

In order to fit the shifts for  $N > Z$  nuclei without adjusting the parameters of the potential it was necessary to increase the rms radii of the neutron distributions beyond those for the protons by unreasonably large amounts, e.g., 0.3 fm for P and Ar. It was therefore decided to adjust the coefficients of  $\rho_n - \rho_p$  in the potential, and this was done in the second stage of the fit. The parameter  $c_1$  was kept at its theoretical value and only  $b_1$  was adjusted, since  $c_0$  and  $c_1$  represent interactions near the nuclear surface and since  $c_1 \approx c_0$  because the  $\pi^-N$  interaction is predominantly with neutrons. On the other hand, the local part of the potential represents interactions which take place throughout the nuclear volume and the parameter  $b_0$  has already been adjusted<sup>2</sup> to include effects due to the Fermi motion. Analysis of the data for Fe, Cu, and Zn showed that the value obtained for  $b_1$  is strongly dependent on the value chosen for  $r_n - r_p$ . However, if the values for  $r_n - r_p$  are constrained to be in the range 0.05 to 0.15 fm, a value of  $(-0.13 \pm 0.02)m_\pi^{-1}$  is obtained for  $b_1$ . If the parameter  $c_1$  is adjusted to fit the data then it differs by  $-200\%$  from the theoretical value. The continuous curve in Fig. 1 shows the calculated values obtained for procedure (b) with  $b_1 = -0.13m_\pi^{-1}$  and  $r_n - r_p = 0.1$  fm where the fit to the data for  $N > Z$  nuclei is greatly improved. The structure displayed by this curve results from the variations of  $(N - Z)/A$ . The value of  $b_1 = (0.13 \pm 0.02)m_\pi^{-1}$  is in good agreement with that obtained previously<sup>10</sup> from fits over the whole of the periodic table. The sign of

$\text{Re}C_0$  appears to disagree with theoretical predictions<sup>3</sup> but we emphasize that there is no significant difference between the fits with  $\text{Re}C_0 = 0$  and  $\text{Re}C_0 > 0$ . However,  $\text{Re}C_0 < 0$  will require larger adjustments in the value of  $c_0$ .

In conclusion, we have made precision measurements of shifts and widths of  $2p$  levels in pionic atoms of Al, Si, S, Ca, Fe, Cu, and Zn. Parameters have been determined for an effective pion-nucleus potential which lead to good agreement with all available experimental results.

We wish to thank Dr. D. M. Asbury and Dr. M. Blecher who participated in early stages of this work.

<sup>1</sup>C. J. Batty *et al.*, Nucl. Phys. **A282**, 487 (1977).

<sup>2</sup>M. Krell and T. E. O. Ericson, Nucl. Phys. **B11**, 521 (1969).

<sup>3</sup>J. Hüfner, Phys. Rep. **21C**, 1 (1975).

<sup>4</sup>C. W. de Jager *et al.*, At. Data Nucl. Data Tables **14**, 479 (1974).

<sup>5</sup>L. Tauscher and W. Schneider, Z. Phys. **271**, 409 (1974).

<sup>6</sup>J. Egger, CERN Report No. NP73-20, 1973 (unpublished).

<sup>7</sup>G. Poelz *et al.*, Phys. Lett. **26B**, 331 (1968).

<sup>8</sup>D. A. Jenkins and R. Kunselman, Phys. Rev. Lett. **17**, 1148 (1966).

<sup>9</sup>R. Abela *et al.*, Z. Phys. **A282**, 93 (1977).

<sup>10</sup>D. K. Anderson, D. A. Jenkins, and R. J. Powers, Phys. Rev. Lett. **24**, 71 (1970).

## Enhanced Stimulated Raman Scattering Does Not Exist

D. Eimerl

Lawrence Livermore Laboratory, University of California, Livermore, California 94550

(Received 19 October 1977)

A recent Letter by Sparks and Sen challenges the viability of a recently proposed design for a commercial laser-fusion power plant. The basis for this challenge is the controversial theory of enhanced stimulated Raman scattering put forward by Sparks. This theory has been examined carefully. It is found to be in direct contradiction to both the classical and quantum theories of stimulated Raman scattering, and it is also in serious conflict with many experimental results.

The recent Letter by Sparks and Sen<sup>1</sup> is the latest in a series of papers<sup>2</sup> dealing with a highly controversial proposal. The essential feature of this proposal is that the gain experienced by the Stokes wave in stimulated Raman scattering

(SRS) is larger than the steady-state gain and that the gain exponent in the steady state depends nonlinearly on the pump intensity. Although the proposal was made originally for SRS in gases it is apparently intended to apply to any process

involving three waves in parametric interaction. Most recently<sup>1</sup> it has been applied to the scattering of a laser by plasma waves in the ionized residual gas in the target chamber of recently proposed laser-driven fusion reactors.<sup>3</sup> In view of the potential importance of this effect, a careful analysis of Sparks's theory has been made. This Letter reports the main results of our investigation, which deals mostly with SRS in gases.

Generally speaking, Sparks claims<sup>2,4-7</sup> (a) that the interaction of three waves may be described by rate equations, (b) that these equations contain a previously overlooked term, and (c) that this term is responsible for the claimed dependence of the gain exponents on the pump intensity. Several arguments have been advanced to support these claims. Although they may appear plausible at first glance, a closer look quickly shows that they are untenable. Not only is Sparks's new rate equation in conflict with both the classical and quantum theories of SRS, or any other three-wave parametric process, but more importantly, it is also in serious conflict with very many experimental results.

In order to illustrate the relevant features of Sparks's theory, first the classical theory of SRS is shown to admit a rate-equation description and the classical rate equations are reviewed. The classical equations of SRS are<sup>8</sup>

$$(\partial/\partial z + \bar{c}^{-1}\partial/\partial t)E_S = \sigma_1 E_L Q^*, \quad (1)$$

$$(\partial/\partial t + \Gamma)Q = \sigma_2 E_L E_S^* + N, \quad (2)$$

where the symbols have their usual meaning and  $N$  is a stochastic force describing the noise sources in the medium. (It may have a dependence on the pump field, but is independent of the Stokes wave.) A detailed mathematical treatment of these equations has been given by Wang.<sup>9</sup> If, with  $\xi = t - z/c$ , we have

$$\theta(\xi, \xi') = 2[\sigma_1 \sigma_2 z \int_{\xi'}^{\xi} d\xi'' |E_L(\xi'')|^2]^{1/2}, \quad (3)$$

then the solution for the Stokes wave is

$$E_S(\xi, z) = E_S(\xi, 0) + \sigma_1 \sigma_2 E_L(\xi) \int_{-\infty}^{\xi} d\xi' e^{-\Gamma(\xi - \xi')} \Phi, \quad (4)$$

where

$$\Phi = E_L(\xi) E_S(\xi, 0) I_1(\theta) 2z/\theta + \int_0^z dz' N I_0(\theta). \quad (5)$$

Thus the mean field is independent of  $N$  but the mean intensity has a component proportional to an integral over the correlation function  $\langle NN^* \rangle$ . However, the *gain* depends only on the pump in-

tensity. From (4) and (5) one may determine which rate equations, if any, reproduce the exact solution. In the transient case, there is no simple equation because the Stokes intensity grows as  $\exp(kI_L z t)^{1/2}$ . However, in the steady state the solution grows as  $\exp(gI_L z)$ . It is useful to rescale the variables  $|Q|^2 = a n_p$ ,  $I_L = b n_L$ ,  $I_S = b' n_S$ , so that the steady-state solutions appear in the form

$$n_S = [n_S(0) + \bar{n}_p] \exp(C n_L z), \quad (6)$$

$$n_p = \bar{n}_p + \Gamma^{-1} C n_L (n_S + \bar{n}_p) \quad (7)$$

for suitably chosen rescaling coefficients. The term  $\bar{n}_p$  represents the terms in  $I_S$  and  $|Q|^2$  which are proportional to  $\langle NN^* \rangle$ . These equations are steady-state solutions of the following equations

$$\begin{aligned} (\partial/\partial z + c^{-1}\partial/\partial t)n_S \\ = c^{-1}[\partial n_p/\partial t + \Gamma(n_p - \bar{n}_p)] = R, \end{aligned} \quad (8)$$

where

$$R = C n_L (n_S + \bar{n}_p). \quad (9)$$

No new physical significance has been given to the rescaled variables. The familiar classical solutions have been written in this unusual form only to allow a direct comparison with Sparks's rate equations.

The new rate equations proposed by Sparks<sup>2,4-7</sup> are obtained from (8) by replacing  $R$  by  $R_S$ , where

$$R_S = C [n_L (n_S + n_p + 1) - n_S n_p], \quad (10)$$

and interpreting the  $n$ 's as mode-occupation numbers. Thus  $n_p$  is interpreted as the number of molecular phonons. Despite these differences in interpretation, in both the classical and the new system  $I_S$  is proportional to  $n_S$ . Therefore, the essential differences follow simply by comparing the behavior of  $n_S$  from (8) and (9) with that from (8) and (10). Clearly the new system (10) is in direct contradiction to the classical theory<sup>10</sup> and gives a totally different behavior for  $I_S$ . Therefore, (10) is either wrong, or contains new physics of a nonclassical nature but which, nonetheless, leads to macroscopic effects. It has been claimed<sup>11</sup> that (9) and (10) may be reconciled if the difference between them is attributed to noise. That is, if (10) describes the growth of both incoherent and coherent energy at the Stokes frequency, whereas (9) describes only the coherent energy and a small portion of the noise. This is equivalent to the assertion that the classical theory does not treat noise correctly, and also has

many experimental consequences, which will be discussed later, along with other predictions of the new rate equation.

The argument given to support (10) is based on perturbation theory for the fully quantized system.<sup>2</sup> The Hilbert space is spanned by the eigenstates of the number operators for the laser, Stokes, and phonon waves. Under the influence of the perturbation  $H' = Ra_L a_S^\dagger a_p^\dagger + \text{H.c.}$ , energy is transferred between the three fields. The error made by Sparks is most clearly illustrated using the density matrix, which satisfies the Heisenberg equations of motion,

$$\begin{aligned} (\partial/\partial t + i\omega_{ij} + \Gamma_{ij})\rho_{ij} \\ = i\sum_l (V_{il}\rho_{lj} - \rho_{il}V_{lj}) + N_{ij}. \end{aligned} \quad (11)$$

Here  $\Gamma$  and  $N$  represent noise effects and decay phenomena and  $V$  is given by the matrix elements of  $H'$ . It connects those states where the laser, Stokes, and phonon numbers all change by unity. Now  $\rho$  may be written as the sum of a diagonal part  $\rho^D$ , and a nearly diagonal part  $\rho^B$ , which contains only  $\langle n_L + 1, n_S - 1, n_p - 1 | \rho | n_L n_S n_p \rangle$  and their Hermitian conjugates, and a remainder  $\rho^C$ . These satisfy the following equations of motion:

$$\partial\rho^D/\partial t = -\Gamma^D\rho^D + i[V, \rho^B]^D + N^D, \quad (12)$$

$$\partial\rho^B/\partial t = -\Gamma^B\rho^B + i[V, \rho^D] + i[V, \rho^C]^B + N^B. \quad (13)$$

Sparks's analysis reduces to solving these equations perturbatively, by setting  $\dot{\rho}^B = 0$ , and solving (13) for  $\rho^B$ , which is then inserted into (12). This gives an equation for the diagonal part  $\rho^D$  in terms of the off-diagonal part  $\rho^C$ . Thus the rate equations for the occupation numbers depend on the quantum correlations. If the latter are small, the second commutator in (13) is small compared to the first, and  $\rho^C$  disappears from the equation for  $\rho^D$ , giving

$$\Gamma^D \dot{\rho}^D = -[V, [V, \rho^D]]^D. \quad (14)$$

This reduces to Sparks's result, Eq. (10). The condition  $\rho^C \ll \rho^D$  implies that all three waves are uncorrelated with each other, and that each wave is described by a diagonal density matrix; namely, it is noise. However, in SRS, it is experimentally established that the waves are highly correlated, and in many experiments they have a well-defined classical phase. Thus in SRS a high degree of quantum coherence<sup>12,13</sup> is found, invalidating (10) and (14). Although it may appear plausible that (10) follows from the "golden rule," as Sparks claims, it is in fact not permissible to replace  $a_L^\dagger$  by  $(\bar{n}_L + 1)^{1/2}$ , etc., where  $\bar{n}_L$

is the average occupation number, as Sparks does,<sup>2</sup> unless special conditions are met. *The golden rule, and the rule-of-thumb replacement just given, follow as special cases of the full Heisenberg equations of motion for the density matrix.* They may not be used unless the condition  $\rho^C \ll \rho^D$  is met, so that  $\dot{\rho}^D$  is determined solely by  $\rho^D$ . It is permitted in systems close to thermal equilibrium, but not in SRS, where  $\rho^C$  is comparable to  $\rho^D$  in its impact on  $\dot{\rho}^D$ .

A second point concerning the golden-rule approach is that the operators  $a_L$ , etc., are functions of both  $z$  and  $t$ . The  $z$  dependence gives  $a_L$  the character of a quantum field, and in fact, the nonlocal commutation relations for the  $a_L$ , etc., invalidate Sparks's result for the propagating case.<sup>14</sup>

Furthermore, it can be shown<sup>15</sup> that the quantum theory of SRS supports the classical results for the mean fields and gives an equation for the noise at the Stokes frequency very similar to the Fokker-Planck equation. The proof, which is too long to be reproduced in a Letter, is based on a very straightforward generalization of the quantum theory of the laser and follows Haken's analysis<sup>16</sup> using the quantum Langevin equations. It can be shown, with use of the technique of quasilinearization,<sup>16</sup> that the noise at the Stokes frequency is a negligible part of the total Stokes intensity (well above threshold) and that the mean field experiences a gain which is bounded by the classical steady-state gain. Thus, the quantum theory does not contain the anomalous gains claimed by Sparks, and (10) is in direct contradiction with the quantum theory of SRS. It also follows that the claim concerning noise at the Stokes frequency is groundless.

Turning to the experimental consequences of Eq. (10), I note the following predictions<sup>3</sup>: (A) The gain at the Stokes frequency is formally  $\gamma_S = \gamma(1 - I_L/I_R)^{-1}$  where  $\gamma$  is the unenhanced gain in centimeters per watt and  $I_R = \Gamma/\gamma$ . (B) For  $I_L \gtrsim I_R$ , the singularity in  $\gamma_S$  is avoided by invoking suppression mechanisms such as anti-Stokes production in the low-dispersion case and by bleaching (i.e., phonon saturation) in the general case. The point  $I_R$  is then interpreted as a threshold for a quasi-discontinuous, almost complete conversion to the Stokes frequency. (C) The Stokes line is broadened in frequency (it is mainly noise) and satisfies  $\Gamma_S/\Gamma \gtrsim (I_L/I_R)^{1/2}$ .

The unenhanced gain  $G$  is  $\gamma I_L z$  where  $z$  is the gain length. Thus, the ratio  $I_L/I_R = G/\Gamma z$ . Typical values in stimulated scattering experiments

are  $G \sim 20-40$ ,  $\Gamma \sim 10^{-2}-1 \text{ cm}^{-1}$ ,  $z \sim 0.1 \text{ mm}$  to  $100 \text{ cm}$ . Thus in most experiments  $I_L \gg I_R$ . However, catastrophic conversions have not been seen in most experiments. Furthermore, anomalies seen in some early work all have reasonable alternative explanations. Therefore, it is extremely unlikely that the discontinuities seen in the early literature have a fundamental origin. For example, in 1 atm of nitrogen, the critical intensity is near  $0.5 \text{ GW/cm}^2$ . Thus any high-intensity visible laser should undergo catastrophic conversion to the Stokes wave in *air*. Such a phenomenon has never been reported, despite the countless occasions when this threshold has been exceeded. The classical theory of transient stimulated Raman scattering has been verified experimentally by Carmen and Mack,<sup>17</sup> Lowdermilk and Kachen,<sup>18,19</sup> and Kachen.<sup>20</sup> In all these experiments,  $I_L \gg I_R$  and no catastrophic instability was observed.

The linewidth of the Stokes wave in SRS has been measured in a number of cases. Patel<sup>21</sup> investigated the linewidths of spin-flip Raman lasers and found that the linewidth was seven orders of magnitude *smaller* than the spontaneous linewidth. These lasers have a very high gain and operate typically at  $I_L/I_R \sim 10^2$  or higher.<sup>22</sup> Similar results on line narrowing have been found in several experiments.<sup>23</sup> A direct measurement on liquid nitrogen by Akhmanov<sup>24</sup> confirms this effect.

The condition  $I_L \gg I_R$  implies that the Stokes wave grows quasidiscontinuously to almost the pump intensity. However, then the condition  $I_S \gg I_R$  is satisfied so that catastrophic or unstable conversion to second Stokes and higher orders should be seen. This is not seen in most experiments, where the first Stokes wave remains unconverted when  $I_L \gg I_R$ . For example, in Ref. 18,  $I_L \sim 15I_R$  and the transient classical theory was verified. Also, experiments quoted by Sparks<sup>25,26</sup> in support of his claims used spectrometers and no depletion of the Stokes wave was seen up to  $I_L \sim 2.2I_R$ .

In summary, the theory of enhanced stimulated Raman scattering has been shown to have no basis in theory and to contradict the classical and quantum theories of SRS. It is also in conflict with experiments on transient Raman scattering, linewidth measurements, and multiple conversion to higher orders. The predictions of this theory apply to almost any high-gain stimulated-scattering experiment, but are not seen. Therefore, Sparks's theory is incorrect for SRS in gases. There is also convincing evidence from

laser heating of solenoidally confined plasmas that his theory fails also for SRS in plasmas.<sup>27</sup> It predicts that the laser in Rukowski's experiment is totally backscattered, penetrating less than  $0.1 \text{ mm}$  into the plasma.<sup>28</sup> It is patently clear that Sparks's challenge of the validity of laser fusion is totally groundless. The real issues for SRS in the fusion chamber plasma will include medium homogeneity and pump nonuniformity, rather than any fundamental theoretical question of the type discussed here.

The author is grateful for stimulating discussions with M. Sparks, A. J. Glass, and J. J. Thomson. This work was performed under the auspices of the U. S. Energy Research and Development Administration under Contract No. W-7405-Eng-48.

<sup>1</sup>M. Sparks and P. N. Sen, Phys. Rev. Lett. **39**, 751 (1977).

<sup>2</sup>M. Sparks, Phys. Rev. Lett. **22**, 450 (1974).

<sup>3</sup>M. Sparks, Phys. Rev. B **11**, 595 (1975).

<sup>4</sup>M. Sparks, J. Appl. Phys. **46**, 2134 (1975).

<sup>5</sup>M. Sparks and J. H. Wilson, Phys. Rev. B **12**, 4493 (1975).

<sup>6</sup>M. Sparks and H. C. Chow, Phys. Rev. B **10**, 1699 (1974).

<sup>7</sup>W. R. Meier and J. A. Maniscalco, UCRL Report No. UCRL-79654 (unpublished).

<sup>8</sup>W. Kaiser and M. Mayer, in *Laser Handbook*, edited by F. T. Arrechi and E. O. Schulz-Dubois (North-Holland, Amsterdam, 1972), p. 1077.

<sup>9</sup>C. S. Wang, Phys. Rev. **182**, 482 (1969).

<sup>10</sup>Reference 5 attempts to derive (10) from the classical theory, but commits an elementary mathematical error in Eq. (2.7).

<sup>11</sup>M. Sparks, private communication.

<sup>12</sup>R. J. Glauber, Phys. Rev. **130**, 2529 (1963).

<sup>13</sup>R. J. Glauber, Phys. Rev. **131**, 2766 (1963).

<sup>14</sup>D. Eimerl, unpublished.

<sup>15</sup>D. Eimerl, UCRL Report No. UCRL-79957, 1977 (to be published).

<sup>16</sup>H. Haken, in *Quantum Optics*, edited by S. M. Kay and A. Maitland (Academic, New York, 1970), p. 242.

<sup>17</sup>R. L. Carman and M. E. Mack, Phys. Rev. A **5**, 341 (1972).

<sup>18</sup>W. H. Lowdermilk and G. I. Kachen, Phys. Rev. A **14**, 1472 (1976).

<sup>19</sup>W. H. Lowdermilk and G. I. Kachen, Appl. Phys. Lett. **27**, 133 (1975).

<sup>20</sup>G. I. Kachen, UCRL Report No. UCRL-51753 (unpublished).

<sup>21</sup>C. K. N. Patel, Phys. Rev. Lett. **28**, 649 (1972).

<sup>22</sup>Y. R. Shen, Rev. Mod. Phys. **48**, 1 (1976).

<sup>23</sup>A. Z. Grasyuk, Sov. J. Quant. Electron. **4**, 269 (1974).

<sup>24</sup>S. A. Akhmanov, Y. E. D'yakov, and L. I. Pavlov, Zh. Eksp. Teor. Fiz. **66**, 520 (1974) [Sov. Phys. JETP **39**, 249 (1974)].

<sup>25</sup>J. B. Grun, A. K. McQuillan, and B. P. Stoicheff, Phys. Rev. **180**, 61 (1969).

<sup>26</sup>E. E. Hagenlocker, R. W. Minck, and W. G. Rado, Phys. Rev. **154**, 225 (1967).

<sup>27</sup>H. L. Rutkowski, D. W. Scudder, Z. A. Pietrzyk, and G. C. Vlases, Appl. Phys. Lett. **26**, 421 (1974).

<sup>28</sup>D. Eimerl, unpublished.

## Stabilization of Drift-Cyclotron Loss-Cone Mode by Low-Frequency Density Fluctuations

Akira Hasegawa

Bell Laboratories, Murray Hill, New Jersey 07974

(Received 15 February 1978)

Low-frequency ( $\omega \ll \omega_{ci}$ ) density fluctuations (which may be produced by the drift-wave instability of the cold-ion species) can stabilize the drift-cyclotron loss-cone (DCLC) mode by scattering electrons and thereby providing anomalous high-frequency resistivity to the DCLC mode.

Injection of cold-ion species to the mirror-confined plasma was found to increase greatly the confinement time of the plasma.<sup>1</sup> This effect is considered to be due to the filling of the loss-cone distribution of hot-ion component by the cold-ion species. Theoretical studies<sup>2,3</sup> of the stabilization effects of the cold ions based on the linear theory revealed, however, that there remain various weaker instabilities even in the presence of fairly large percentage of cold-ion density. One of the residual instabilities is the drift-cyclotron instability<sup>4</sup> which is driven only by the density gradient. The result of the quasilinear theory<sup>5</sup> shows also that the quasilinear diffusion process is not sufficient to stabilize this instability. Recent experimental results in fact indicate the existence of the drift-cyclotron mode even though the level of the fluctuations is greatly reduced by the presence of the cold ions.<sup>6</sup>

Most recently Aamodt *et al.*<sup>7</sup> suggested a stabilization mechanism of the drift-cyclotron mode by the nonlinear frequency shift by noting that the instability is sharply peaked at the ion cyclotron frequency. I present here an alternative mechanism of stabilization of the drift-cyclotron mode [as well as the drift-cyclotron loss-cone (DCLC) mode]. The idea is to introduce anomalous high-frequency resistivity by the scattering of electrons off from low-frequency density fluctuations.

The low-frequency fluctuations at  $\omega \approx 10^{-1}\omega_{ci}$  are in fact observed by the previously mentioned experiment.<sup>6</sup> One candidate for the origin of such fluctuations is the drift-wave (universal) instability caused by the cold-ion density gradient as will be described later. In the presence of such

fluctuations the drift-cyclotron mode (or DCLC mode) and the low-frequency mode are strongly coupled by the electrons which satisfy the resonant condition,  $\omega_H - \omega_L \approx k_{\parallel}v_e$ , where  $\omega_H$  is the drift-cyclotron frequency ( $\sim\omega_{ci}$ ),  $\omega_L$  and  $k_{\parallel}$  are the angular frequency and the parallel wave number of the low-frequency fluctuations, and  $v_e$  is the electron thermal speed. The coupling produces a high-frequency resistivity in the drift-cyclotron mode or DCLC mode.

To study the coupling process, I take the plasma to be uniform in the direction of the magnetic field, which is justified if  $k_{\parallel}^{-1} \ll L_p$ , the length of the mirror. I assume that  $\omega_L$  and  $k_{\parallel}$  satisfy the usual condition,

$$k_{\parallel}v_{ic} \ll \omega_L \ll k_{\parallel}v_e \sim k_{\parallel}v_{ih},$$

where  $v_{ic}$  and  $v_{ih}$  are, respectively, the cold- and hot-ion thermal speeds. Then the low-frequency electron density is given by the Boltzmann distribution  $f_L^{(1)} = (e\phi_L/T_e)f_0$ . The  $\vec{E} \times \vec{B}$  coupling is found to be most efficient as in other cases of ion frequency modes.<sup>8</sup> The second-order density perturbation of electrons at the beat frequency  $\omega_{bk} = \omega_H - \omega_{Lk}$  is given by

$$f_b^{(2)} = \sum_k \frac{f_0}{i(\omega_{bk} - k_{\parallel}v_{\parallel})} \frac{e(\vec{k}_H \times \vec{k}) \cdot \hat{z}}{B_0 T_e} \phi_H \phi_{Lk}^*, \quad (1)$$

where  $T_e$  is the electron temperature;  $\hat{z}$  is the unit vector in the direction of the magnetic field  $\vec{B}_0$ ;  $\vec{k}$  and  $\phi$  are the wave number and the Fourier amplitude of the electrostatic potential, with subscripts  $H$  and  $L$  indicating the drift-cyclotron or DCLC mode and the low-frequency mode, respectively. With use of Eq. (1) and Poisson's equa-