tons from the $Z=50$ closed shell are now being thoroughly mapped over a large region of Z and N, a complete theoretical understanding is required for the coexistence of this unusual collective feature.

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Two-Dimensional Integral-Equation Solution of the Four-Nucleon System

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The ^{4}He binding energy and the n-^{3}H and p-^{3}H phase shifts below the $d+d$ threshold are calculated from Alt-Grassberger-Sandhas equations taking into account the full subamplitudes. As two-body interaction a separable s-wave spin-dependent potential is used.

Recent results of four-nucleon calculations based on the integral-equation approach have usually been achieved by means of separable expansion of the $1+3$ and $2+2$ subamplitudes in the integral kernel, thus reducing the original four-body equations to one-dimensional effective two-body equations.[1-8] Using separable two-body potentials but avoiding the expansion of the three-body subamplitudes one is left with *two*-dimensional integral equations.

On this basis we have calculated the ^{4}He binding energy[9] and four-nucleon phase shifts below the $d+d$ threshold. As two-nucleon interaction we have chosen a separable potential consisting of s-wave Gaussian form factors in the deuteron and antibound-state channel. Compared with the conventional Yamaguchi potential, which also could be handled, the Gaussian choice was preferred since it yields better low-energy three-body data[10] with parameters adjusted to deuteron binding energy, singlet and triplet scattering lengths, and singlet effective range. Moreover, there is indication that the Gaussian potential works well also in the four-body case.[8]

In Fig. 1 the $1+3$ and $2+2$ channels are illustrated for the special case of a (3, 4) cluster symbolized by a small circle and described by the s-wave form factors used in this paper. The sub-

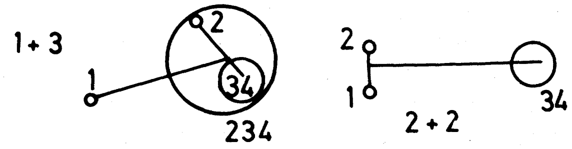

FIG. 1. Relative motions in $1+3$ and $2+2$ intermediate channels.

amplitudes depend on two further relative angular momenta. It is one of our main purposes to study the contributions of p waves in *both* these relative motions. We mention that in the $2+2$ case only the s wave between the two free particles (1 and 2 in Fig. 1) contributes to the corresponding amplitude in the integral kernel after symmetrization. For total angular momenta 0 or 1, eighteen angular-momentum-spin-isospin channels are coupled in the fully symmetrized resulting equations.

The numerical solution of this highly coupled set of integral equations was accomplished with the Padé method which turned out to be most powerful for this kind of problem.[11] We have applied this technique twice, namely for calculating the $1+3$ and $2+2$ subamplitudes, and, on the other hand, for calculating the four-body amplitudes. In most cases sufficient accuracy was reached with the [5,5] approximant.

We confined our investigation to energies below the $d+d$ threshold. Even in this region the numerical effort increases rapidly with energy. Therefore, only the $L=1$ phase shifts shown in

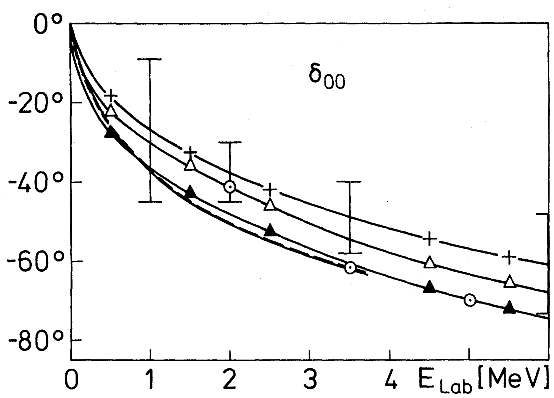

FIG. 2. Singlet s-wave phase shifts for n-^{3}H. The full line corresponds to a purely s-wave $1+3$ subamplitude while the dashed line includes the p waves. Resonating-group results are from Ref. 12 (△) and Ref. 13 (▲); integral-equation results are from Ref. 6 (⊙). The experimental points are taken from Ref. 14 (+).

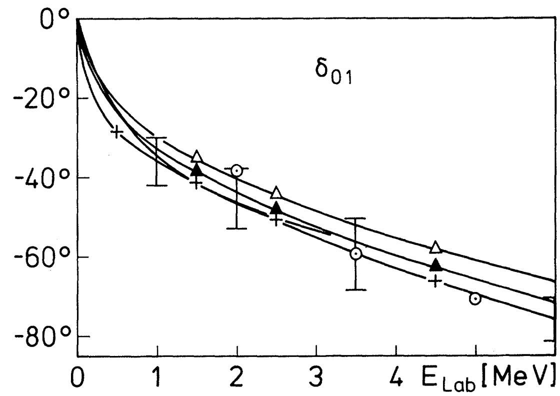

FIG. 3. The same as Fig. 2 in the triplet s-wave channel.

Fig. 5 were extended up to the $d+d$ threshold. The $1+3$ subamplitude contains in the cited energy region only the triton pole. For its position we got the value 8.99 MeV in relatively good agreement with Bakker's value[10] of 8.96 MeV for the same potential. In the $2+2$ subamplitude a pole shows up at 4.45 MeV in agreement with the expected value of twice the deuteron binding energy.

Numerical integration was performed introducing a set of ten Gaussian mesh points for all continuous variables. For explicit integration in the neighborhood of poles, standard subtraction techniques have been applied. In order to give some feeling for the numerical effort, we mention that about 10 h on an IBM 370/168 computer were needed for providing one point in Figs. 2–5.

We computed the ^{4}He ground-state binding energy and found 33.3 MeV if only s waves were in-

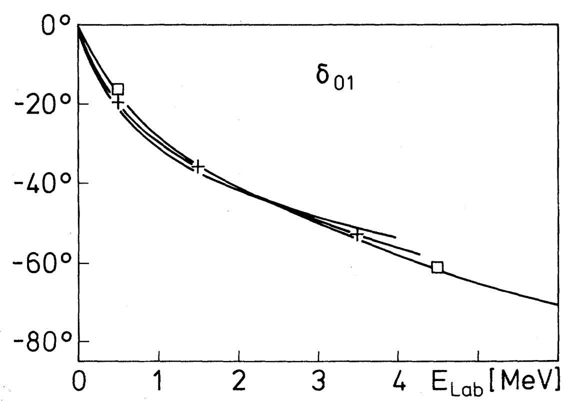

FIG. 4. Triplet s-wave phase shift for p-^{3}H. The full line corresponds to a purely s-wave contribution in the $1+3$ subamplitude. Variational results are given in Ref. 15 (□); experimental points are from Ref. 16 (+).

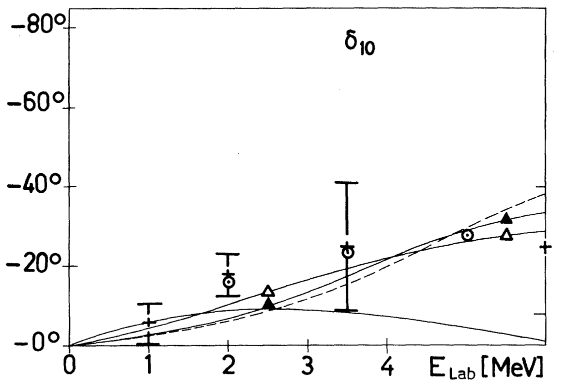

FIG. 5. The same as Fig. 2 in the singlet p-wave channel.

cluded in the three-body subsystem of the $1+3$ channel. Inclusion of p and also of d waves in this subsystem gives contributions of less than 0.1 MeV. s-wave phase shifts are given in Figs. 2–4, showing the typical shape of hard-sphere scattering. For the n-^{3}H singlet channel the influence of the $(1+3)$-subamplitude p waves was studied (Fig. 2). The deviation from the results obtained solely with s-wave subamplitudes turned out to be at most 1 deg. For this channel our numbers miss the error bars of the experiment.[14] The triplet channel of the same reaction seems to be less sensitive because all calculations[6,13] are close to each other. Note that in this channel we also achieve the best fit to experimental data (Fig. 3). Figure 4 shows the triplet channel of the p-^{3}H reaction, where the situation looks similar to the n-^{3}H case. Finally the n-^{3}H phase shift in the singlet p-wave channel is shown in Fig. 5. The $(1+3)$-subamplitude p wave gives here a remarkable contribution.

The above considerations demonstrate that the two-dimensional four-body equations are amenable to numerical solutions in the bound state and in the low-energy scattering region. For the α particle we found an overbinding related to the triton overbinding, in full agreement with the Gauss line in the Tjon plot of Ref. 8. The phase shifts obtained show qualitative agreement with experiment[14,16] and resonating-group calculations.[12,13,15] Of particular relevance is the good agreement with solutions of four-body integral equations using separable expansion of the subamplitudes.[6] Especially the effect of the subsystem p waves found by Tjon is reproduced. This agreement is, on the one hand, an additional justification of the conjecture[10] that the Gaussian separable potential used in our approach simulates rather well the Malfliet-Tjon potential used in Ref. 6. It is, moreover, a good hint on the convergence of the separable-expansion method. In this respect comparison of our results with expansion-method calculations, both for the Gaussian separable potential, would be desirable. Such calculations are under way.[17]

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