Dissipation in Two-Dimensional Superfluids

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Dissipation of energy by a thin film of ^{4}He on an oscillating planar substrate arises both from motion of free vortices and from polarization of bound pairs. Starting from a Langevin equation for vortex diffusion, and taking into account production of free vortices from bound pairs, we estimate this dissipation in various regimes of frequency, amplitude of vibration, and temperature.

Some years ago, Kosterlitz and Thouless[1] proposed a physically appealing picture of superfluidity in two dimensions. They suggested that in the superfluid state, smoothly varying phase fluctuations coexist with a dilute gas of bound vortex pairs with opposite vorticity. With increasing temperatures, a finite fraction of the pairs dissociates, destroying the superfluidity and causing a transition to a normal phase. Kosterlitz[2] subsequently made quantitative calculations of the static properties of a superfluid which support this picture. Recent work by José *et al.*[3] suggests that the model used by Kosterlitz, as well as the results obtained by him, provides a universal long-wavelength description of two-dimensional superfluidity. These theories predict[4] that the superfluid density drops discontinuously to zero by a universal amount as the temperature is raised to the transition temperature, T_c.

Since both the superfluid density and the dissipation contribute to the inertia and absorption of energy of a ^{4}He film on an oscillating substrate,[5] precise experimental tests of the physical ideas described above require theoretical predictions for the dissipation as a function of frequency, amplitude of oscillation, and temperature. In this Letter, we determine the behavior of this dissipation, which arises from the motion of vortices across the superflow. Our analysis of the vortex motion builds on the Kosterlitz-Thouless picture of the static properties, and has been aided by an analogy with the dynamics of a plasma, confined between capacitor plates and subjected to an oscillating electric field, in which charges move by diffusion. Contributions from free vortices and bound pairs enter in the various regimes. A central result of our calculations is that the dissipation is concentrated in a narrow temperature interval about T_c, and tends to zero, at all temperatures except T_c, in the limit of vanishing frequency and substrate velocity.

Consider a helium film of uniform thickness, on a substrate which has an infinite length L in the x direction and a large but finite width W in the y direction. The substrate is driven sinusoidally at frequency ω, with velocity $\vec{v}_n(t)$ in the x direction. To calculate the response of the system, we start with the equations of motion for the positions $\vec{r}_i(t)$ of a collection of N vortices, namely

$$\frac{d\vec{r}_i}{dt} = n_i\left(\frac{Dh\rho_0}{mk_BT}\right)\hat{z}\times(\vec{v}_n - \vec{v}_s{}^i) + C(\vec{v}_n - \vec{v}_s{}^i) + \vec{v}_s{}^i + \vec{\eta}_i(t), \tag{1}$$

where D is a diffusion constant, $n_i = \pm 1$ is the sign of the vortex, m is the helium mass, and $\hat{z}$ is a unit vector perpendicular to the plane of the film. The quantity $\vec{v}_s{}^i$ is the local superfluid velocity at $\vec{r}_i$ (excluding the divergent self-field of the vortex at $\vec{r}_i$); ρ_0 is the background superfluid density, integrated across the film thickness, which would be present in the absence of vortices; and the number C entering the convective terms $[C(\vec{v}_n - \vec{v}_s{}^i) + \vec{v}_s{}^i]$ of Eq. (1) is a constant between 0 and 1. The $\vec{\eta}_i(t)$ are fluctuating Gaussian noise sources whose components $\eta_i^{(\alpha)}(t)$ satisfy

$$\langle \eta_i^{(\alpha)}(t)\eta_j^{(\beta)}(t')\rangle = 2D\delta_{ij}\delta_{\alpha\beta}\delta(t-t'). \tag{2}$$

The local superfluid velocity may be written in the form

$$\vec{v}_s(\vec{r}) = \vec{u}_s + (h/m)\hat{z}\times\sum_j n_j \nabla G(\vec{r},\vec{r}_j), \tag{3}$$

where $\vec{u}_s$ is a uniform flow in the x direction, and $G(\vec{r},\vec{r}_j)$ is the two-dimensional Coulomb potential at point $\vec{r}$ due to a charge $(-4\pi)^{-1}$ at point $\vec{r}_j$, together with image charges such that $G(\vec{r},\vec{r}_j) = 0$ for $\vec{r}$ at the film edges. These boundary conditions ensure that there is no superflow across the lateral edges of the film. Far from the edges, $G\approx(2\pi)^{-1}\ln|\vec{r}-\vec{r}_j|$. Because G vanishes for $\vec{r}$ at the boundaries, the spatial average of $\vec{v}_s(\vec{r})$ is just equal to $\vec{u}_s$, and the momentum density of the film is $\vec{g} = \rho_0\vec{u}_s + (\rho-\rho_0)\vec{v}_n$. The time dependence of the average superfluid velocity $\vec{u}_s(t)$ produced by the motion of vortices is given by

$$d\vec{u}_s/dt = -\sum_i (h/mLW)n_i\hat{z}\times d\vec{r}_i/dt. \tag{4}$$

Equation (1) is obtained by balancing drag forces against the Magnus force and solving for the resulting line velocity, $d\vec{r}_i/dt$.[6] The diffusion constant D, together with C and ρ_0, is determined by interactions with the substrate and with excitations such as rotons and ripplons. Equation (4) is a microscopic restatement of the equation for the decay of two-dimensional superflows due to vortex motion, as discussed by Langer and Reppy.[7]

Kosterlitz[2] has shown that on large enough length scales, the static properties of our system can be related to those of a dilute gas of renormalized vortices. We will assume that critical fluctuations only enter through static quantities and calculate the dissipation by considering only the effects of free vortices and isolated vortex pairs. We also assume that the complicated effects of vortex collisions and substrate interactions can be incorporated into the parameter D which we estimate to be $\lesssim \hbar/m \approx 10^{-4}$ cm^2/sec. With these assumptions the convective part of (1) may be ignored, because it carries an isolated vortex in the direction of $\vec{v}_n$, and leaves unchanged the separation vector of an isolated pair. These processes give no contribution to $d\vec{u}_s/dt$ in (4).

It will prove useful to restate a number of results from Refs. 1 and 2 in terms of a "length-dependent dielectric constant" $\tilde{\epsilon}(r)$ that includes the screening effect only of vortex pairs whose separation is less than r.[8] At large separations, below T_c, $\tilde{\epsilon}(r)$ tends to a constant $\tilde{\epsilon}_\infty$, which is the macroscopic zero-frequency dielectric constant in the plasma analog. This quantity is equal to $\rho_0/\rho_s(T)$, where $\rho_s(T)$ is the integrated superfluid density across the thickness of the film. Above the critical temperature, $\tilde{\epsilon}(r)$ tends to infinity at large r.

The leading corrections to $\tilde{\epsilon}_\infty$ below T_c allow one to define a correlation length $\xi_-(T)\approx a\exp[1/x(T)]$, where a is an interatomic distance and $x(T) = -4 + 2\pi\hbar^2\rho_s(T)/m^2k_BT$. For $r \gg \xi_-(T)$, one finds that

$$\tilde{\epsilon}^{-1}(r)\approx\tilde{\epsilon}_\infty^{-1}\{1+\tfrac{1}{2}x(T)\exp[-\ln(r/a)/\ln(\xi_-/a)]\}. \tag{5a}$$

Note that $\tilde{\epsilon}_r^{-1}-\tilde{\epsilon}_\infty^{-1}\propto r^{-x(T)}$ in this region. Near T_c, $x(T)\approx 2b^{-1}(1-T/T_c)^{1/2}$, where b is a nonuniversal constant. For $a<r<\xi_-$, below and at T_c, we have

$$\tilde{\epsilon}^{-1}(r)\approx\epsilon_c^{-1}[1+0.5/\ln(r/a)], \tag{5b}$$

where $\epsilon_c = \rho_0/\rho_s(T_c)$ is the value of $\tilde{\epsilon}_\infty$ for $T\to T_c^-$.

Above T_c, Kosterlitz has defined a correlation or screening length, $\xi_+(T)\approx a\exp[b\pi(T/T_c-1)^{-1/2}]$, in terms of which the density of free vortices becomes $n_f\sim\xi_+^{-2}$. Note that $\xi_+\sim a(\xi_-/a)^{2\pi}$ for corresponding reduced temperatures. For $1<\ln(r/a)\ll 1/x(2T_c-T)$, one finds that $\tilde{\epsilon}(r)$ obeys (5b), while more generally for $\ln(r/a)<2\pi/x(2T_c-T)$,

$$\tilde{\epsilon}^{-1}(r)=\epsilon_c^{-1}\{1+\tfrac{1}{2}\pi\cot[\pi\ln(r/a)/\ln(\xi_+/a)]/\ln(\xi_+/a)\}. \tag{5c}$$

To study the dissipation at finite frequencies and at low substrate velocities, we define a frequency-dependent "dielectric constant" $\epsilon(\omega)$, in terms of the Fourier transforms of u_s and v_n, by $v_n(\omega)/\epsilon(\omega) = v_n(\omega)-u_s(\omega)$. [In the plasma analog $\hat{z}\times(\vec{v}_n-\vec{u}_s)$ plays the role of electric field.] The power dissipated per unit area in the film is the time average of $\vec{v}_n\cdot d\vec{g}/dt$, namely

$$P=\tfrac{1}{2}\rho_0(v_n^{\max})^2\omega\,\mathrm{Im}[-\epsilon^{-1}(\omega)], \tag{6}$$

where $v_n^{\max}$ is the amplitude of the substrate velocity. The real part of $\epsilon^{-1}(\omega)$ determines the effective superfluid density at frequency ω. In general, both bound and free vortices will contribute to $\epsilon(\omega)$.

With this in mind, we break the summation over vortices in (4) into two parts. At low frequencies ($\omega \ll D\xi_+^{-2}$) above T_c, relaxation effects are dominated by free vortices at a density $n_f \sim \xi_+^{-2}$. The value of $d\vec{r}_i/dt$ for the free vortices is calculated from (1) with $\vec{v}_s^{\,i}$ replaced by the average quantity $\vec{u}_s$. We replace the bound-vortex contribution to (4) by a polarization term $(\epsilon_b - 1)d(\vec{v}_n - \vec{u}_s)/dt$, where ϵ_b, the dielectric constant due to the bound pairs, is approximately equal in this regime to $\epsilon_c \equiv \rho_0/\rho_s(T_c)$ introduced above. In this way, we find $d\vec{u}_s/dt = -\gamma(\vec{u}_s - \vec{v}_n) + (1-\epsilon_c^{-1})d\vec{v}_n/dt$ with the relaxation rate $\gamma = h^2\rho_0 D n_f/m^2\epsilon_c k_B T$. If we introduce an effective charge $e = (\pi h^2\rho_0/m^2)^{1/2}$, then $\epsilon(\omega)$ takes the familiar form

$$\epsilon(\omega) \approx \epsilon_c + 4\pi i e^2 D n_f/k_B T\omega. \tag{7}$$

The power dissipated at low frequencies becomes $P \simeq \frac{1}{2}\rho_s(T_c)(v_n^{\max})^2\omega^2/\gamma$.

For any fixed finite ω, (7) must break down as $T \to T_c$ from above. Just above T_c, for $Da^{-2} \gg \omega \gg D\xi_+^{-2}$, and for *all* $\omega \ll Da^{-2}$ *below* T_c, the principal dissipative contribution to $\epsilon(\omega)$ arises from the polarization of bound pairs. We assume that the orientation of a vortex pair with separation r relaxes to equilibrium at a characteristic rate $\nu(r)$, which we estimate as $\approx 2Dr^{-2}$. Assuming a simple exponential relaxation, the generalization of the analysis of Ref. 1 to finite frequencies is

$$\epsilon(\omega) \approx 1 + \int_a^\infty dr\, \frac{(d\tilde{\epsilon}/dr)\times 2Dr^{-2}}{-i\omega + 2Dr^{-2}}, \tag{8}$$

where $(d\tilde{\epsilon}/dr)dr$ is the contribution to the static dielectric constant from pairs of separation r, and is proportional to r^2 times the number of pairs at separation r.[1] We obtain $d\tilde{\epsilon}(r)/dr$ by differentiating the results for $\tilde{\epsilon}^{-1}$ collected in Eq. (5) above. Since $r(d\tilde{\epsilon}/dr)$ is a slowly varying function of T and r near T_c,[1-3] we can estimate the integral in (8) to be

$$\mathrm{Re}\,\epsilon(\omega) \approx \tilde{\epsilon}((2D/\omega)^{1/2}), \tag{9a}$$

$$\mathrm{Im}\,\epsilon(\omega) \approx \tfrac{1}{4}\pi[r(d\tilde{\epsilon}/dr)]_{r=(2D/\omega)^{1/2}}. \tag{9b}$$

Within the domains of validity of Eqs. (5a)–(5c), one observed that (9a) exceeds (9b), so that we may write the power dissipated as

$$P = \tfrac{1}{8}\pi\rho_0(v_n^{\max})^2\omega|r(d\tilde{\epsilon}^{-1}/dr)|_{r=(2D/\omega)^{1/2}}, \tag{10}$$

with $\tilde{\epsilon}^{-1}$ given by the appropriate formula of (5a)–(5c). At low frequencies below T_c when $\xi_- \ll (2D/\omega)^{1/2}$, one finds $P \propto \omega(T_c - T)(\omega/2Da^{-2})^{x(T)/2}$; near T_c and for $(2D/\omega)^{1/2} \ll \xi_-$ or $\ll (\xi_+/a)^{\pi/2}$, one obtains $P \propto \omega/\ln^2(\omega/Da^{-2})$. Note that (9) is in approximate agreement with (7) at the borderline frequency $\omega \approx D\xi_+^{-2}$, for $T > T_c$.

Thus far we have calculated only the linear response to the substrate velocity. Our approximations are invalidated for sufficiently large v_n by an unbinding of the pairs. Standard results obtained in studies of the decay of persistent supercurrents imply that there is a critical radius $r_c \approx \hbar/mv_n$ beyond which pairs will rapidly separate.[7,9-11] For fixed ω and T the creation of additional free vortices enhances $\mathrm{Im}\,\epsilon$, which increases the dissipation below T_c.

The creation of free vortices from pairs that diffuse apart can be determined from the Langevin equation for the separation vector $\vec{r}$ of a pair,

$$\frac{d\vec{r}}{dt} = \frac{-2D}{k_B T}\left[\frac{\rho_0 h^2}{2\pi M^2}\,\frac{\vec{r}}{r^2\tilde{\epsilon}(r)} - \rho_0\frac{h}{m}\hat{z}\times(\vec{v}_n - \vec{u}_s)\right] + \vec{\eta}(t), \tag{11}$$

where $\langle\eta^\alpha(t)\eta^\beta(t)\rangle = 4D\delta_{\alpha\beta}\delta(t-t')$. Equation (11) follows from (1) if we absorb the effects of all other vortices into $\tilde{\epsilon}(r)$ and u_s. The calculation is carried out using methods described previously.[12] Below T_c, when $\xi_- \ll r_c \ll (D/\omega)^{1/2}$, the rate of generation of free vortices per unit area goes as $R \sim Da^{-4}[x(T)]^2|ma\hbar^{-1}(v_n - u_s)|^{4+x(T)}$. In a wide film where the steady-state density of vortices is limited by the recombination of vortices passing within r_c of each other,[12] one finds that n_f is proportional to $(R/D)^{1/2}$. The analysis that leads to (7) can be repeated, with

$$\gamma(t) \sim Da^{-2}x(T)|ma\hbar^{-1}[v_n(t) - u_s(t)]|^{2+x(T)/2}. \tag{12}$$

In the present case, however, if the frequency ω is not too low, we will have $\gamma \ll \omega$ and a dissipation rate $P \approx \rho_s(T_c)\langle v_n^2(t)\gamma(t)\rangle$, where the average is taken over a cycle.

In a sufficiently narrow sample below T_c (but still assuming $r_c \ll W$), vortex annihilation may occur predominantly at the walls rather than through pair recombination. Under such conditions, however, *generation* of free vortices at the edges will also dominate pair dissociation in the "bulk," and the resulting density of free vortices turns out to be the same as the $W \to \infty$ case.

For temperatures sufficiently close to T_c and v_n fixed, when $\xi_-(T) \gg r_c$ Eq. (12) must be modified. The quantity $x(T)$ must now be replaced by $1/\ln[\hbar/ma(v_n - u_s)]$.

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Superconducting Al-PbBi Tunnel Junction as a Phonon Spectrometer

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Phonons incident on an Al-PbBi tunnel junction were found to be detectable only if their frequency exceeds a voltage-tunable threshold. Modulating this threshold yielded a phonon spectrometer with a resolution of 10 GHz at a phonon power of 10^{-6} W and a frequency range from 100 GHz up to several hundred GHz.

A complete description of any solid-state system which interacts with high-frequency phonons (> 100 GHz) requires knowledge not only about the frequency of the incident phonons but also of the emitted ones. While it is no longer difficult to generate monochromatic phonons with frequencies tunable up to the terahertz range,[1-3] the analysis of phonon frequencies in this range is still a major obstacle because a phonon spectrometer is lacking.

Several solutions have already been proposed. Some impurities in crystals exhibit frequency-selective phonon absorption which "burns a hole" in the phonon spectrum. Tuning the absorption frequency with an external magnetic field[4,5] or with stress[6] yields information about the phonon frequency distribution. Selective phonon detection was achieved by probing the excited states of selectively absorbing impurities by optical techniques.[7-9] Sidebands of optical luminescence lines can also be used to investigate the frequency distribution of nonequilibrium phonons.[10]

Although these experiments yielded important results, none of the suggested spectrometers found widespread application, mainly because all of them use special bulk materials which cannot readily be applied to the study of all phonon sources of interest. Furthermore, none of them satisfactorily combines resolution, detection speed, and sensitivity.

In this Letter, I report on investigations of tunnel junctions consisting of two superconductors

