous fluctuations is determined by the kinetics of heat exchange between the resistor and nearby materials. These kinetics are not well known *a priori*[4] but have been measured by current-jump techniques, in which the Joule heating $P(t)$ is changed and the resulting resistance change $\Delta R(t)$ monitored.[4,6] Using the spectral shape obtained from these current-jump experiments and setting the integrated spectral intensity equal to the thermodynamic prediction, several authors have concluded that the observed noise is much less than predicted in the 100-Hz region.[4,6]

The relation used in comparisons of noise data with current-jump data was

$$G(\tau)=[\Delta R(\tau)/\Delta R(0)](k/c_v W)\gamma^2, \quad (1)$$

where $G(\tau)=\langle \delta R(t)\,\delta R(t+\tau)/R^2\rangle$, $\Delta R(\tau)$ is the response to a brief pulse of heating, k is Boltzmann's constant, c_v is the heat capacity per unit volume of the resistor, W is the resistor volume, and $\gamma=(T/R)\partial R/\partial T$, with T being the absolute temperature. In addition to the ordinary assumptions used in analyzing fluctuations,[8,9] the derivation of Eq. (1) used the assumptions that both the fluctuation weighting function[9] and the power dissipation density were uniform over the volume W and 0 elsewhere and that the heat transport was by diffusion with a uniform diffusion coefficient in the resistor and its surroundings.[4]

A more general relation

$$G(\tau)=[\Delta R(\tau)/R](kT/\Delta H)\gamma \quad (2)$$

has recently been derived.[10] (ΔH is the heat deposited by the current pulse.) The derivation of this relation required only one special assumption—that there be only one type of conducting material in the resistor, as is the case in all the examples that we shall discuss. It should be mentioned that one of the standard assumptions is that the enthalpy fluctuations are instantaneously spatially uncorrelated, which is equivalent to saying that the heat capacity of small volumes (e.g., 100 nm cubed) is defined by the macroscopic c_p.[8] Such an assumption may break down near a phase transition, such as the superconducting transi-

tion, but certainly applies to most metal films.

Equation (2) may be expressed as

$$kT\gamma \frac{\Delta R(t)}{R} = \int_{-\infty}^{t} P(t')G(t-t')dt' \qquad (3)$$

by considering $P(t')$ as a continuum of heat pulses. For constant P one obtains the steady-state solution

$$kT\gamma\Delta R/R = S(0)P/4, \qquad (4)$$

where the spectral density of the fractional resistance fluctuations, $S(f)$, is given by[9] $4\int_0^\infty G(\tau) \times\cos(2\pi f\tau)d\tau$. Then, since $P = V^2/R$ (where V is voltage) and for a constant-current experiment the voltage noise from enthalpy fluctuations is given by $S_v(f) = V^2S(f)$,[4] we obtain

$$S_v(0)/4kTR = (\Delta R/R)\gamma. \qquad (5)$$

The expression $4kTR$ will be immediately recognized as the equilibrium resistor noise (Johnson or Nyquist noise.)

Equation (5) has been confirmed in an experiment on enthalpy fluctuations in an electrolyte solution.[10] By allowing for a slight correction (12%) for the finite current source impedance in that experiment, the reported extrapolated value of $S_v(0)/4kTR$ was 4.8. The value $\gamma = -20.5$ was given as were data sufficient to calculate[11] $\Delta R/R = -0.27$, giving a prediction of $S_v(0)/4kTR = 5.5$ by Eq. (5). The agreement is very good, given the uncertainty (~30%) of extrapolating $S_v(f)$ to $f = 0$. Perfect agreement is not expected, since Eq. (5) is exact only to first order in $\Delta R/R$.[12]

Voss and Clarke[4] also report sufficient data to calculate an expected $S_v(0)$ for the sample in which they observed noise less than their predicted value. They give $\gamma = 1.05$ and, from the I-V curve of their Fig. 3 and the voltage given in Fig. 2, $R \simeq 130\ \Omega$, $\Delta R/R \simeq 0.03$. These data imply $S_v(f) \leq S_v(0) = 7\times10^{-20}\ \mathrm{V^2/Hz}$ for the spontaneous fluctuations. This value is more than an order of magnitude less than the smallest excess noise (at 1 kHz) reported in this sample, so that no contradiction of the standard thermodynamic prediction was observed.

The predicted low-frequency $S_v(f)$ of Voss and Clarke was, however, about four orders of magnitude larger than that predicted here and exceeded the observed noise for some frequencies. The discrepancy results from an improper use of Eq. (1). Since in this equation (or rather its Fourier transform) the pulse response is used only to determine the spectral shape, not the magnitude of the expected fluctuations, normalization requires setting the integral of the spectrum over all frequencies equal to the total predicted mean-square fluctuations. Thus if the width of the spectrum (i.e., the characteristic high-frequency rolloff points) is underestimated, the expected spectral density at low frequencies is overestimated. The width of the spectrum was seriously underestimated in Ref. 4 because the pulse response was measured with a 1-kHz ac bridge,[4] setting a minimum response time of 1 msec. In fact, a heat exchange time of 1 msec would not have been possible in this experiment, since, given the reported Joule heating V^2/R and the sample heat capacity, it would imply a temperature rise of $\sim10^5$ °C in steady state. Much faster kinetics are expected from a simple diffusion model[4,9,10] which gives, from the reported sample dimensions and heat diffusion coefficients, the two shortest characteristic diffusion times as ~100 μsec and less than 10 nsec, respectively, consistent with the observed temperature rise of roughly 10°C.[13]

Dutta, Eberhard, and Horn[6] have also reported noise below the predictions of the standard thermodynamic model. Although they did not give sufficient data to recalculate an exact upper bound on the predicted $S_v(f)$, their techniques were essentially similar to those of Voss and Clarke and are subject to the same corrections.

Recent measurements[14] on Sn films at the superconducting transition, with $\gamma > 1000$, show noise spectra of the shape given by Eqs. (1) and (2) with a magnitude given to within about a factor of 2 by Eq. (1). Data reported were insufficient for a comparison of the magnitude with that of Eq. (5), since the nonlinearity of the I-V curves was primarily due to field effects, not heating. As the authors point out, the spectral agreement indicates that despite the onset of long-range order, the instantaneous spatial correlation of the enthalpy fluctuations is negligible.

One may conclude that the resistor-noise observations reported so far are consistent with the straightforward thermodynamic prediction for spontaneous enthalpy fluctuations. In resistors for which $|\gamma| \lesssim 1$, as in most metals, thermodynamic enthalpy fluctuation noise will always be small compared to Johnson noise for all frequencies unless the applied voltage is so large that the resistor is very severely heated. In an electrolyte resistor with $|\gamma| \simeq 20$, spontaneous enthalpy fluctuation noise is measurable and fits the standard model precisely.

The question of the origin of the $1/f$ noise ob-

served in metal films remains. The evidence (particularly the spatial cross-correlation experiment[1]) presented by Voss and Clarke that this noise results from "temperature" fluctuations remains convincing. The arguments of Horn and co-workers[5-7] that the noise magnitude does not have the right temperature dependence to be the predicted spontaneous enthalpy fluctuations are also convincing and agree with our conclusion that the spontaneous noise should be scarcely measurable in ordinary metal films. The conclusion of Voss and Clarke that the spectrum of the spontaneous fluctuations is skewed by unknown forces[4] appears to be unjustified, as is the conclusion of Horn and co-workers that the predominant source of the $1/f$ noise is not temperature fluctuations at all.[6,7] An intermediate conclusion which seems consistent with all the data would be that the $1/f$ noise comes from "temperature" fluctuations of unknown origin in excess of those predicted by thermodynamics.

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$$\tau_H = (\Delta R/R)c_p WT(P\gamma)^{-1} = S(0)/4\int_0^\infty S(f)\,df$$

is on the order of the geometric mean of the two shortest diffusion time, as may be easily verified by integrating the model spectra of Ref. 1.

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Problem of $D^0 \to K^-\pi^+\pi^0$ in Nonleptonic Charm Decay

S. P. Rosen
Purdue University, West Lafayette, Indiana 47907
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If the branching ratio for $D^0 \to K^-\pi^+\pi^0$, which currently stands at $(12 \pm 6)\%$, remains much larger than 3% in future data, then either the $\Delta T = 1$ rule breaks down, or the final-state $\bar{K}\pi\pi$ contains much structure. The possibility that this structure is due to $\bar{K}^*$ and ρ resonances is briefly examined.

Recent data[1-3] on $D \to \bar{K}\pi\pi$ are examined in the light of the $\Delta T = 1$ rule for nonleptonic charm decay.[4] It is argued that if the branching ratio for $D^0 \to K^-\pi^+\pi^0$, which currently stands at $(12 \pm 6)\%$,[2] remains significantly larger than 3%, then the $\bar{K}\pi\pi$ final state must be a highly structured object, and that this structure could well be due to resonances such as $\bar{K}^*$ and ρ. Implications for the ratio of the total D^+ and D^0 widths and for $D \to \bar{K}\pi$ are also considered.

Although the observed sample of $D \to \bar{K}\pi\pi$ is small, it seems to indicate that the corresponding decay matrix element is uniform over the Dalitz plot.[3] If this is indeed the case, then the $\Delta T = 1$ rule predicts a simple relationship[5] between the rates for $D^+ \to K^-\pi^+\pi^+$ and $D^0 \to K^-\pi^+\pi^0$:

$$\Gamma(K^-\pi^+\pi^+) = 4\Gamma(K^-\pi^+\pi^0), \tag{1}$$

where $\Gamma(K^a\pi^b\pi^c)$ denotes the rate for a specific decay mode of the appropriate D meson. Accord-

