quires a detailed knowledge of both the pion and proton structure function, and a prescription for how the scattering occurs. In the constituent-interchange model, for example, entirely different subprocesses may dominate.

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Unification of the Basic Particle Forces at a Mass Scale of Order $1000m_W$

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By considering a semisimple group [e.g., $SU(4)^4$] subject to discrete symmetries which ensure one coupling constant in the unification limit, we show that the unification mass scale need be no higher than $1000m_W$. The possible emergence of a light octet of *axial* color gluons is noted.

The hypothesis[1,2] that the fundamental particles and their interactions are unified at a basic level through a Lagrangian characterized by a single gauge coupling constant raises two important questions: (1) At what energy scale would this "complete" unification[3] (lost at low energies through spontaneous breaking of the symmetry) exhibit itself? (2) What is the value of the renormalized weak angle $\sin^2\theta_W$?

In attempting to answer these questions several authors[4-6] have claimed that the so-called superunifying mass scale M needs to be ultraheavy ($\gtrsim 10^{15}$ GeV) for the "strong" interactions to be strong at low energies. The purpose of this Letter is to show that there exist simple patterns of a hierarchical breakdown of a unifying group G {for example, the previously proposed semisimple group[7] $[SU(4)\otimes]^4$}, which permit unification at a mass scale ten to twenty orders of magnitude below previous estimates. This relatively low, unifying mass scale raises the exciting possibility that the unification hypothesis may be tested through ongoing cosmic-ray experiments and perhaps also with the next generation of accelerators. We obtain a value for renormalized $\sin^2\theta_W \approx \frac{2}{7}$ consistent with the value allowed presently by experiments.[8] Implications of possible lower values[9] of $\sin^2\theta_W$ ($\simeq \frac{1}{4}$) are also stated.

The reason why the unification mass M encountered by previous authors is so large may be traced to the following underlying special assumption: The embedding of the low-energy weak (G_W) and strong $[SU(3)_{\rm col}]$ groups within the unifying symmetry G is such that the gauge coupling constants associated with G_W and $SU(3)_{\rm col}$ are *equal* to each other in the (bare) symmetric limit.

The embeddings presented in this Letter permit a departure from this assumption—here the coupling constants of G_W are smaller (e.g., by a factor of $1/\sqrt{2}$) than SU(3)-color coupling. Correspondingly, the unification mass scale is found to be dramatically lower than those obtained in previous estimates.[10]

We now proceed to demonstrate the role which embedding plays in the determination of the unifying mass scale M by considering the unifying symmetry $[SU(4)\otimes]^4 \equiv SU(4)_A \otimes SU(4)_B \otimes SU(4)_{C}' \otimes SU(4)_D'$. The discrete symmetries $A \leftrightarrow B \leftrightarrow C \leftrightarrow D$ ensure that the theory starts with one basic coupling constant. For notational purposes, we recall the following salient features of this symmetry.[7] The symmetry operates on a set of 4×4 four-component "*basis*" *fermions* $F_{L,R}$ possessing four flavors (u,d,s,c) and four colors, plus an analogous set of *mirror fermions* $F_{L,R}{}^m$ needed for the cancellation of anomalies. There are two possible ways to gauge the fermions: (I) *(Chiral flavor* $\otimes$ *chiral color) gauging:* Assume that F_L and F_R^m transform as $(\underline{4},\underline{1};\underline{4}^*,\underline{1})$, while F_R and F_L^m as $(\underline{1},\underline{4};\underline{1},\underline{4}^*)$ under $[\overline{\mathrm{SU}(4)\otimes}]^4$. Here A and B refer to chiral flavor, and C and D to chiral color gauging. (II) (*Chiral flavor ⊗ vectorial color) gauging:* Alternatively assume[11] $F_L=(\underline{4},\underline{1};\underline{4}^*,\underline{1})$, $F_R^m=(\underline{4},\underline{1};\underline{1},\underline{4}^*)$, $F_R=(\underline{1},\underline{4};\underline{4}^*,\underline{1})$, and $F_L^m=(\underline{1},\underline{4};\underline{1},\underline{4}^*)$. Here the subscripts A and B refer to chiral flavor (as in Case I), but C and D refer to *vectorial color gauging* of (F) and (F^m), respectively.

We shall assume (first) for simplicity that the symmetry $[\mathrm{SU}(4)\otimes]^4$ descends spontaneously at the primary stage of symmetry breaking through a single superheavy mass scale M ($\gg m_W$) to a "low-energy" symmetry having the form $G_W \otimes G_{\mathrm{col}}$, where G_W is either (i) the left-right–asymmetric $G_L \equiv \mathrm{SU}(2)_A \otimes \mathrm{U}(1)$, or (ii) the left-right–symmetric $G_{LR} \equiv \mathrm{SU}(2)_A \otimes \mathrm{SU}(2)_B \otimes \mathrm{U}(1)_{C+D}$. G_{col} is specified below, while $\mathrm{SU}(2)_A$ $[\mathrm{SU}(2)_B]$ is the *Glashow-Iliopoulos-Maiani–diagonal* subgroup of of $\mathrm{SU}(2)_A{}^{\mathrm{I}}$ and $\mathrm{SU}(2)_A{}^{\mathrm{II}}$ operating on the doublets $(u,d)_L$ and $(c,s)_L$, respectively [similarly for $\mathrm{SU}(2)_B$]. $\mathrm{U}(1)_{C+D}$ is the vector-diagonal sum[12] of $\mathrm{U}(1)_C$ and $\mathrm{U}(1)_D$ representing the fifteenth generator of $\mathrm{SU}(4)_C{}'$ and $\mathrm{SU}(4)_D{}'$, respectively. The $\mathrm{U}(1)$ generator in G_L is a normalized linear combination of $\mathrm{U}(1)_{C+D}$ and the neutral generator of $\mathrm{SU}(2)_B$. Self-consistent generator normalization[4,5] requires that the bare-coupling constant g_2 [associated with $\mathrm{SU}(2)_{A,B}$] be related to the coupling constant g_G of the unifying group $[\mathrm{SU}(4)\otimes]^4$ by $g_2 = g_G/\sqrt{2}$. For both G_L and G_{LR}, the $\mathrm{U}(1)$ coupling constant g_1 is also related to the unifying-group coupling constant g_G by $g_1 = g_G/\sqrt{2}$.

Renormalization in the weak sector (G_W).—Using familiar decoupling-theorem arguments,[4,13] the running coupling constants $g_{1,2}(\mu)$ for momenta $\mu \le M$ are given by

$$g_i{}^{-2}(\mu) = 2g_G{}^{-2}(M) + 2b_i \ln(M/\mu) \qquad (\text{for } i=1,2), \tag{1}$$

where $b_2 = -22/[3(4\pi)^2] + b_1$. We shall first consider the weak group G_W to be $G_L = \mathrm{SU}(2)_A \otimes \mathrm{U}(1)$ and assume that quarks are fractionally charged. (The results for the alternative cases of $G_W = G_{LR}$ and integral-charge quarks are given later.) The electromagnetic coupling constant is related to g_1 and g_2 by[4,5] $e^{-2} = g_2{}^{-2} + \frac{5}{3} g_1{}^{-2}$; and the weak angle is given by $\sin^2\theta_W = 3g_1{}^2/(3g_1{}^2 + 5g_2{}^2)$. We use these expressions together with Eq. (1) to obtain

$$g_2{}^{-2} - g_1{}^{-2} = (8\sin^2\theta_W - 3)/5e^2 = -(11/12\pi^2)\ln(M/\mu). \tag{2}$$

Emergence of chromodynamics.—Assume that the basic four-color symmetry $G_{CD} \equiv \mathrm{SU}(4)_C{}' \otimes \mathrm{SU}(4)_D{}'$ breaks through a heavy mass scale M ($\gg m_W$) either directly into the three-color diagonal-sum symmetry $\mathrm{SU}(3)_{C+D}{}' \otimes \mathrm{U}(1)_{C+D}$ (case 1) or into the "split" color symmetry $\mathrm{SU}(3)_C{}' \otimes \mathrm{SU}(3)_D{}' \otimes \mathrm{U}(1)_{C+D}$ (case 2) [the latter subsequently breaks into $\mathrm{SU}(3)_{C+D} \otimes \mathrm{U}(1)_{C+D}$ through a *relatively light mass scale* M' $(0 \le M' \lesssim m_W)$]. Consider for both cases the effect of primary symmetry breaking through the heavy mass M:

$$G_{CD} \xrightarrow{M} \mathrm{SU}(3)_{C+D}{}' \otimes \mathrm{U}(1)_{C+D} \quad (\text{case 1}); \tag{3a}$$

$$G_{CD} \xrightarrow{M} \mathrm{SU}(3)_C{}' \otimes \mathrm{SU}(3)_D{}' \otimes \mathrm{U}(1)_{C+D} \quad (\text{case 2}). \tag{3b}$$

Case (1) (the conventional case).—In this case the chromodynamic coupling constant g_S, associated with the vectorial-*diagonal* symmetry $\mathrm{SU}(3)_{C+D}{}'$, is related to the unifying-group constant g_G by $g_S = g_G/\sqrt{2}$ (note the *conventional*[4-6] symmetric-limit equality between g_S and the weak-coupling constants $g_{1,2}$). The renormalization-group equation for $g_S(\mu)$ for $\mu \le M$ reads

$$g_S{}^{-2}(\mu) = 2g_G{}^{-2}(M) + 2b_3 \ln(M/\mu), \tag{4}$$

where $b_3 = -11/(4\pi)^2 + b_1$. Using Eqs. (2) and (4), with $u_S \equiv g_S{}^2/e^2 = \alpha_s/\alpha$, we find that $g_2{}^{-2} + \frac{5}{3}g_1{}^{-2} - \frac{8}{3}g_S{}^{-2} = e^{-2}(1 - 8/3u_S) = (11/4\pi^2)\ln(M/\mu)$. Combining this expression with Eq. (2), we find

$$u_S = \tfrac{8}{3}[1 - (11\alpha/\pi)\ln(M/\mu)]^{-1}, \tag{5}$$

$$\sin^2\theta_W = \tfrac{1}{6} + 5/9u_S. \tag{6}$$

Case (2) (split color).—For simplicity, first assume that the split color symmetry $\mathrm{SU}(3)_C{}' \otimes \mathrm{SU}(3)_D{}'$ is broken, if at all, only *softly* into its diagonal sum $\mathrm{SU}(3)_{C+D}{}'$, by a mass scale M' (~1 GeV), low compared to electroproduction vertex momenta. [Note that at least a soft breaking of split color symmetry is necessary, if color gauging is chiral (pattern I).] With such soft symmetry breaking, electroproduction would still be described by an effective strong-interaction coupling constant $g_S(\mathrm{QCD}) = g_C = g_D = g_G$ (in the symmetric limit; here, QCD stands for quantum chromodynamics), to be contrasted with $g_1 = g_2 = g_G/\sqrt{2}$ (in the same limit). Hence, g_S satisfies

$$g_S^{-2}(\mu)=g_C^{-2}(\mu)=g_G^{-2}(M)+2b_3'\ln(M/\mu), \qquad (7)$$

where[14] $b_3'=11/(4\pi)^2+\frac{1}{2}b_1$. Using Eqs. (2) and (8) we find that $g_2^{-2}+\frac{5}{3}g_1^{-2}-\frac{16}{3}g_S^{-2}=e^{-2}(1-16/3u_S)=(77/12\pi^2)\ln(M/\mu)$ in which case

$$u_S=2\times\tfrac{8}{3}[1-(77\alpha/3\pi)\ln(M/\mu]^{-1}, \qquad (8)$$

$$\sin^2\theta_W=\tfrac{2}{7}+10/21u_S. \qquad (9)$$

Note the crucial distinctions between the coefficients of $\ln(M/\mu)$ in Eqs. (5) and (8).

The results for case (1) [Eqs. (5) and (6)] correspond to the conventional results[4,5] for embedding $SU(2)_A\otimes U(1)\otimes SU(3)_{\rm col}'$ in a simple group. Setting $\mu\approx 3$ GeV we obtain (in this case) the standard result[4] that the unification mass scale M needs to be ultraheavy ($\approx 6.5\times10^{15}$ GeV) for $\alpha_s(\mu=3\text{ GeV})$ to equal ~ 0.2, as suggested by QCD fits to electroproduction data. For case (2), we see from Eq. (8) that $M\approx 2\times10^6$ GeV for $\alpha_s(\mu=3\text{ GeV})\approx 0.2$. This is the low unification mass scale advertised in the beginning of this Letter.

The origin of this enormous difference between the unifying mass scale for case (1) and case (2) is easy to trace. The difference stems from the *dichotomy* between the manner of descent of the low-energy flavor versus low-energy color symmetries. For case (1), both the flavor $SU(2)_A^{I+II}$ and the color $SU(3)_{C+D}$ descend through "diagonal summing" of relevant gauges with the same heavy mass scale M. Contrast this with case (2), where flavor $SU(2)_A^{I+II}$ descends through *diagonal summing,* as in case (1), but where color $SU(3)_C'\otimes SU(3)_D'$ descends *without diagonal summing to* low energies ($0\leq M'\lesssim m_W$). *This differential color-flavor gauge descent is the key to low-mass unification.*[15]

Case (3)—We not consider the possibility that the split symmetry $SU(3)_C'\otimes SU(3)_D'$ breaks at the *secondary step* of symmetry breaking by a moderately heavy mass scale $M'\sim m_W$ to the diagonal sum $SU(3)_{C+D}'$. In this case, known chromodynamics is generated essentially by the diagonal sum of $SU(3)_C'$ and $SU(3)_D'$.[16] Thus $g_S=g_C/\sqrt{2}=g_G/\sqrt{2}$ (in the symmetric limit) as in case (1), but now $\frac{1}{2}g_S^{-2}(\mu)=g_C^{-2}(\mu)=g_D^{-2}(\mu)=g_G^{-2}(M)+2b_3'\ln(M/\mu)$. Comparing with Eqs. (7)–(9), we see that the present case is analogous to case (2) except that u_S is replaced by $2u_S$; correspondingly, the unification mass scale is $\sim10^7$ GeV.

It should be stressed that case (2), involving only a soft breaking of split color symmetry presents the novel possibility of the existence of *light* color-octet gluons (mass $\sim$ 1–2 GeV) in addition to the massless vector QCD octet. These would be *axial color gluons* if the color gauging $SU(4)_C'\otimes SU(4)_D'$ is chiral.

We have presented results for cases (1)–(3) by assuming that the weak and electromagnetic symmetry G_W is $SU(2)_A\otimes U(1)$. The essential features of our results are unaltered for the left-right-symmetric choice $G_{W'}=G_{LR}=SU(2)_A\otimes SU(2)_B\otimes U(1)_{C+D}$, for which the corresponding results are

$$u_S=\tfrac{8}{3}\kappa[1-\rho(22\alpha/3\pi)\ln M/\mu]^{-1}; \qquad (10)$$

$$\sin^2\theta_W=\tfrac{1}{4}+1/3u_S,\quad \kappa=1,\ \rho=1\quad\text{(case 1)}; \qquad (11)$$

$$\sin^2\theta_W=\tfrac{1}{3}+\kappa/9u_S,\quad \kappa=2,\ \rho=3\quad\text{(case 2)}; \qquad (12)$$

$$\sin^2\theta_W=\tfrac{1}{3}+\kappa/9u_S,\quad \kappa=1,\ \rho=3\quad\text{(case 3)}. \qquad (13)$$

Analogous considerations apply to the case of integral-charge quarks. Values of $\sin^2\theta_W$ for this case are found to be the same in the large-u_S limit as those for fractionally charged quarks for a given G_W. The only change (relevant for the estimate of M) is that the factor $\frac{8}{3}$ on the right-hand side of Eqs. (5), (8), and (10) is replaced by 4. This permits unification at mass scale M as low as about 10^5 GeV for $\alpha_s(\mu=3\text{ GeV})\approx 0.2$. Such a mass scale coincides with the estimate of lepton-quark X-gauge-meson masses obtained in Ref. 1 from independent considerations. This in turn strengthens the compatibility of the hypothesis of unconfined unstable integral-charge quarks within a *unified* theory.

The hypothesis of low-mass unification suggests that vector chromodynamics may be supplemented by axial chromodynamics mediated by light-mass axial gluons [case (4)]. It needs to be examined whether the existence of such light axial gluons would help remove some of the lingering discrepancies[17] between QCD and observed charmonium physics.

We have assumed for simplicity that all four SU(4) sectors (A,B,C,D) are broken by one universal mass scale M to their low-energy components.[18] This leads to the result $\sin^2\theta_W\approx0.3$. If we permit the mass scales M_A, M_B, and M_C and M_D associated with the primary breakdown of flavor $SU(4)_{A,B}$, and color $SU(4)_C'$ and $SU(4)_D'$ to be *different* from each other, the embedding presented in this Letter would permit a lowering of $\sin^2\theta_W$ to about $\frac{1}{4}$. In this case, one of the mass-scales (M_C or M_D) needs to be heavy ($\gtrsim10^{14}$ GeV) while the other two could be relatively light ($\sim10^4$ GeV

A low *universal* unifying mass scale $M\approx10^5$–10^6

GeV raises the hope that perhaps only one mass scale triggers spontaneous symmetry breaking with radiative corrections[19] of order α and α^2, providing mass to the lighter gauge bosons (such as the familiar weak intermediate bosons W's).

To conclude, we have shown that embedding plays a dramatic role in the determination of the unifying mass scale M. Discrete symmetries, for example within the semisimple unifying group $[SU(4)\otimes]^4$, together with the embedding mechanism presented here, play a potent role in unifying the basic particle forces at a mass scale of order $1000\,m_W$, *in sharp contrast to all previous estimates which place this mass scale ten orders of magnitude higher.* This low unifying mass scale is amenable to study by cosmic-ray experiments and hopefully also by experiments at accelerators to be built in the future.

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[11]This pattern is consistent with equality of the four SU(4) coupling constants provided fermions are massless. Such a possibility can therefore, be entertained, if fermions acquire mass through dynamical spontaneous breakdown of the symmetry. We present pattern II, for its novelty; our considerations, however, would be centered around the more "conventional" pattern of chiral color gauging (Ref. 7), for which fermions can acquire mass through the vacuum expectation value of the flavor-color– and left-right–symmetric scalar multiplet $(4,\underline{4}^*,\underline{4}^*,4)$.

[12]Our results here do not change if $U(1)_C\otimes U(1)_D$ remains a good low-energy symmetry. The important phenomenological consequences of this possibility permitting a light neutral Z^0 in the mass range 25–40 GeV will be considered separately.

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[14]The light-multiplet contribution (due to fermion multiplets F and F^m as well as any light scalar field multiplets) to $g_C^{-2}(\mu)$ is $b_1 \ln(M/\mu)$.

[15]Our recipe for low-mass unification is to assume that weak (G_W) and strong ($G_{\rm col}$) symmetries descend from G such that the bare-coupling constants of G_W are smaller than that of $G_{\rm col}$ by an embedding factor $\delta<1$; for case (2) described in the text, $\delta=1/\sqrt{2}$. *Note that such a differential descent is* not *permissible within the unifying groups SU(5) and SO(10) (Ref. 2).*

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