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Comparison of the Reactions $^{12}C(\pi^+,p)^{11}C$ and $^{12}C(p,d)^{11}C$ near the Same Momentum Transfer

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(Received 2 February 1978)

Observed transitions in the reaction $^{12}C(\pi^+,p)^{11}C$ at 49.3 MeV confirm the importance of multistep processes in the reaction mechanism. The relative intensity of the transitions is nearly the same as in the reaction $^{12}C(p,d)C^{11}$ at 700 MeV, except for an apparent isospin selection rule. Transitions to low-lying states in ^{10}C via the reaction $^{12}C(\pi^+,d)$ have also been observed.

Angular distributions have been measured for several transitions in the reaction $^{12}C(\pi^+,p)^{11}C$ at $T_\pi = 49.3$ MeV and have been compared to the $^{12}C(p,d)^{11}C$ data at $T_p = 700$ MeV.[1] Both reactions are classified as neutron-pickup reactions, but the similarity in their reaction mechanism is not adequately known. The comparison is made at the same momentum transfer, between 2.3 and 3.2 fm^{-1}, for both reactions. The probability is small that a bound neutron in the target has so large a momentum. Therefore, rather than a single-step pickup process it is more likely that the momentum transfer occurs in multiple steps involving off-shell particles. For example, the incoming projectile experiencing the strong nuclear field may acquire an off-shell momentum ($p^2 \neq E^2 - m^2$). This effect on the incoming and outgoing projectile is included in a standard distorted-wave Born-approximation (DWBA) calculation using optical potentials.[2] One goal of this experiment is to add the (π^+,p) data to the extensive study[3,4] of the pion optical potential near $T_\pi = 50$ MeV. Various optical potentials which explain the available elastic- and inelastic-pion-scattering data have very different off-shell behavior and predict widely different cross sections for the (π^+,p) reactions.[2]

The large momentum transfer is also likely to result in multiple changes in the structure of the nucleus. There are states of ^{11}C that are predominantly a neutron hole weakly coupled to the collective 2^+ (4.44 MeV) or 3^- (9.63 MeV) states of ^{12}C. A reasonable model of the transitions to these states is a second-order DWBA calculation coupling the inelastic (collective) channels to the ^{11}C neutron-pickup channels. Such a model has been used successfully[1,5] for the reaction $^{12}C(p,d)^{11}C$ at $T_p = 700$ MeV. An alternative approach[6] to a coupled-channels calculation for the (π^+,p) reaction is to use an effective pion–two-nucleon interaction, $H_{NN\pi}$, that includes single-rescattering contributions. The latter approach is also applicable to two-step transtions which do not have strong soupling to well-defined intermediate states, such as the two-step transition to the $T = \frac{3}{2}$, 12.5-MeV state of ^{11}C (see below). The study of these various transitions will enhance our understanding of the (π^+,p) reaction process.

The experiment was performed at the LEP channel at the Clinton P. Anderson Meson Physics Facility (LAMPF), using a stack of eight intrinsic germanium crystals.[7] Approximately 5×10^6 pions per second with 50 MeV kinetic energy bombarded natural carbon targets with thicknesses between 0.35 and 0.7 g/cm^2. The proton spectra, using proton identification from the range-energy relationship, are about 99.9% pure. Three multiwire proportional chambers were placed in front of the detector to trace the trajectory of each particle. The absolute cross section was normalized to the known[8] $d(\pi^+,p)p$ cross section at $\theta = 40°$ and 55° using a deuterated polyethylene, CD_2, target. The C + π^+ elastic peak was used to monitor the relative integrated beam current, target thickness, solid angle, and dead time allowing the normalization of the actual runs to the calibration runs. This procedure required knowing the shape, but not the magnitude, of the C + π^+ elastic angular distribution. Independently, the published elastic-scattering angular distributions[4] were rechecked and confirmed. For the above system the detection of 160-MeV protons from the reaction $^{12}C(\pi^+,p)^{11}C$ was about 20% less efficient than the detection of 110-MeV protons observed in the reaction $d(\pi^+,p)p$, because of nuclear reactions in and multiple scattering out of the detector crystals. The decreased efficiency was calculated from the available Ge nuclear reaction cross sections giving an approximate 8% difference in efficiency and from a Monte Carlo calculation of the outscattering giving an approximate 12% additional reduction. The Monte Carlo calculation was checked for consistency by varying the radius of acceptance of events hitting the first germanium crystal.

A spectrum at $\theta_{lab} = 45°$ with a resolution full width at half-maximum of 1.65 MeV is shown in Fig. 1. For other spectra the resolution varied between 1.2 and 2.0 MeV. Using all the spectra, we obtain clear indication of peaks centered at excitation energies 0.0, 2.0 ± 0.2, 4.4 ± 0.2, 6.4 ± 0.2, 8.5 ± 0.2 and 12.5 ± 0.3 MeV. Excellent fits to all spectra are obtained assuming these peak locations and a steeply rising background. The shape of the background above 7 MeV excitation appears to be dominated by the three-body continuum $p + \alpha + {}^7Be$ and $p + p + {}^{10}B$ starting at 7.5 and 8.7 MeV, respectively. In addition, the many two-body states in this region may be contributing to the background leading to some uncertainty in its shape and contributing to the error in the cross section of the 12.5-MeV peak. The only significant variation in peak shapes appeared at $\theta_{lab} = 30°$ where the peak at 8.5 MeV was 200 keV wider than the ground state, indicating some contributions from both the 8.425-MeV $\frac{5}{2}^-$ state and the 8.7-MeV $\frac{7}{2}^+, \frac{5}{2}^+$ doublet.

Also shown in Fig. 1 are the relative intensities of peaks observed in the reaction $^{12}C(p,d)^{11}C$ at the same momentum transfer, 2.44 fm^{-1}. The relative intensities of the peaks below 9 MeV excitation are similar for both reactions except for the 6.4-MeV peak. However, at $\theta_{lab} = 75°$ and 110°, the 6.4-MeV peak becomes the dominant peak in the $^{12}C(\pi^+,p)^{11}C$ spectra in similarity to the 6.4-MeV peak in the (p,d) data.

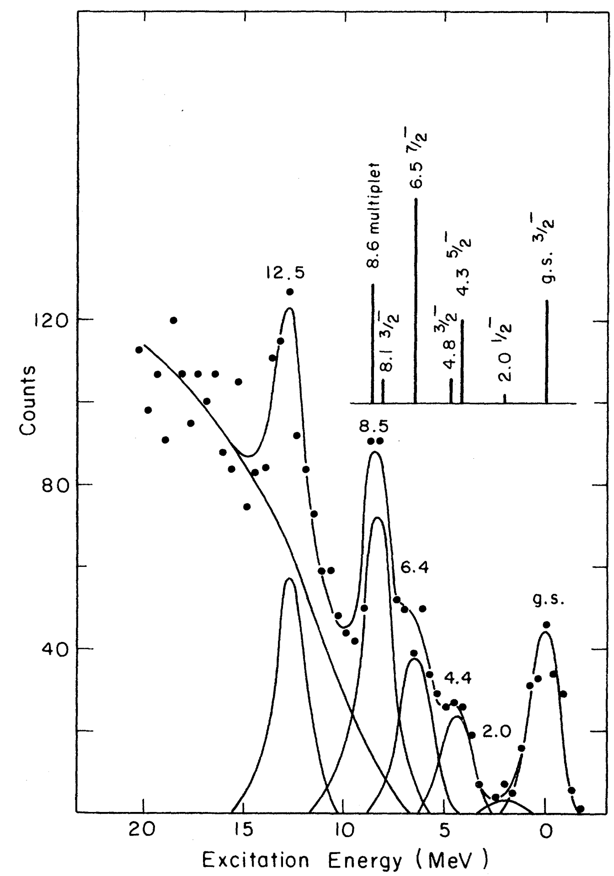

FIG. 1. The $^{12}C(\pi^+,p)^{11}C$ spectrum at $T_\pi = 49.3$ MeV, $\theta_{lab} = 45°$. The solid lines show the individual peak shapes, background, and total sum. In the upper right-hand corner, the lengths of the lines indicate the relative intensity of states populated in the reaction $^{12}C(p,d)^{11}C$ with the same momentum transfer, $\Delta q = 2.44 \pm 0.24$ fm^{-1} ($T_p = 700$ MeV, $\theta_{lab} = 11.3°$, Ref. 1).

These results are in sharp contrast to the low-momentum-transfer neutron-pickup reactions in which the $\frac{3}{2}^-$ ground state and to a lesser extent the $\frac{1}{2}^-$ 2.0-MeV, and $\frac{3}{2}^-$ 4.8-MeV states dominate the spectra. Because these states have the largest single-neutron spectroscopic strength,[9] they have the largest one-step reaction amplitudes. The (π^+,p) data, in agreement with the (p,d) data, indicates that the one-step process is less important at high momentum transfer. The similarity of the 700-MeV (p,d) to the (π^+,p) spectra suggests that the peak centered at 6.4 ± 0.2 MeV is predominantly the known 6.48-MeV $\frac{7}{2}^-$ state, as observed in the (p,d) experiments,[1,5] and not the 6.34-MeV $\frac{1}{2}^+$ state. The 6.48-MeV state has a dominant configuration of $1p_{3/2}$ hole coupled to the $^{12}C(4.44)$ 2^+. This result is very similar to a strong peak observed in the reaction[10] $^{12}C(p,\pi^+)^{13}C$ at (7.57 ± 0.16)-MeV excitation in ^{13}C which is likely to be predominantly the $\frac{5}{2}^-$ 7.55-MeV state with the configuration $1p_{1/2}$ particle coupled to the $^{12}C^*(4.44)\ 2^+$.[11]

In comparison to the $^{12}C(p,d)$ spectra, the $^{12}C(\pi^+,p)$ spectra have a much larger three-body continuum. This result is expected because pion mass-to-energy conversion is most favored when shared by more than one nucleon. A second dissimilarity between the two reactions is the strong peak centered at 12.5 ± 0.3 MeV excitation in the (π^+,p) reaction. This peak is not observed in the (p,d) reaction at any bombarding energy or angle including both 700 and 800 MeV[12] (p,d) data. This suggest that the peak is the 12.5-MeV $\frac{1}{2}^-$, $T=\frac{3}{2}$ state which is isospin forbidden in the (p,d) reaction. This state has a large one-particle, two-hole component $1p_{1/2}\ (1p_{3/2})^{-2}$ based on the ^{12}C core and is strongly populated in the reaction[13] $^{13}C(p,t)^{11}C$

Angular distributions are shown in Fig. 2. The differential cross sections to the $\frac{3}{2}^-$ ground state is nearly twice as large but similar in shape to the $^{16}O(\pi^+,p)^{15}O^*$ 6.18-MeV $\frac{3}{2}^-$ state at $T_\pi=66$ MeV.[14] The $\frac{3}{2}^-$ ^{11}C ground state takes about[9] 80% of the $1p_{3/2}$ neutron spectroscopic strength. The

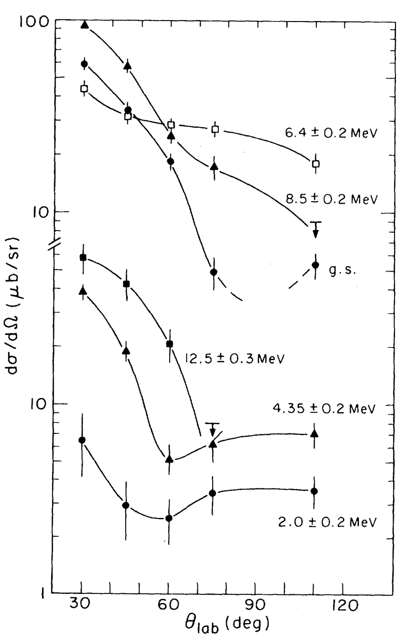

FIG. 2. Angular distributions for the reaction $^{12}C(\pi^+,p)^{11}C$ at 49.3 MeV. Statistical errors including background errors are shown. The overall normalization error is about 15%. Lines through the data are freely drawn.

angular distributions to the 2.0 and the 6.4 MeV-peaks are much flatter than the other angular distributions. The cross section of the ground state relative to the 2.0-MeV state is in the ratio 10 to 1 at 30° in agreement with the spectroscopic strengths, $(2J+2)S$. At back angles the ratio is only 1.5 to 1 suggesting that multistep processes to these states become more important at back angles.

In summary, the reactions $^{12}C(p,d)C^{11}$ and $^{12}C(\pi^+,p)C^{11}$ compared at approximately the same momentum transfer give similar intensities to the low-energy states of ^{11}C. Although energy resolution is inadequate to uniquely identify peaks above 4 MeV excitation in the (π^+,p) spectra, comparison with other data suggests the identification of the 6.48- and 12.5-MeV states. These states have significantly different nuclear structure from the ground state and will provide a severe test of the various models of the (π^+,p) reaction mechanism. The intriguing possibility that the (π^+,p) reaction strongly populates states with $T=1+|N-Z|/2$ offers a new tool for studying these higher-isospin states.

Simultaneously with the measurement of the (π^+,p) cross sections, deuterons from the reaction[15] $^{12}C(\pi^+,d)^{10}C$ were observed. The laboratory cross section to the $^{10}C(0^+$ g.s. $+2^+$ 3.35 MeV) states is 650 ± 250 nb/sr at $\theta=30°$. This is in excellent agreement with the DWBA calculation of Betz and Kerman,[15] which includes both the distortion of the incoming pion and outgoing deuteron waves. The (π^+,d) reaction is sensitive to those neutron pair correlations with small relative momentum and high total momentun with respect to the rest of the nucleus.

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Theory of a Free-Electron Laser

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(Received 18 August 1977)

A classical, linear theory is given for the gain of a short-wavelength free-electron laser. Waves propagating along the relativistic beam are unamplified when the beam is cold. Two of the six geometric optics modes have phase velocities $< c$. When thermal effects are included, the theory predicts the possibility of substantial amplification of these two modes for asymmetric electron distribution functions, due to wave-particle resonance.

Two recent experiments on the Stanford Linear Accelerator Center's superconductive linear accelerator have rekindled wide interest in the possibility of free-electron lasers.[1,2] A classical single-particle radiation analysis for this system was published in 1951 by Motz,[3] who later carried out experiments[4] with 100-MeV electrons. Phillips[5] posed the problem as an O-type traveling-wave interaction, and successfully built and operated microwave devices (Ubitrons) of high efficiency (~ 10%) and high power output (~ 800 kW). Quantum-mechanical calculations, posing the interaction as stimulated Compton scattering—a nonlinear process—have been published by Madey,[6] Sukhatme and Wolf,[7] and by Colson.[8] Hopf *et al.*[9] have reproduced Madey's result using a classical theory based on the relativistic collisionless Boltzmann equation, and have also published a strong-signal theory.[10] General features of stimulated processes leading to gain have been summarized by Granatstein and Sprangle.[11] Kwan, Dawson, and Lin[12] have carried out a linear stability analysis for a cold beam employing periodic boundary conditions and compared the predicted temporal growth rates with numerical simulations.

The linearized theory presented here invokes a mechanism not included in the above, namely wave-particle resonance, but neglects the free-streaming effects which underlie prior work. It indicates that within the geometric-optics approximation, a cold relativistic beam propagating along the axis of a helical static magnetic field does not amplify short-wavelength radiation propagating along the beam. Moreover, it suggests that sizable gain can be obtained using a beam with a finite momentum spread, but that this spread must be asymmetric. When the gain mechanism described here is operative it is likely that the above nonlinear theories must be modified so as to incorporate it.

We take the periodic transverse static magnetic field to be the curl of a vector potential

$$\vec{A}(z) = (mc^2/e)\vec{V}(z), \tag{1}$$

where $\vec{V}(z) = \vec{e}_x V_x(z) + \vec{e}_y V_y(z)$ is the dimension-