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Instantons and Spin Forces between Massive Quarks

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A generalization of Wilson's loop prescription to include spin is proposed. Instantons are shown to generate a spin-spin interaction between quarks. The large splitting between $J/\psi(3100)$ and $\eta_c(2830)$ may, in fact, be dominated by this mechanism and thus may provide a vivid demonstration of the physical effects of instantons. Heuristic arguments are presented which explain the sign and form of the interaction.

Recent developments in instanton physics[1] have generated a new understanding of gauge field theory. In addition to considering small quantum fluctuations about the perturbation-theoretic vacuum, one must also take into account large classical fluctuations. The essentially quiescent vacuum of the traditional perturbative treatment has been replaced by a coherent quantum superposition of vacua with different topological character. As a result, certain nonperturbative features of the theory have been discovered. A promising picture of strong interactions may emerge along these lines.[2]

We would like to suggest here an instanton-induced effect which has immediate physical implication for the hadronic spectrum. We will calcu-

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late the effective spin force between two static, classical, spinning, fixed sources, generated by instantons. To the extent that the interaction of massive quarks is adequately described in the static approximation, our results will be directly relevant to the spectroscopy of charmonium and of the upsilon[3] family. A relatively clean application would be to the splitting between 1^- and 0^- states. Experimentally, the splitting between $J/\psi(3100)$ and $\eta_c(2830)$ appears to be too large to be explicable by the spin-spin Breit interaction deduced from simple one-gluon exchange.[4] We would like to suggest that this unexpectedly large splitting may be credited to instantons.

Before launching into computation we find it illuminating to present an intuitive physical argument which anticipates our conclusions below. Firstly, what would the sign of the interaction be? Imagine an electron and a positron in the presence of a fluctuating electromagnetic field (of either thermal or quantum origin). Their magnetic moments would evidently follow the direction of the local and instantaneous magnetic field; this means that their spins would tend to be antiparallel on the average. Similarly, a large fluctuation in Yang-Mills fields resulting from the tunneling between vacua of different topological characters would effectively tend to *antialign the spins in a quark-antiquark bound state.* Secondly, we can understand the nature of the interaction. In electrodynamics the quantum fluctuations about the standard vacuum are localized and rapid, and it should not be surprising that the spin interaction should average out if the electron and positron are separated (hence the δ-function spin-spin Breit interaction.) In contrast, in chromodynamics the (classical) fluctuations are slow and spatially extended (a fact traceable to the scale invariance of the theory leading to instantons of all sizes). Thus, we expect a term in the spin-spin interaction independent of the separation R between the quark and the antiquark [Fig. 1(a)]. Thirdly, the classical field fluctuations are large and explicit of order $1/g$ thus offsetting the small coupling g between quark and gluon. Instanton effects are nonperturbative and are small only in the regime of low instanton density. Thus we will not be surprised if the instanton-generated spin-spin interaction actually overwhelms in magnitude the order-g^2 effect of one-gluon exchange.

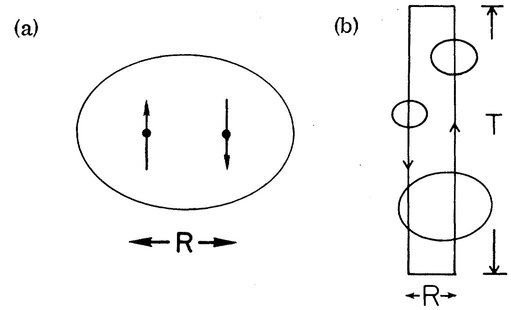

FIG. 1. (a) Spinning quarks in an instanton background. (b) Space-time description of the situation in (a). The instantons are supposed to be far apart.

We now turn to a quantitative discussion. Our first task is to formulate a gauge-invariant procedure of computing the effective spin interaction. We propose the following generalization of the Wilson[5] loop integral to include spin:

$$\lim_{T\to\infty} e^{-E(R)T} = \lim_{T\to\infty} \langle P \exp\{ig \oint dx^\mu [A_\mu + (i/m)\tilde{F}_{\mu\nu} S^\nu]\}\rangle. \quad (1)$$

Here $E(R)$ is the effective interaction between two static sources. The loop integral is to be taken over the path shown in Fig. 1(b). The mass of the classical sources is denoted by m. $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma}$ is the dual Yang-Mills field and S^ν is a classical spin four-vector. P is an ordering prescription (A_μ and $\tilde{F}_{\mu\nu}$ are matrices in color space). To readers familiar with the static source interpretation of the Wilson loop integral the above is evidently a natural and gauge-invariant way of describing the interaction between two classical massive (colored) billiard balls with spins $\vec{S}_1$ and $\vec{S}_2$ held fixed at a separation of $\boldsymbol{R}$. The corresponding Hamiltonian is the color generalization of the static electromagnetic Hamiltonian $H = eA_0 - (e/m)\vec{S}\cdot\vec{B}$. A noteworthy feature of Eq. (1) is the relative factor of i between spin term and the A_μ term. This is because upon rotation to Euclidean space $dx_0 \to idx_4$ and $A_0 \to iA_4$ while the antisymmetric symbol $\epsilon_{\mu\nu\lambda\sigma}$ does not change.

We take the spins to be fixed in time. Thus $S^0(t) = 0$, $\vec{S}(t) = \vec{S}_1$ on the $+t$ leg of the path in Fig. 1(b) and $\vec{S}(t) = \vec{S}_2$ on the $-t$ leg. With T large and with the instantons far apart from each other we can disentangle the multi-instanton contributions, following the authors of Ref. 2, to obtain

$$E(R) = -\int (d\rho/\rho^5) d(\rho) \int d^3x \, \tfrac{1}{3}\, \mathrm{tr}[U^-(\vec{x} - \vec{R}) U^+(\vec{x}) - 1]. \quad (2)$$

This formula involves computing with the single instanton only. Here

$$U^+(\vec{x}) = T\exp\{ig\int_{-\infty}^{\infty}dt[A_0(\vec{x},t) + (i/2m)F_{jk}(\vec{x},t)S_{1i}\epsilon_{ijk}]\}, \tag{3}$$

where A_0 and F_{jk} corresponding to the single instanton located at $\vec{x}$. $U^-(\vec{x})$ by a sign change in the exponential and by replacing $\vec{S}_1$ by $\vec{S}_2$. Equation (2) calls for an integration over the position of the instanton $\vec{x}$ and its size ρ. The distribution function $d(\rho)$ is defined in Ref. 2. Recall that the spin-independent interaction vanishes[2] as $R\to 0$ and indeed $\sim R^2$ for small R. In contrast the spin-spin interaction does not vanish as $R\to 0$. In principle we can certainly evaluate $E(R)$ as a function of R. In this Letter we content ourselves with evaluating the term independent of R. The computation simplifies significantly. Evaluating the trace in Eq. (2) in the limit of large m we obtain

$$E(R) = +\frac{16\pi^3}{9}\xi\,\vec{S}_1\cdot\vec{S}_2\frac{1}{m^2}\int_0^{\rho_c}\frac{d\rho}{\rho^4}d(\rho) + O(R^2). \tag{4}$$

Here ξ is a pure number defined by the integral

$$\xi \equiv \int_0^{\infty}dr\,\frac{r^2}{(r^2+1)^3}\left[\tfrac{1}{2} + \frac{(r^2+1)^3}{\pi^2 r^2}\sin^2\frac{\pi r}{(r^2+1)^{1/2}}\right] \tag{5}$$

$$\equiv \sim 0.2\,. \tag{6}$$

The sign and spatial dependence of the interaction is evidently in accord with our intuitively motivated remarks above.

We are hampered in our attempt to evaluate the splitting between J/ψ and η_c precisely by several considerations. Firstly, the size distribution function $d(\rho)$ is not known very accurately, especially as ρ approaches a certain cut-off size ρ_c and instantons start overlapping. At this point the coupling $g^2(\rho\mu)/8\pi^2$ rises rapidly and other effects (merons, etc.) may become important. Indeed, Callan, Dashen, and Gross[2] identify ρ_c as the confinement scale. We have thus cut off the integral in Eq. (4) by ρ_c. In any case, the derivation of Eq. (2), based on the dilute gas approximation, breaks down. Secondly, it is not clear to what extent quark spin could be described by classical spins. Our prescription is to replace the classical quantity $\vec{S}_1\cdot\vec{S}_2$ by the corresponding quantum version appropriate for spin $\frac{1}{2}$:

$$\tfrac{1}{2}S(S+1) - \tfrac{3}{4} = \{-\tfrac{3}{4} \text{ for } S=0,\ \tfrac{1}{4} \text{ for } S=1\}. \tag{7}$$

Thirdly, light quarks (up, down, and perhaps strange) suppress tunneling effects induced by instantons of small size. Thus, there should be a cutoff on the lower limit of the ρ integral in Eq. (4) as well; however, this is expected to be a small correction since $d(\rho)$ vanishes rapidly as $\rho\to 0$. Because of these caveats, our evaluation below should be regarded as providing an order-of-magnitude estimate at best.

We note that if the radius of a heavy-quark bound state is small enough so that the terms of order R^2 and higher in Eq. (4) can be neglected then we do not need to know the detailed form of the wave function to evaluate the spin-spin splitting. So finally, using[2]

$$d(\rho) = (0.06)\left(\frac{8\pi^2}{g^2(\rho\mu)}\right)^6\exp\left(-\frac{8\pi^2}{g^2(\rho\mu)}\right) \tag{8}$$

and including a factor of 2 to account for anti-instantons as well as instantons, we obtain for the splitting

$$\Delta E(1^- - 0^-) \sim 3\pi^3\mu^3/m^2 \tag{9}$$

Here μ denotes the usual renormalization scale appearing in $g^2(\rho\mu)$, that is, $8\pi^2/g^2(\rho\mu)\to 11\ln(1/\rho\mu)$ for small ρ (neglecting quarks). For a numerical estimate we take $\mu\sim 200$ MeV and, for charmonium, $m\sim 1500$ MeV, and obtain

$$\Delta E(\text{charmonium}) \sim 450\ \text{MeV}. \tag{10}$$

As remarked above, this numerical result can be trusted only to within a factor of a few. We took $8\pi^2/g^2(\rho_c\mu)\sim 13$. A different choice of ρ_c easily changes

$$\int_0^{\rho_c}\frac{d\rho}{\rho^4}d(\rho)$$

by a factor of 2. The numerical result is clearly rather sensitive to the choice of μ. The precise form of $d(\rho)$ also suffers from large uncertainties as discussed in Ref. 2.

What is noteworthy is that the spin-spin splitting can be so large. This may be traced to the fact that there are so many different paths available for the gauge fields to tunnel along. In other words, the integration over the position of the instantons and their sizes in Eq. (2) is largely responsible for the magnitude of the effect.

We conclude with few comments.

(1) We have not taken into account contributions due to merons and other configurations. However, according to the qualitative arguments presented above, they could only add to the spin-spin interaction. The R-dependent spin-spin force, which we have not evaluated here, would probably decrease the interaction as R increases.

(2) To evaluate the splitting between P-wave states and other physically interesting quantities we will need to compute the spin-orbit interaction generated by instantons. We have not yet extracted this and other effects (such as recoil) in a gauge-invariant way. One possibility may be to evaluate the Wilson loop for curved paths, matching the action of the paths to that calculated with an effective potential with coefficients to be determined. More generally, we could calculate the entire instanton-generated interaction between massive quarks, including all spin- and velocity-dependent terms.

(3) Since the spin-spin force is to first approximation independent of the distance R between quarks, the magnitude of the hyperfine splitting should scale as $1/m_q^2$ (once the quark mass is large enough to concentrate the wave function in a region small compared with $1/\mu$) independent of the details of the wave function. This contrasts markedly from the behavior expected from the Breit potential, which gives a splitting which scales with m_q, or in general, from pictures in which the spin-spin interaction is strongly localized.

(4) We have also computed the spin-spin interaction in the limit of large separation R between the quarks. The interaction is generated by instantons far away from both quarks as well as by instantons overlapping one of the quarks spatially. We find the interaction vanishes as $1/R^3$.

(5) We note that the numerical integral in Eq. (5) is the same as the corresponding integral appearing in the spin-independent potential (Ref. 2). This suggests that the spin-spin interaction and the spin-independent interaction may be intimately related.

(6) While we were able to give only an order-of-magnitude estimate of the splitting between J/ψ and η_c we believe that the mechanism presented here may be largely responsible for the large splitting. It is certainly a large effect of the right sign. The direction of the splitting is entirely determined by an intuitive quasiclassical argument rather reminiscent of the Casimir effect.[6] If our interpretation is correct, then it provides an indication of the physical reality of instantons.

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