Deformed $\frac{9}{2}^+$ Proton-Hole States on Odd-A $^{119-125}$Cs

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$\Delta J = 1$ bands built on deformed $\frac{9}{2}^+$ proton-hole states have been observed in $^{119-125}$Cs. The $\frac{9}{2}^+$ bandhead energies drop phenomenally to become the ground state in ^{119}Cs($N = 64$). The band systematics for N(62–74) and Z(51–55) are defined. A deformed-structure relationship is suggested between the excitations of a $1g_{9/2}$ proton and a pair of $1g_{9/2}$ protons across the $Z = 50$ closed shell.

The coexistence of strongly deformed states along with the more spherical states in nuclei just above the $Z = 50$ closed proton shell has generated considerable theoretical and experimental interest in the nature of the collectivity in this region. Of particular interest are the $\Delta J = 1$ bands built on low-lying deformed $\frac{9}{2}^+$ states, which have been observed in five odd-A Sb($Z = 51$) nuclei[1-3] and in six odd-A I($Z = 53$) nuclei.[4,5] These $\frac{9}{2}^+$ bandheads are believed to result from the excitation of a $1g_{9/2}$ proton across the $Z = 50$ closed shell. This structure has been implied for the Sb isotopes by $L = 4$ Te(t, α) pickup strength[6] and by $L = 0$ In(^{3}He,n) transfer.[7] The excitation energies of such states would be expected at ~3–4 MeV for a spherical system but instead have been observed to follow for each isotope a parabolalike curve as a function of neutron number N with minima at surprisingly low energies near the middle of the neutron shell [950 keV in ^{121}Sb($N = 70$) and 307 keV in ^{119}I($N = 66$)]. This unexpected drop in energy has been attributed[5,8] to the deformation of the core. In the present study, a search has been made for these $\frac{9}{2}^+$ proton-hole states in Cs($Z = 55$) nuclei; the $\Delta J = 1$ bands were observed in four isotopes with the $\frac{9}{2}^+$ excitation energies dropping phenomenally to become the ground state in ^{119}Cs($N = 64$).

The ability of a $g_{9/2}$ proton excited across the $Z = 50$ closed shell to induce large deformations is documented by the systematics of the $\frac{9}{2}^+$ $\Delta J = 1$ bands as a function of N(62–74) and Z(51–55). These systematics imply deformations which achieve a maximum near the middle of the $N = 50$–82 neutron shell and increase with Z. The present odd-A Cs study defines the Z dependence of the systematics in that the deformation of these $\frac{9}{2}^+$ proton-hole states in the Cs nuclei has become dominant in the determination of their energies. A deformed-structure relationship between the excitation of a $1g_{9/2}$ proton and the excitation of a pair of $1g_{9/2}$ protons has recently been made. Low-lying deformed 0_2^+ states in even Sn($Z = 50$) nuclei have been identified by the observation of their rotational bands via (α, $2n\gamma$) studies.[9] These 0_2^+ states have a large $L = 0$ Cd(^{3}He,n) strength[10] implying a significant $(1g_{9/2})^{-2}$ proton component. The deformations extracted from these bands have β greater than or approximately equal to those for the $\frac{9}{2}^+$ bands in the neighboring Sb nuclei, and the excitation-energy parabolas for the 0_2^+ Sn and $\frac{9}{2}^+$ Sb states have similar shapes and minima. An intriguing aspect of these Sn 0_2^+ states is their significant population strength recently observed via Te(d, ^{6}Li) α-pickup measurements.[11] The systematics for the deformed $\frac{9}{2}^+$ proton-hole states established out to the Cs($Z = 55$) nuclei predict, by the above relationship, the coexistence of low-lying deformed 0^+ (proton-hole pair) states in the even Te($Z = 52$) and Xe($Z = 54$) nuclei. These states have not been identified. A detailed mapping of the properties of the $1g_{9/2}$ proton excitation is theoretically important for the understanding of their potential energy surfaces.

To study the collective excitations in the odd-A Cs isotopes, several fusion-evaporation reactions were employed using ^{6}Li, ^{10}B, ^{14}N, and ^{16}O heavy-ion beams along with isotopically enriched self-supporting targets. The following reactions were used to populate the specific $^{119-133}$Cs residual nuclei: ^{106}Cd(^{16}O,$p2n$)^{119}Cs; ^{110,112}Cd(^{14}N, $3n$)^{121,123}Cs; $^{116-124}$Sn(^{10}B, $3n$)$^{123-131}$Cs; and $^{126-130}$Te(^{6}Li, $3n$)$^{129-133}$Cs. These reactions favor population with large alignment of high-spin states whose dominant decay modes are via stretched γ-ray cascades. In order to determine the level structure of these nuclei, the following set of γ-ray measurements using Ge(Li) detectors were performed: γ excitation, γ-γ coincidence, and γ angular distribution $W(\theta)$. The details of the experimental techniques have been described earlier.[12] The excitation measurements taken over an energy range from the Coulomb barrier to 34, 51, 59, and 75 MeV for ^{6}Li, ^{10}B, ^{14}N, and ^{16}O beams, respectively, were compared with reaction calculations[13] as a means of selecting the optimal beam energies and identifying the odd-A Cs reaction channels. The γ-γ coincidence measurements were used to determine the γ-ray cascades and to establish the identity of the residual nuclei via the connection of unknown γ rays with those known from previous β-decay work. To obtain information on the spins of the levels and the γ-ray intensities I_γ, γ-ray $W(\theta)$ were measured in singles at four angles. Spin assignments were made on the basis of the $W(\theta)$, lifetime, and I_γ results.[12]

The γ-ray cascades deduced from these odd-A nuclei have a considerable similarity; three different bands of levels were observed with near repetition in several of the Cs isotopes. The first involves stretched $J\to J-2$ quadrupole ($E2$) transitions that end on low-lying $\frac{11}{2}^-$ isomers. They were extracted from all of the odd-A $^{119-133}$Cs nuclei implying $\Delta J=2$ bands built on the $1h_{11/2}$ quasi-proton states. The level spacings for these bands are similar to those for the even Xe core nuclei, a feature consistent with decoupled bands[14] based on the $1h_{11/2}$ orbital. The second type were similar $\Delta J=2$ bands built on the $1g_{7/2}$ quasi-proton states which were observed in the $^{125-133}$Cs isotopes. The third type of band involved $J\to J-1$ ($M1-E2$) γ-ray cascades which are corroborated by the existence of $E2$ crossover transitions. These $\Delta J=1$ bands built on $\frac{9}{2}^+$ states were observed in the $^{119-125}$Cs isotopes as shown in Fig. 1.

For ^{123,125}Cs, the energies and the $\frac{9}{2}^+$ assignments for these bandheads were deduced from the γ-γ coincidence and the $W(\theta)$ data, respectively, for the decay γ rays. The $\frac{9}{2}^+$ assignments for the ^{119}Cs ground state and the ^{121}Cs β-decaying isomer have been determined recently by atomic-beam measurements.[15] The observed $\Delta J=1$ bands in ^{119,121}Cs were associated with the $\frac{9}{2}^+$ states as bandheads since no decay γ rays were found in coincidence with the $J\to J-1$ cascades and because of the $\Delta J=1$ band systematics. These $\Delta J=1$ bands define the nuclear structure which is the focus of the present Letter. The $\frac{9}{2}^+$ bandheads are believed to be the deformed proton-hole states ($6p$-$1h$) created by the excitation of a $1g_{9/2}$ proton from the $Z=50$ closed core. The complete γ-ray study for all of the Cs isotopes will be published later; several preliminary reports have been made.[16]

The similarity of the $\frac{9}{2}^+$ $\Delta J=1$ bands in the odd-A Cs isotopes and those previously observed in the odd-A Sb and I isotopes suggest a common collective structure. The band spacings and the $\frac{9}{2}^+$ bandhead energies decrease continuously with N in going from ^{125}Cs to ^{119}Cs. The excitation energies of the Cs $\frac{9}{2}^+$ bandheads are compared with those for the Sb and I isotopes in Fig. 2; as shown, in ^{119}Cs it has been dropped to become the ground state. The bandhead energies for the Cs isotopes appear to be following the parabola-

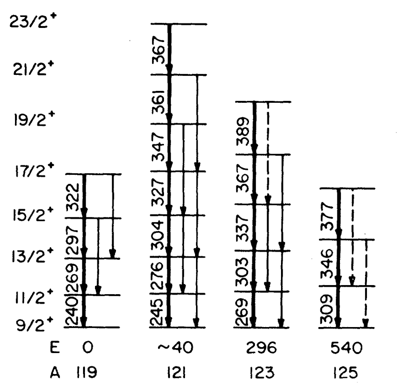

FIG. 1. Observed $\Delta J=1$ bands on $\frac{9}{2}^+$ states in odd-A Cs nuclei. The γ-ray and bandhead energies are in keV.

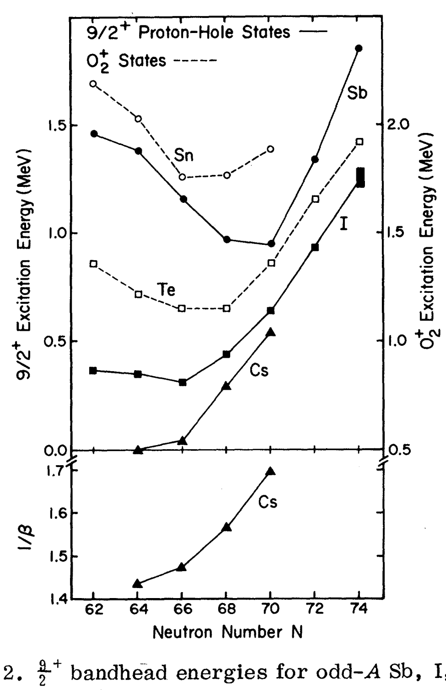

FIG. 2. $\frac{9}{2}^+$ bandhead energies for odd-A Sb, I, and Cs nuclei and 0_2^+ energies for even-A Sn and Te nuclei plotted against N. The lower part shows $1/\beta$ for the Cs $\frac{9}{2}^+$ bands. Preliminary data for ^{115}I are from P. Chowdhury *et al.* (unpublished).

like curves of the Sb and I isotopes (see Fig. 2) with the minimum energy occuring at $N \approx 64$.

A strongly coupled deformed (prolate) rotor interpretation[17] for these Cs $\frac{9}{2}^+$ bands is implied by the band spacings, the $E2$-$M1$ mixing ratios, the direct to crossover intensity ratios, and the $\frac{9}{2}^+$ bandhead energies, as previously shown[8,5] for the Sb and I $\frac{9}{2}^+$ bands (an anharmonic-core decription contains several equivalent features[18]). The $\Delta J=1$ band spacings in the Cs isotopes yield moments of inertia $\mathcal{J}$ that are consistent with large deformations. The maximum value of $(2\mathcal{J}/\hbar^2)$ for the Cs isotopes extracted from the energy spacings of the $\frac{13}{2}^+$ and $\frac{9}{2}^+$ band members (this choice avoids possible shifts in the $\frac{11}{2}^+$ member due to deformation asymmetries) is 47.2 MeV^{-1} in ^{119}Cs. This compares with the maximum values of 39.5 and 34.2 MeV^{-1} obtained for ^{119}I and ^{121}Sb, respectively, showing a continuous increase in $\mathcal{J}$ with an increase in Z. The $E2$-$M1$ mixing ratios for the $J \to J-1$ transitions and the intensity ratios of the $J \to J-1$ to $J \to J-2$ crossover transitions yield enhanced $E2$ strengths which imply intrinsic quadrupole moments and large prolate deformations that are also consistent with the band spacings.

The Cs $\frac{9}{2}^+$ bandhead energies, which unexpectedly drop to the ground state in ^{119}Cs, can be understood on the basis of the Nilsson proton-hole orbitals available at prolate deformations. Calculations of the total potential energy of these $\frac{9}{2}^+$ states including the [404] $\frac{9}{2}^+$ proton-hole energy, which decreases with deformation, and the potential energy surface of the $A+1$ core (even Ba nuclei for the Cs $\frac{9}{2}^+$ states) have revealed minima at significant prolate deformations ($\beta \approx 0.2$); the parabolalike curves representing the $\frac{9}{2}^+$ bandhead energies as a function of N as well as the $\Delta J=1$ band spacings for the Sb and I isotopes are in qualitative agreement with these calculations.[8,5] The lowest bandhead energies correspond with the largest deformations. This is manifest experimentally by a decrease in the band spacings as the bandhead energies decrease.

A comparison as a function of N of the $\frac{9}{2}^+$ bandhead energies and the deformations extracted from the $\Delta J=1$ band spacings can show the extent to which the deformation influences these energies. The deformation parameter β is related to the moment of inertia $\mathcal{J}$ by the equation,[19] $\beta = 5.4(A^{-5/6})(2\mathcal{J}/\hbar^2)^{1/2}$. The values of $2\mathcal{J}/\hbar^2$ were extracted from the observed band spacings of the Cs isotopes as discussed above. In the lower part of Fig. 2, the resulting values of $1/\beta$ are plotted as a function of N. The curves drawn through the $\frac{9}{2}^+$ bandhead energies and the $1/\beta$ values for the Cs isotopes are correlated in shape implying that the deformation of these states is a dominant factor in the reduction of the excitation energies from the large values expected for spherical $1g_{9/2}$ proton-hole states. Similar comparisons for the Sb and I isotopes show slight differences in the shapes of the two curves, suggesting that detailed neutron-proton and pairing interactions are also causing some variations as a function of N.

The systematic properties of the $\frac{9}{2}^+$ proton-hole states for the odd-A Sb, I, and Cs isotopes imply large deformations that maximize near the middle the neutron shell and increase with Z outside the $Z=50$ closed shell. The common feature in these deformed states is the excitation of a $1g_{9/2}$ proton across the $Z=50$ closed shell, namely $2p$-$1h$, and $6p$-$1h$ proton structures in the Sb($Z=51$), I($Z=53$), and Cs($Z=55$) isotopes, respectively.

The deformed structure relationship between the excitation of a $1g_{9/2}$ proton and the excitation of a pair of $1g_{9/2}$ protons has been established by the observation[9] of rotational bands on the low-lying 0_2^+ states in the even Sn($Z=50$) nuclei. These $2p$-$2h$ deformed states have $\mathcal{J}$ greater than or approximately equal to those of the $2p$-$1h$ $\frac{9}{2}^+$ states in the Sb isotopes and the parabolalike curves connecting the bandhead energies for the 0_2^+ Sn and $\frac{9}{2}^+$ Sb states have similar shapes and minima as a function of N. The Sn 0_2^+ energies are plotted in Fig. 2 for comparison. By this relationship, the deformed $4p$-$1h$ and $6p$-$1h$ $\frac{9}{2}^+$ states in the I and Cs isotopes would then suggest deformed $4p$-$2h$ and $6p$-$2h$ 0^+ states in the even Te($Z=52$) and Xe($Z=54$) isotopes. No rotational bands based on 0^+ states have been observed in these nuclei. The known 0_2^+ states in the Te isotopes,[20] however, have an energy dependence similar to the $\frac{9}{2}^+$ states in the I isotopes; these states, which possibly include a significant $4p$-$2h$ component, are also shown in Fig. 2. Of course, 0^+ states of a different structure are also possible at these energies in the Te nuclei. Only preliminary information is available on 0^+ states in the Xe nuclei.[21] The complete systematics of the deformed $\frac{9}{2}^+$ proton-hole states determined out to the Cs isotopes point to the need for a search of low-lying deformed 0^+ states in the Te and Xe isotopes.

In conclusion, since the deformed properties of the states involving the excitation of $1g_{9/2}$ protons from the $Z=50$ closed shell are now being thoroughly mapped over a large region of Z and N, a complete theoretical understanding is required for the coexistence of this unusual collective feature.

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Two-Dimensional Integral-Equation Solution of the Four-Nucleon System

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The ^{4}He binding energy and the n-^{3}H and p-^{3}H phase shifts below the $d+d$ threshold are calculated from Alt-Grassberger-Sandhas equations taking into account the full subamplitudes. As two-body interaction a separable s-wave spin-dependent potential is used.

Recent results of four-nucleon calculations based on the integral-equation approach have usually been achieved by means of separable expansion of the $1+3$ and $2+2$ subamplitudes in the integral kernel, thus reducing the original four-body equations to one-dimensional effective two-body equations.[1-8] Using separable two-body potentials but avoiding the expansion of the three-body subamplitudes one is left with *two*-dimensional integral equations.

On this basis we have calculated the ^{4}He binding energy[9] and four-nucleon phase shifts below the $d+d$ threshold. As two-nucleon interaction we have chosen a separable potential consisting of s-wave Gaussian form factors in the deuteron and antibound-state channel. Compared with the conventional Yamaguchi potential, which also could be handled, the Gaussian choice was preferred since it yields better low-energy three-body data[10] with parameters adjusted to deuteron binding energy, singlet and triplet scattering lengths, and singlet effective range. Moreover, there is indication that the Gaussian potential works well also in the four-body case.[8]

In Fig. 1 the $1+3$ and $2+2$ channels are illustrated for the special case of a (3, 4) cluster symbolized by a small circle and described by the s-wave form factors used in this paper. The sub-