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Spontaneous Symmetry Breaking and Blocking of Metastable States

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A mechanism for spontaneous symmetry breaking and related phenomena and the corresponding blocking of metastable states is discussed. It is based on the interaction of an object with a background of "probes" like photons or particles, etc., in its natural surrounding. Applications include quasilocalization of macroscopic bodies, spontaneous parity nonconservation of sugar crystals, localization of atoms in molecules (Born-Oppenheimer approximation), stability of metastable compounds, and perhaps also intrinsic symmetries of elementary particles.

The object of this Letter is to discuss a quantum mechanical mechanism of spontaneous symmetry breaking and related phenomena and the corresponding blocking of states which are not eigenstates of the Hamiltonian of the object in question. This mechanism differs from, and is much more powerful than, the one usually discussed in the current literature based on nonsymmetric solutions to symmetric equations.

Consider an object O and a probe P described in Hilbert spaces H^O and H^P, respectively. For simplicity H^O is assumed to be two dimensional; generalizations will be mentioned at the end. Consider further two orthogonal states φ_1^O and φ_2^O of the object and an interaction between object and probe leading to the following transitions of the combined system:

$$\varphi_i^O\otimes\varphi_0^P\rightarrow\varphi_i^O\otimes\varphi_i^P,\quad i=1,\ 2, \tag{1}$$

where φ_0^P is the assumed initial state of the probe. By linearity this gives the transition for arbitrary initial state $a\varphi_1^O+b\varphi_2^O$. If no further observation is performed on the probe, and the object alone is considered after separation of the two, the object has to be described by a density matrix ρ^O on H^O and the trace has to be taken over H^P.

Taking φ_1^O,φ_2^O as basis in H^O and assuming $(\varphi_1^P,\varphi_2^P)=0$, the transition (1) for an initial superposition $a\varphi_1^O+b\varphi_2^O$ with $|a|^2+|b|^2=1$ leads then for the density matrix ρ^O of the object to

$$\begin{pmatrix}|a|^2 & ab^*\\ a^*b & |b|^2\end{pmatrix}\rightarrow\begin{pmatrix}|a|^2 & ab^*(\varphi_2^P,\varphi_1^P)\\ a^*b(\varphi_1^P\varphi_2^P) & |b|^2\end{pmatrix}=\begin{pmatrix}|a|^2 & 0\\ 0 & |b|^2\end{pmatrix}. \tag{2}$$

This constitutes a "reduction of the state vector" of a pure initial to a mixed final state of the object.

There are two important aspects in this connection. One is the compatibility of this reduction with the linearity of the law of motion. This is the case by construction. The second is the assessment of the relevance and frequency of occurrence of this phenomenon. This may be judged from its connection to the process of measurement. Indeed Eq. (1) is a (simplified) model of a measurement, in which information is transferred from the object to the probe in such a way that subsequent observation of the probe alone *could* discriminate exactly between the two cases where the object is initially in the state φ_1^O or φ_2^O. The *possibility* of this discrimination requires φ_1^P and φ_2^P in Eq. (1) to be orthogonal and thus leads to the exact depletion of the off-diagonal elements of ρ^O in Eq. (2) if the object alone is considered, i.e., even, and in particular, *in* *the absence of any actual observation of the* probe.[1] The following two examples illustrate some important application. (I omit the superscript O for the object where this does not lead to confusion.)

Example 1.—The object is a macroscopic body. φ_1 and φ_2 are two localized spacially well-separated wave functions. The probe consists of one or or several photons in the region where φ_1 or φ_2 is localized. By observation of the photons one could discriminate between the case where the object is in the state φ_1 or φ_2. It follows that the passing, emission, or absorption of such photons destroys any coherence which might have prevailed before. Thus under the usual conditions of the macroscopic world where one can see the objects, i.e., discriminate between different locations by observing photons, it is not possible to preserve the coherence between states with macroscopically different localization.[2] There is no particular (e.g., cosmological) initial condition needed for this. This is responsible also, for instance, for the appearance of bubbles, localized in space, in an overheated liquid.

Example 2.—Consider sugar. It can be in a "right" (φ_1) and "left" (φ_2) state. If the sugar is crystallized, the two may be discriminated by eye (or microscope) by observing the light scattered from it. Thus, again, any coherence between the two states φ_1 and φ_2 is immediately destroyed by the interaction with this light. Now if parity is conserved and the ground state is nondegenerate it is of the form $\varphi = \alpha\varphi_1 + \beta\varphi_2$ with $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$. Symmetry breaking cannot be due to lack of symmetry of the ground state of a symmetric Hamiltonian unless the ground state is degenerate, which is not expected to be the case in general. In both examples the probes could as well be electrons or molecules from the surrounding (in particular for sugar dissolved in a liquid).

An important question which remains to be answered is, what singles out the particular states φ_1 and φ_2 for the reduction (2), i.e., why does the reduction in the examples above not sometimes produce an incoherent mixture between $\varphi_+ = (\sqrt{2})^{-1}(\varphi_1 + \varphi_2)$ and $\varphi_- = (\sqrt{2})^{-1}(\varphi_1 - \varphi_2)$, say, instead of φ_1 and φ_2. But this would imply that if the object is originally in the state φ_1, say, it could be found afterwards with 50% probability in the state φ_2. This means that the interaction with the probe, for which I took photons in my examples, could cause with 50% probability a transition between φ_1 and φ_2 which for large enough distance between the two cannot be the case for a massive body. Thus the inertia of massive objects singles out quasilocalized states for a reduction by interactions with light probes.

I now turn to an important consequence of reduction (2) in case it occurs frequently (in a sense to be discussed), namely the *blocking* of the states φ_1 and φ_2 by stochastic (repeated) reduction.

Consider for concreteness example 2 with a parity-conserving Hamiltonian H. For proper choice of the phases of φ_1 and φ_2 the two eigenstates of H are $\varphi_\pm = (\varphi_1 \pm \varphi_2)/\sqrt{2}$. Their energy difference is $\hbar\omega$. Then, after a reduction the density matrix on the right-hand side of (2) will evolve in a time interval τ according to $\rho \to \exp\{-i\tau H/\hbar\}\rho\exp\{i\tau H/\hbar\}$ to

$$\frac{1}{2}\begin{pmatrix} 1+\delta_0\cos\omega\tau & i\delta_0\sin\omega\tau \\ -i\delta_0\sin\omega\tau & 1-\delta_0\cos\omega\tau \end{pmatrix}, \tag{3}$$

where $\delta_0 = |a|^2 - |b|^2$. For $\delta_0 \neq 0$, in particular for $\delta_0 = \pm 1$, i.e., pure initial state φ_1 or φ_2, this exhibits the expected oscillations with circular frequency ω. Thus the original property (right-handedness of some sugar crystals) disappears after a time $T \sim \omega^{-1}$. The fact that this time is long is usually assumed to be the reason for stability. It will be seen that this is only part of the truth.

If after a time interval τ the object is again hit by a probe leading to a reduction of the density matrix to diagonal form, then

$$\rho \to \frac{1}{2}\begin{pmatrix} 1+\delta & 0 \\ 0 & 1-\delta \end{pmatrix}, \text{ with } \delta = \delta_0\cos\omega t. \tag{4}$$

If this is repeated n times, say, then, after a time $t = \sum_\nu \tau_\nu \approx n\bar{\tau}$, where τ_ν are the time intervals between subsequent reductions and $\bar{\tau}$ their mean value, one obtains $\delta = \delta_t = \delta_0 \prod_\nu \cos\omega\tau_\nu$ in Eq. (4).

I now consider the important cases where[3]

$$\omega\tau_\nu \ll 1, \text{ i.e., } \tau_\nu \ll T = 1/\omega. \tag{5}$$

Then

$$\prod_{\nu=1}^{n}\cos\omega\tau_\nu \approx \prod_{\nu=1}^{n}\left[1 - \tfrac{1}{2}(\omega\tau_\nu)^2\right] \approx \exp\left(-\tfrac{1}{2}\omega^2\sum_{\nu=1}^{n}\tau_\nu^2\right).$$

If all τ_ν were equal this would lead to $\exp\{-n\omega^2\bar{\tau}^2/2\}$. Assuming more realistically a stochastic distribution with distribution function $e^{-\tau/\bar{\tau}}$ corresponding to a Poisson distribution for the counting rate within a given time interval, then $\sum_\nu \tau_\nu^2 = 2n\bar{\tau}^2 = 2\bar{\tau}t$ for large n and one obtains

$$\delta_t = \delta_0 e^{-\lambda t}, \text{ with } \lambda = \omega \times \omega\bar{\tau} = \omega \times \bar{\tau}/T. \tag{6}$$

Thus the relaxation time $1/\lambda = T \times T/\bar{\tau} \gg T$ is much larger than $T = 1/\omega$ if (5) holds. This blocking of the states φ_1 and φ_2 by stochastic "reduction of the state vector" prevents oscillation between φ_1 and φ_2 and is responsible for enhanced stability.

The same analysis may be performed also if the Hamiltonian has not the high symmetry (parity conservation) with respect to φ_1 and φ_2 as assumed above. If its eigenstates are $\varphi = \alpha\varphi_1 + \beta\varphi_2$ and $\varphi' = -\beta^*\varphi_1 + \alpha^*\varphi_2$ with $|\alpha|^2 + |\beta|^2 = 1$, one obtains $\delta_t = \delta_0 \prod_\nu (1 - 8|\alpha\beta|^2 \sin^2 \frac{1}{2}\omega\tau_\nu)$. With (5) this leads to the replacement of λ in (6) by $\lambda = 4|\alpha\beta|^2 \times \omega\bar{\tau}/T \leqslant \omega\bar{\tau}/T$ where one notes that $|\alpha|^2 + |\beta|^2 = 1$ implies $4|\alpha\beta|^2 \leqslant 1$ with equality holding in the symmetric case $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$.

The model considered so far is rather schematic in several respects. First of all it is clear that, in particular for microscopic systems, the interaction with a probe like a photon does not necessarily lead to orthogonal states φ_1^P and φ_2^P for given incoming state φ_0^P. If it does not, then one obtains only a *partial reduction* of the density matrix ρ:

$$\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \to \begin{pmatrix} \rho_{11} & \zeta\rho_{12} \\ \zeta^*\rho_{21} & \rho_{22} \end{pmatrix}, \tag{7}$$

where $\zeta = (\varphi_2^P, \varphi_1^P)$. Since the states are normalized to unity, $|\zeta| < 1$ unless $\varphi_1 = \alpha\varphi_2$. After m reduction with ζ_μ the off-diagonal elements are multiplied by

$$\zeta^{(m)} = \prod_{\mu=1}^{m} \zeta_\mu,$$

which approaches zero for large m if $|\zeta_\mu| < 1$. This is in line with the previously discussed relation to the process of measurement since even if the object may not be localized with one photon, in example 1 for instance, this may still be done with many of them, which then may be considered as one probe[1] leading to a (almost) complete reduction, or as many individual probes each leading to a partial reduction. From the physics involved one thus expects the blocking effect to be working similarly where, of course, one has to require now that m partial reductions with $|\zeta^{(m)}| \ll 1$ should take place in a time interval which is short compared to $T = 1/\omega$. This is borne out also by the formal analysis whose result is given here for symmetric (parity-conserving) Hamiltonian as in Eqs. (3)–(6). Under the condition that subsequent reductions are statistically independent one can—for the analysis of the average relaxation—replaced τ_ν by $\bar{\tau}$, τ_ν^2 by $2\bar{\tau}$ (see above), and the ζ_ν by their *average* ζ with $|\zeta| < 1$. The relevant condition replacing (5) turns out to be

$$\omega\bar{\tau}/(1 - |\zeta|) \equiv \omega\tau_0 \ll 1. \tag{8}$$

After a sequence of n evolutions under H as in (3) and subsequent reductions according to (7), an initial density matrix $\rho(0)$ turns into $\rho(t)$, where $t = \sum_\nu \tau_\nu = n\bar{\tau}$, with[4]

$$\delta_t = e^{-\lambda' t}\left\{\delta_0 - 2\,\mathrm{Im}\left[\frac{\omega\bar{\tau}}{1-\zeta}(1-\zeta^n)\rho_{12}(0)\right] + O_1(\omega^2\tau_0^2)\right\},$$

$$\rho_{12}(t) = e^{-\lambda' t}\left\{\zeta^n\rho_{12}(0) + \frac{i}{2}\frac{\omega\bar{\tau}\zeta}{1-\zeta}(1-\zeta^n)\delta_0 + O_2(\omega^2\tau_0^2)\right\}, \quad \lambda' = \omega^2\bar{\tau}\,\mathrm{Re}\,\frac{1}{1-\zeta} \equiv \omega^2\tau_{\mathrm{eff}}, \tag{9}$$

where $\delta_t = \rho_{11}(t) - \rho_{22}(t)$ as in Eq. (4). Now $|\zeta|^n = \exp(t \ln|\zeta|/\bar{\tau}) < \exp(-t/\tau_0)$, implying that the off-diagonal element ρ_{12} is effectively reduced to zero after a run-in time of a few times τ_0. The condition (8) implies $\tau_0 \ll \omega^{-1} \ll \lambda^{-1}$, and $|\omega\bar{\tau}/(1-\zeta)| \ll 1$ in (9) so that spontaneous symmetry breaking and blocking is recovered with modified relaxation time $1/\lambda'$ as expected. This may, of course, be generalized to nonsymmetric evolution as discussed for complete reduction.

In conclusion it is seen that even if single reductions are very weak, i.e., if $(1 - |\zeta|) \ll 1$, spontaneous symmetry breaking and blocking persist as long as (8) holds. This ensures a wide applicability of this phenomenon.

The second oversimplification of the model analyzed is that H^O is two dimensional and that φ_1^O and φ_2^O are reproduced exactly. Of course, the whole discussion can be generalized to l different mutually orthogonal states $\varphi_1^O \cdots \varphi_l^O$ without any principal change. Of some interest, however, is the generalization to groups of states or subspaces H_i^O of H^O such that starting from given initial states $\varphi_i^O \in H_i^O$ subsequent interactions with probes not necessarily reproduce φ_i^O but still lead to (not necessarily pure) states which are again in H_i^O for all i. This interaction then leads to the depletion of matrix elements of the density matrix between states from different H_i^O in the same way and under the same circumstances as for the case of one-dimensional H_i^O analyzed explicitly above. The general features of the blocking effect are expected to prevail.

In conclusion I restate the main ingredients of the mechanism presented here. It is based on the interaction of the object under consideration with a background of probes in its natural surrounding. The relevant background may consist for instance of electromagnetic or corpuscular radiation and perhaps even neutrinos, or molecules within a gas or fluid, and eventually also phonons, etc., in a solid, depending on the kind of object studied. The characteristic behavior of the object under the influence of this background is governed by two criteria:

(i) *The inertness criterion* (a) singles out the *inert states* to which reduction takes place such that (b) the interaction with the background (probes) does not induce transitions between these states. It thus defines the axes of spontaneous symmetry breaking. For macroscopic bodies it obviously singles out macroscopically localized states. If some interactions violate condition (b) independent of the choice of inert states, the state relaxes to unit density matrix, i.e., "total chaos." Of course, small violation of condition (b) can be tolerated as long as the relaxation time obtained from these transitions is long as compared to the other characteristic times of the problem at hand.

(ii) The *frequency criterion* for the blocking effect defines the regime in which the latter dominates over the free evolution under the Hamiltonian describing the object. In the cases calculated here it is given by Eqs. (5) and (8).

From the thermodynamical point of view the frequency criterion implies that the object is in a temperature bath with $kT \gg \hbar\omega$ (T = absolute temperature, k = Boltzmann constant) implying unit equilibrium density matrix. However, under the conditions of the inertness criterion, *the shorter* $\bar{\tau}$ *and the smaller* ζ, *i.e., the more intense the coupling, the longer is the relaxation time* $1/\lambda$ *or* $1/\lambda'$.

The mechanism of spontaneous symmetry breaking and blocking of metastable states so obtained applies to many macroscopic systems and compounds which would not be stable otherwise and thus seems to play a crucial role for the stability and classical property of the macroscopic world.[5] It eliminates the need for special (cosmological) initial conditions in order to obtain macroscopically localized states.[6] It is expected to be important also in molecular physics for the localization of atoms in molecules (and thus the applicability in the widely used Born-Oppenheimer approximation), guaranteeing in particular the stability of steroisomers, chiral molecules (such as sugar), etc. To what extent it operates also for intrinsic degrees of freedom (including P and/or CP) of elementary particles is an open question, which seems, however, certainly worthwhile studying.

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[1]The connection with the problem of measurement and the interpretation of quantum mechanics will be discussed in more detail elsewhere. Here I only remark that starting from a purely statistical (ensemble) interpretation of quantum mechanics one obtains in this way under macroscopic conditions (see example 1 below) a description of individual systems.

[2]In *principle* one could recover coherence by including in the observation the final states of *all* such photons.

[3]Of course I implicitly assume also that the collision time between object and probe is short compared to τ_v. This is, however, relevant only for the exact calculation, not for the general feature of the effect discussed. (See the partial reduction discussed below.)

[4]$|O_i(x^2)| \leq C_i x^2$ for $x \ll 1$ with C_i independent of n.

[5]For instance, $T = 1/\omega \approx 1$ yr and $\tau \approx 10^{-5}$ sec imply $1/\lambda \approx 3 \times 10^{12}$ yr according to (6). On the other hand, for molecules with $T = \omega^{-1} \lesssim 10^{-2}$ sec it may be possible to study the blocking effect experimentally, though perhaps not in the extreme limit (8), for instance in a molecular beam crossing laser light or going through a gas of appropriate pressure.

[6]For a recent synopsis (with references) of other approaches to establish quantum mechanics as a general theory whose applicability includes the classical domain see J.-M. Lévy-Leblond, in *Quantum Mechanics, A Half Century Later*, edited by J. Leite Lopes and M. Paty (Reidel, Dordrecht, 1977), pp. 187–206. Note, however, that no attempt is made here to prove a reduction of the form (2) for the combined system described on $H^O \otimes H^P$. Unlike other approaches the probe can therefore be a simple microscopic system, which is obviously an important feature. On the other hand, no coherence can prevail between different "pointer" positions (or living and dead cats, etc., so to speak) in the case of a *macroscopic* apparatus under usual conditions, i.e., in the presence of radiation, air, etc. (necessary for living cats).