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Dissipation near the Critical Point of a Two-Dimensional Superfluid

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We show that in a nonequilibrium, two-dimensional superfluid near the critical point, the energy for dissipative fluctuations is greatly reduced as a result of screening by the gas of bound vortex pairs, leading to an enhanced decay of the superfluid velocity. We also show that the superfluid density is reduced by a velocity-dependent amount as T_c is approached from below.

^{4}He films of few monolayers in thickness are of considerable interest in view of current progress in the understanding of critical phenomena in the two-dimensional X-Y model.[1,2] Experiments by Telschow and Hallock[3] and Bishop and Reppy[4] on the low-temperature properties of few-monolayer He films on a Mylar substrate suggest that dissipation of the two-dimensional (2-D) superflow in the onset transition region is much larger than the dissipation of dilute three-dimensional-type superfluid in Vycor substrates.

The purpose of this Letter is to point out that this qualitative difference between the dissipation for two-dimensional versus three-dimensional systems can be understood in terms of the fact that the critical fluctuations in a 2-D superfluid may be represented by the thermodynamics of a gas of vortex pairs interacting through long-range logarithmic potentials.

The decay of the nonequilibrium current-carrying state of a superfluid in three dimensions proceeds via the nucleation of vortex rings of a certain critical size.[5] In two dimensions the analogous mechanism is that of nucleation of vortex pairs with a critical separation normal to the superflow. We show that for a two-dimensional superfluid near the critical point the energy for creation of vortex pairs is greatly reduced as a result of screening by the gas of bound vortex pairs in a way similar to the 2-D Coulomb gas. It is this screening which reduces the free-energy barrier for nucleation of vortex pairs of critical size and hence increases the dissipation of two-dimensional superflow. We also show that the average number of vortex pairs whose size is below the critical distance in the metastable state is an increasing function of superfluid velocity, so that the superfluid density is reduced by a velocity-dependent amount as the critical temperature is approached from below.

We consider a 2-D superfluid[6] in a metastable state characterized by a superfluid velocity V_s. In the spirit of the Vinen theory[7] as elaborated by Iordanskii[8] and Langer and Fisher[9] we assume that the excitation of a vortex pair with mean separation $y>y_0$ perpendicular to the direction of superflow will reduce V_s by an amount h/mA, with A the dimensions of the film. The mechanism behind this decay process is analogous to the droplet model of homogeneous nucleation, i.e., fluctuation vortex pairs with distances $y<y_0$ tend to coalesce whereas those pairs generated at distances beyond y_0 will tend to expand indefinitely and crash against the boundaries, thereby providing a channel for flow dissipation. The critical distance y_0 is determined by considering

the energy of a vortex pair in the laboratory frame, which is given by[5]

$$E = (\rho_0/2\pi)K^2 \ln(y/a) - \rho_0 K V_s y \tag{1}$$

with K the vorticity, a the vortex core radius, ρ_0 the superfluid density in units of mass/area, and y the distance, along an axis perpendicular to the superfluid velocity, between the vortices making up the pair. Since the maximum of (1) determines the saddle point of the fluctuation barrier, it can be used to determine the critical distance y_0, i.e.,

$$y_0 = K/2\pi V_s. \tag{2}$$

In the absence of any pair interactions $E(y_0)$ determines the energy required to nucleate a vortex pair of critical radius. The rate at which vortex pairs fly apart is then given by

$$R = \nu_0 V_s^2 \int_{y_0(1-\sigma)}^{y_0(1+\sigma)} d^2r \exp[-E(r)/k_B T] \tag{3}$$

with ν_0 a prefactor that depends on the substrate and with no critical-point singularities, k_B the Boltzmann constant, and $\sigma \ll 1$. Far below the critical temperature this picture correctly describes the decay of the metastable states. As T_c is approached from below, however, a 2-D superfluid in equilibrium is threaded with vortex pairs whose average distance increases with temperature and diverges at T_c.[1] It is therefore obvious that in the metastable state, the energy required to produce a vortex-pair fluctuation within a distance y_0 will be strongly renormalized by the presence of an equilibrium distribution of vorticity, since the latter acts so as to screen the long-range velocity fields of the fluctuations.

We can calculate the renormalized activation energy by exploiting the isomorphism of the vortex model of liquid helium to the 2-D Coulomb gas, a system that displays an insulating-to-metallic transition as the temperature is increased.[10-12] The creation of a vortex pair in the 2-D superfluid is analogous to the introduction of an extra pair of opposite charges in the 2-D Coulomb gas; and since as the system gets closer to the insulating-metallic transition temperature its polarizability increases, the energy required to add them is reduced by electrostatic screening.

The energy E required to inject a pair of opposite charges in a Coulomb gas is given by

$$E = (-2e^2 \ln r)/\epsilon \tag{4}$$

with ϵ the $q=0$ dielectric constant of the 2-D Coulomb gas. The dielectric constant in the insulating, low-temperature regime is given by[13]

$$\epsilon(T) = e^2(k_B T\beta)^{-1} \tag{5}$$

with β a temperature-dependent quantity which near T_c behaves as

$$\beta \simeq 2 + C(1 - T/T_c)^{1/2}, \tag{6}$$

while T_c is related to the superfluid density through

$$T_c = \pi\hbar^2\rho_s/2k_B m^2. \tag{7}$$

At low velocities we assume that the vortex distribution is not changed appreciably, and remembering that the mapping into the 2-D superfluid can be carried out formally by replacing e^2 by $2\pi\hbar^2\rho/m^2$, with m the mass of a helium atom, we obtain after an elementary integration

$$R = \nu_0 V_s^{2\beta}. \tag{8}$$

The rate at which the superfluid velocity decays, which is given by[14]

$$dV_s/dt = -V_s(\nu_0 R)^{1/2}, \tag{9}$$

then becomes

$$dV_s/dt = -\nu_0 V_s V_s^{\beta}. \tag{10}$$

In all these equations the superfluid velocity is in units of an inverse vortex core size, i.e., $V_s = (\text{cm/sec}) \times m/\hbar \simeq (\text{cm/sec}) \times 10^{-4}$. This means that since[15] $C \simeq 1$, dissipation will become significant within a percent of the critical point. This result follows from the $\frac{1}{2}$-power law of Eq. (6). This behavior is in qualitative agreement with the experiments of Ref. 4.

These ideas can also be applied to the superfluid density of a moving 2-D superfluid in the following manner. Those vortex pairs separated by distances smaller than the instability threshold y_0 are assumed to be in quasiequilibrium with a distribution determined by their energy in the rest frame of the substrate. Since in the presence of a flow velocity V_s the distribution of the metastable bound pairs is not symmetric, the total current will be diminished by a backflow which is dominated by the contribution from the number of pairs near the instability threshold. Since this contribution to the backflow is given by $V_s V_s^{2(2-\beta)}$ we can obtain ρ_s by dividing the metastable current by V_s. We therefore obtain

$$\rho_s(V_s) \simeq \rho_s(V_s = 0) - B V_s^{2(2-\beta)} \tag{11}$$

with B a numerical constant. As can be seen, no matter how small the superfluid velocity is, close to T_c finite-velocity effects on ρ_s will be detectable.

One of us (R.J.M.)[13] has extended the virtually rigorous renormalization-group technique of Kosterlitz[15] to this quasiequilibrium picture. Although the details will be published elsewhere, the conclusions can be stated in a simple manner. The calculations show that the superfluid density is now given by

$$\rho_s(V_s) = (m^2 k_B T)/\pi\hbar^2)[\beta - DV_s^{(2\beta-4)}] \tag{12}$$

with D a substrate-dependent constant of order $D \simeq 4$.

Several points deserve comment concerning Eqs. (10) and (12). (i) Near T_c there is a marked enhancement in the dissipation rate and a pronounced drop in ρ_s which should be observable in the temperature range $T \simeq 0.01T_c$. (ii) The temperature range over which finite-velocity effects occur should broaden with increasing V_s, a fact which is consistent with the observations of Bishop and Reppy.[4] (iii) For $T < T_c$ the temperature dependence of the velocity exponent is the same as the one that determines the temperature dependence of $\rho_s(V_s = 0)$ near T_c. (iv) For $V_s = 0$ the temperature dependence of ρ_s agrees with the stationary film results of Nelson and Kosterlitz[16] so that Eq. (12) provides a quantitative formula for extrapolating to zero-velocity persistent-current measurements.

As we have shown, in a nonequilibrium 2-D superfluid the proximity to the critical point decreases significantly the energy barrier for dissipative fluctuations, thereby increasing the rate at which the superfluid current decays. Moreover, the superfluid density as determined by persistent-current measurements becomes velocity dependent near T_c, masking in part the universal behavior of the critical value of $\rho_s(V_s = 0)/T$.

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Note added.—Ambegaokar, Halperin, Nelson, and Siggia[17] have independently derived these results and extended them to high frequencies. In their work, the characteristic distance traveled per cycle by a diffusing vortex plays the role of $1/V_s$ in this Letter.

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