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Linearity of $1/f$ Noise Mechanisms

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Measurements of statistical quantities other than the spectrum can distinguish between different $1/f$ noise mechanisms. One such quantity, the average behavior before and after a given fluctuation amplitude, is used to determine if the noise mechanism is linear. Measurements on different sources show that $1/f$ noise in some systems, such as carbon resistors and field-effect transistors, is due to a linear mechanism, while other systems, such as p-n junction devices, require a nonlinear mechanism.

$1/f$ noise represents the fluctuations of some physical quantity (usually resistance) about its steady-state value. It is found in a wide variety of quite different systems,[1] and in some cases has been shown to be an equilibrium property.[2] A $1/f$ spectral density does not, however, uniquely define a random process. It is important, therefore, to determine experimentally to what extent $1/f$ noise is universal, either in terms of physical mechanism or the underlying mathematics. Previous attempts have been made to characterize better $1/f$ noise by measuring such quantities as amplitude distributions,[3] variance fluctuations,[4] or zero-crossing statistics,[5] but the results of different researchers often show greater variation than those on different physical systems. In this Letter it is shown that a new statistical measurement can differentiate between linear and nonlinear noise mechanisms, and between the $1/f$ noise in different physical systems.

The theoretical difficulty of achieving a $1/f$ spectrum for the fluctuations in simple linear systems (other than as an *ad hoc* distribution of time constants) has led some researchers to propose nonlinear models for the origin of $1/f$ noise.[6,7] Measurements of the spectrum and the amplitude distribution, however, have been unable to determine whether or not nonlinear effects are important.[3] The observed Gaussian distributions are possible for both linear and nonlinear processes.[8] Although all systems may be expected to shown some nonlinearity for perturbations sufficiently far from the steady state, it is important to determine if nonlinearities affect the statistical properties of the spontaneous fluctuations. For nonlinearities to be important, a fluctuation far from the steady-state value should, on the average, have a different behavior in time from a fluctuation that is close to the steady-state value. The measured average behavior before and after a given fluctuation as a function of the amplitude of that fluctuation can be used to determine if the noise mechanism is linear.

Specifically, a stationary fluctuating quantity, $V(t)$, with zero average may be divided into an ensemble of samples each of length $2T$. Each member of the ensemble represents a possible $V(t)$ from time $-T$ to time T. Consider those members of the ensemble for which $V(0)$ is in some range about a specific value V_0. The average time behavior over this subset of the ensemble, $\langle V(t) \mid V(0)=V_0\rangle$, is then the average behavior before and after a fluctuation of amplitude V_0. $\langle V(t) \mid V(0)=V_0\rangle \to V_0$ as $t\to 0$ and $\langle V(t) \mid V(0)=V_0\rangle \to 0$ as $t\to\pm\infty$. For a linear system, $\langle V(t) \mid V(0)=V_0\rangle/V_0$ is independent of V_0 and proportional to the autocorrelation function $\langle V(0)V(t)\rangle$. For nonlinear systems, $\langle V(t)\,|\,V(0)=V_0\rangle/V_0$ is a function of V_0.

According to Onsager,[9] $\langle V(t)\,|\,V(0)=V_0\rangle$ is the same as the macroscopic response to a perturbation such that $V(0)=V_0$. Thus, the linearity could also be determined from the response to different external perturbations of the same order of magnitude as the fluctuations. The major difficulty with this procedure is that it may be possible to produce the same perturbation by different methods. For example, the resistance is not a fundamental thermodynamic variable but is a function of other quantities such as temperature or carrier density, that may be externally perturbed. However, comparing the response to specific perturbations with the average response of the fluctuations can be used to determine which quantities are important to the $1/f$ noise.

Measurements on five different noise sources are reported here. Most of them produced what is generally considered $1/f$ noise at a level well above amplifier background. Source A was the current noise near threshold in a commercial MOSFET (metal-oxide-semiconductor field-effect transistor) (2N4351). Source B was the voltage fluctuations across a 1-MΩ carbon resistor with $\langle V\rangle = 45$ V. Source C was the current noise in the base-collector junction of a reverse-biased commercial n-p-n Si transistor (2N498). Source D was the voltage fluctuations of the output of a Si n-p-n transistor configured as a common-emitter amplifier. Source E was the reverse-biased current noise in a commercial p-n diode (1N4818). Source E differed from the others in that it showed a large amount of burst noise.

Experimentally, the desired noise voltage, $V(t)$, was amplified, bandpass limited to the range 0.03 Hz to 5 kHz, and digitally sampled every 200 μsec by an analog-to-digital converter attached to an IBM System-7 computer. The 0.03-Hz high-pass filter assured fluctuations with zero average, while the 5-kHz low-pass filter reduced aliasing. A consecutive string of $2N+1$ of these digitized noise values, $\{V_n\}$, where n varies from $-N$ to N, represents a quantized member of the ensemble of $V(t)$ from time $-T$ to time T ($T=200N$ μsec). Each of the digitized values was scaled by the same amount to produce integer v_n in a small range about zero (usually -75 to 75). $\langle v_n\,|\,v_0=V_0\rangle$ is then measured by averaging all members of the ensemble, $\{v_n\}$, that have the same V_0. This process is demonstrated in Fig. 1. Figure 1(a) shows five members of the ensemble,

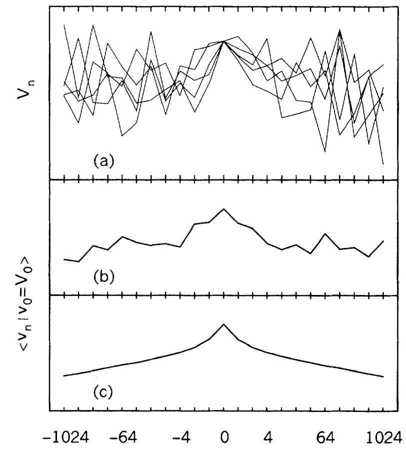

FIG. 1. (a) Five samples of a noise voltage vs delay n with a specified value at $n=0$; (b) average of sixteen such samples; (c) average of 32 768 samples.

$\{v_n\}$, that have the same value at $n=0$. Note that not all of the digitized values are shown for each member of the ensemble but only those for which $n=0$, ±1, ±2, ±4, ±8, ±16, ±32, ±64, ±128, ±256, ±512, and ±1024. This choice of delay times gives roughly a logarithmic time scale that is appropriate for the $1/f$ noise autocorrelation function. Figures 1(b) and 1(c) show how $\langle v_n\,|\,v_0=V_0\rangle$ forms a smooth curve as successive members of the ensemble having the same V_0 are averaged. This smooth curve is then the average manner in which a fluctuation of amplitude V_0 builds up and decays in time.

To determine the linearity of the noise mechanism, $\langle v_n\,|\,v_0=V_0\rangle$ was measured for each of the values of V_0 and the shapes compared. In practice, $\langle v_n\,|\,v_0=V_0\rangle$ was accumulated for all values of V_0 simultaneously. The probability distribution of possible values for v_n was also accumulated. Experiments ran for up to 4 days with at least 10^8 members being required of the ensemble for a good average.

Figure 2 shows the measured spectral density for each of the noise sources with the 0.03-Hz high-pass filter. The spectra all show a general power-law behavior. Individual differences are, however, noticeable. The carbon resistor (B) remains closest to the exact $1/f$ law. The FET (field-effect transistor) (A) and the base-collector junction (C) both show slight flattening of the spectra at higher frequencies, while the burst noise (E) is significantly steeper than $1/f$.

Figure 3 shows the measured amplitude distri-

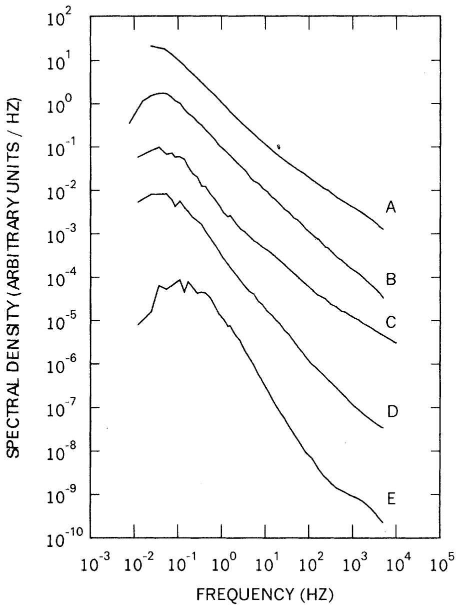

FIG. 2. Spectral densities for the five noise sources. The spectra have been offset for display.

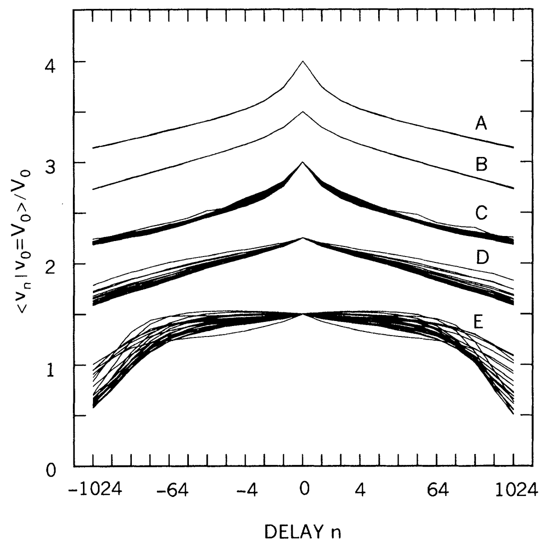

FIG. 4. Superposition of $\langle v_n|v_0=V_0\rangle/V_0$ vs delay n for all V_0.

butions for each of the five noise sources. The log of the probability of occurrence is plotted versus X^2, where $X=\Delta V/V_{\rm rms}$. Negative values correspond to $\Delta V<0$. Sources A and B show exact Gaussian distributions out to amplitudes having a probability of occurrence 10^6 times smaller than the average. Source C shows a very slight departure from the Gaussian distribution for large positive fluctuations. Source D shows non-Gaussian tails, while the burst noise (E) shows two distinct peaks.

Figure 4 shows the superposition of $\langle v_n|v_0=V_0\rangle/V_0$ vs delay n for all V_0 for the noise sources. Only values with small V_0 or too few members to give a meaningful average were eliminated. The delay times used are the same as in Fig. 1. For linear systems each of the $\langle v_n|v_0=V_0\rangle/V_0$ will fall on the same curve. This is the case for the FET (A) and the carbon resistor (B). For nonlinear systems the $\langle v_n|v_0=V_0\rangle/V_0$ will not superimpose. This is the case for the n-p-n transistor (C) and (D) and the burst noise (E).

Figure 5 shows $\langle v_0|v_0=V_0\rangle/V_0$ vs V_0 for the noise sources at a delay of $n=\pm16$ (±3.2 msec). Similar plots are possible for the other delays. As in Fig. 4, the differences between linear and

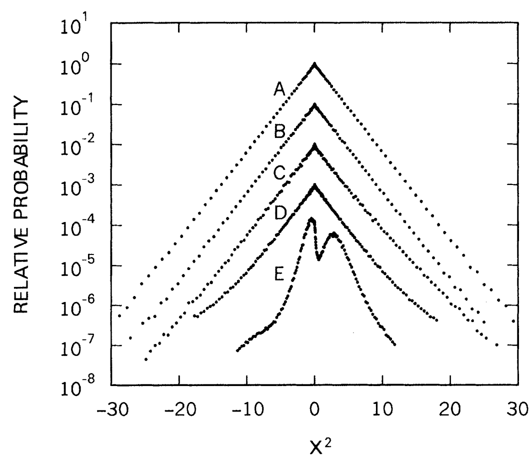

FIG. 3. Relative probability of occurrence vs X^2, where $X=\Delta V/V_{\rm rms}$. Successive distributions have been offset by one decade.

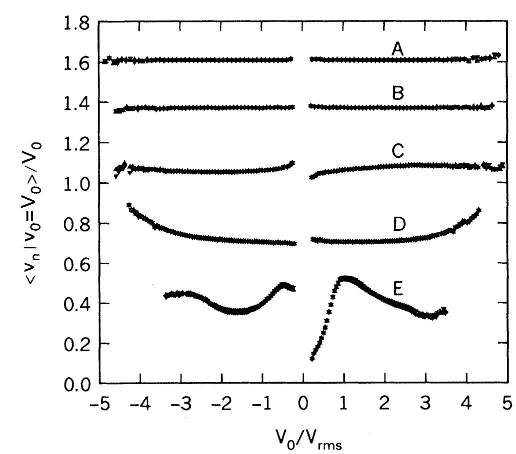

FIG. 5. $\langle v_n|v_0=V_0\rangle/V_0$ vs $V_0/V_{\rm rms}$ for delays of $n=16$ (▲) and $n=-16$ (▼).

nonlinear systems are apparent. Once again A and B, which are independent of V_0, are characteristic of linear systems, while C, D, and E show the dependence on V_0 of a nonlinear system. Although A and B, the linear systems, show the spectra closest to an exact $1/f$ law, preliminary results indicate that this correspondence is not preserved in other systems.

It is interesting to note that in Figs. 4 and 5 there is no significant difference between positive and negative delays even for the systems that show nonlinearities. As far as this statistical test is concerned, all of the $1/f$ noise sources appear time reversible. One might have expected otherwise if the noise mechanism was a causal response to a random input.[10]

In summary, the experimental study of statistical quantities other than the spectrum can be of great help in suggesting or eliminating possible general mechanisms for the noise. An initial study of one such quantity, $\langle V(t) | V(0) = V_0 \rangle$, for different noise sources shows that not all $1/f$ noise is the same. In certain classes of systems (semiconductor p-n junction devices) nonlinear mechanisms play an important role in the spontaneous fluctuations. Other systems (carbon resistors and FET's) seem to be adequately described by linear mechanisms. A single physical theory cannot account for both types.

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