

Then, the tunneling probability between the classical vacua  $\alpha$  and  $\beta$  becomes

$$p(\beta, t|\alpha, u) = [u_E(\beta)/u_E(\alpha)] \sum_{n=0}^{\infty} u_n(\beta)u_n(\alpha) \exp[-(E_n - E)(t - u)/\hbar]. \quad (12)$$

For sufficiently large  $t - u$ , we can approximate Eq. (12) by taking only one counter term into account, obtaining

$$p(\beta, t|\alpha, u) \simeq [u_E(\beta)u_0(\beta)u_0(\alpha)/u_E(\alpha)] \exp[(E - E_0)(t - u)/\hbar]. \quad (13)$$

Thus we have found that the penetration rate of the system with energy  $E < V(\gamma)$  from the "false" vacuum  $\alpha$  to the "true" one  $\beta$  is given by  $(E - E_0)/\hbar$ , which can be computed immediately in each practical case.

I conclude with the following comments.

- (i) Quantized motion of the system in the energy eigenstate is subjected to a Markov process.
- (ii) Tunneling probability of the system with energy  $E < V(\gamma)$  through the local potential barrier  $\gamma$  is given by the transition probability density of the Markov process.
- (iii) In the semiclassical limit, the tunneling probability thus obtained reduces to the well-known WKB prescription.
- (iv) Penetration rate through the local potential barrier  $\gamma$  is proportional to the energy of the system measured from the lowest energy eigenvalue.
- (v) My approach to quantum mechanical tunneling phenomena has a close analogy with Langer's analysis<sup>10</sup> of the nucleation process in classical statistical mechanics.
- (vi) My formulation is valid not only for one-dimensional systems but also for higher-dimensional ones.

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## Quantum Nondemolition Measurements of Harmonic Oscillators

Kip S. Thorne, Ronald W. P. Drever,<sup>(a)</sup> Carlton M. Caves,  
Mark Zimmermann, and Vernon D. Sandberg  
*California Institute of Technology, Pasadena, California 91125*

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The complex amplitude  $X_1 + iX_2 \equiv (\mathbf{x} + i\mathbf{p}/m\omega)e^{i\omega t}$  of a harmonic oscillator is constant in the absence of driving forces. Although the uncertainty principle forbids precise measurements of  $X_1$  and  $X_2$  simultaneously ( $\Delta X_1 \Delta X_2 \geq \hbar/2m\omega$ ),  $X_1$  alone can be measured precisely and continuously ("quantum nondemolition measurement"). Examples are given of measuring systems that do this job. Such systems might play a crucial role in gravitational-wave detection and elsewhere.

A standard technique for measuring very weak forces is to let them act on a high- $Q$  harmonic oscillator, and then to monitor the motion of the oscillator.<sup>1</sup> Examples are Dicke-Eötvös experiments and gravitational-wave detectors.<sup>1</sup> Some

future gravitational-wave detectors may use massive ( $m \sim 100$  kg) dielectric crystals (sapphire or silicon) with eigenfrequencies  $\omega/2\pi \sim 5000$  Hz, cooled to a few millidegrees where their fundamental modes would contain  $N = kT/\hbar\omega \sim 10^4$  quanta

(phonons) and might have  $Q \sim 10^{13,2}$ . The number of quanta would random walk due to Nyquist forces, with a mean time between jumps ( $\Delta N = 1$ ) of  $\Delta t \approx Q/\omega N^2 \sim 3$  sec—which might be long compared to the readout time of the detectors. Clearly, such oscillators are quantum mechanical devices.<sup>1</sup>

In quantum theory, one describes an oscillator by a generalized coordinate  $\hat{x}$  and a generalized momentum  $\hat{p}$  with commutator  $[\hat{x}, \hat{p}] = i\hbar$ . It is useful to introduce the complex amplitude,

$$\hat{X}_1 + i\hat{X}_2 \equiv (\hat{x} + i\hat{p}/m\omega)e^{i\omega t}, \quad (1)$$

where  $t$  is time,  $\omega/2\pi$  is eigenfrequency, and  $m$  is generalized mass. Note that  $(m\omega)^{1/2}\hat{X}_1$  and  $(m\omega)^{1/2}\hat{X}_2$  are canonically conjugate observables; i.e.,  $[\hat{X}_1, \hat{X}_2] = i\hbar/m\omega$ . In the Heisenberg picture  $d\hat{X}_j/dt = (-i/\hbar)[\hat{X}_j, \hat{H}_I]$ , where  $\hat{H}_I$  is the interaction part of the Hamiltonian. Thus, in the absence of interactions,  $\hat{X}_1$  and  $\hat{X}_2$  are constant.

All standard systems for measuring an oscillator attempt to measure  $\hat{X}_1$  and  $\hat{X}_2$  “simultaneously” and with roughly equal precision—usually by monitoring  $\hat{x}(t)$  over one or more cycles. The uncertainty principle,  $\Delta X_1 \Delta X_2 \geq \frac{1}{2} |[\hat{X}_1, \hat{X}_2]|$ , limits the precision of such measurements to  $\Delta X_1 \approx \Delta X_2 \approx (\hbar/2m\omega)^{1/2}$ , which corresponds to an uncertainty  $\Delta N \approx (N + \frac{1}{4})^{1/2}$  in the number of quanta  $\hat{N} = (m\omega/2\hbar)(\hat{X}_1^2 + \hat{X}_2^2) - \frac{1}{2}$ . This limit was first derived by Braginsky<sup>3</sup> for a Fabry-Perot coordinate sensor and an optical lever, and later by Giffard<sup>4</sup> for any measuring system whose output  $V(t)$  is a linear function of  $x(t)$ . The above derivation generalizes it to any system which tries to measure both  $\hat{X}_1$  and  $\hat{X}_2$ .<sup>5</sup>

Braginsky<sup>6</sup> has pointed out that the above “quantum limits” on  $\Delta X_1$ ,  $\Delta X_2$ , and  $\Delta N$  pose serious obstacles for gravitational-wave detection: To encounter at least three supernovae per year, one must reach out to the Virgo cluster of galaxies. But gravitational waves from supernovae at that distance will produce  $|\Delta X_1| \approx |\Delta X_2| \approx 0.3 \times [m/(10 \text{ tons})]^{1/2} (\hbar/m\omega)^{1/2}$  in a mechanical oscillator on earth, corresponding to  $\Delta N \approx 0.4(N + \frac{1}{2})^{1/2} [m/(10 \text{ tons})]$ . For detectors of reasonable mass this signal is below the quantum limit.

Braginsky<sup>6,7</sup> has suggested a way out of this impasse: Instead of attempting to measure  $\hat{X}_1$  and  $\hat{X}_2$ , measure the number of quanta  $\hat{N}$  directly—and give up all information about the phase  $\hat{\psi} = \tan^{-1}(\hat{X}_2/\hat{X}_1)$ . Unfortunately, direct measurements of  $\hat{N}$  suffer from two problems: (i) To avoid changing the number of quanta while measuring it (“quantum nondemolition measurement”),

the interaction part of the Hamiltonian  $\hat{H}_I$  must commute with  $\hat{N}$ —which means that  $H_I$  must be quadratic (or higher order) in the coordinate and momentum of the oscillator. However, quadratic couplings are much more difficult to achieve in practice than linear couplings—especially when the signals are so weak. (ii) From a sequence of precise measurements of  $\hat{N}$  one cannot infer the precise time dependence  $F(t)$  of a weak force (signal) driving the oscillator. One can only get an estimate of the spectral density of  $F(t)$  near the oscillator’s resonant frequency. [From sequences of measurements of  $\hat{N}$  on an infinite number of oscillators, all coupled to the same  $F(t)$ , one can infer  $|F_\omega(t)| = |\int_0^t F(t')e^{-i\omega t'} dt'|$ ].

This Letter proposes a new type of quantum nondemolition measurement—one which circumvents the above two problems. The idea is simple: Measure  $\hat{X}_1$ , and give up all information about  $\hat{X}_2$  and about  $\hat{N}$ . A linearly coupled system to do this must have

$$\hat{H}_I = K\hat{X}_1\hat{\phi} = K\hat{x}\hat{\phi}\cos\omega t - K(\hat{p}/m\omega)\hat{\phi}\sin\omega t, \quad (2)$$

where  $\hat{\phi}$  is an observable of the measuring system and  $K$  is a coupling constant. In making its measurement, the system perturbs  $\hat{X}_2$  but leaves  $\hat{X}_1$  unaffected:  $d\hat{X}_1/dt = 0$ ,  $d\hat{X}_2/dt = -(K/m\omega)\hat{\phi}$  [Heisenberg picture]. The output of the system is some suitable observable  $\hat{J}$  which fails to commute with  $\hat{\phi}$ , and which therefore evolves as

$$\frac{d\hat{J}}{dt} = \frac{-i}{\hbar}[\hat{J}, \hat{H}_M] + \frac{\partial \hat{J}}{\partial t} - \frac{i}{\hbar}K[\hat{J}, \hat{\phi}]\hat{X}_1,$$

where  $\hat{H}_M$  is the Hamiltonian of the measuring system. The readout for a suitably chosen  $\hat{J}$  can give a precise result for the value of  $\hat{X}_1$ .

Suppose that a measurement with such a system at time  $t=0$  gives a precise value  $\xi_0$  for  $\hat{X}_1$ , and thereby puts the oscillator into the eigenstate  $|\xi_0\rangle$  of  $\hat{X}_1(t=0) \equiv \hat{X}_1(0)$ . In the absence of driving forces,  $\hat{X}_1$  remains constant, so that subsequent measurements give the same precise value  $\xi_0$ .

Now couple the oscillator to a weak, classical driving force  $F(t)$ —e.g., a gravitational wave. The interaction Hamiltonian is  $\hat{H}_I = -\hat{x}F(t)$ ; and the resulting evolution of  $\hat{X}_1$  is  $\hat{X}_1(t) = \hat{X}_1(0) - \int_0^t [F(t')/m\omega] \sin(\omega t') dt'$ . The initial state  $|\xi_0\rangle$  is an eigenstate of  $\hat{X}_1(t)$  with eigenvalue  $\xi(t) = \xi_0 - \int_0^t [F(t')/m\omega] \sin(\omega t') dt'$ . Consequently, subsequent precise measurements of  $\hat{X}_1$  leave the state of the oscillator unchanged and give the result  $\xi(t)$ . By a suitable choice of measuring system, in principle one could monitor  $\hat{X}_1$  precisely and continuously<sup>8</sup>; and from the results one could

infer, precisely, the time dependence  $F(t)$  of the driving force:  $F(t) \sin \omega t = -m\omega d\xi/dt$ . [In practice, one might want to couple two oscillators to the driving force, and measure  $\hat{X}_1$  for one oscillator, getting  $F(t) \sin \omega t$ , and  $\hat{X}_2$  for the other, getting  $F(t) \cos \omega t$ .]

Systems that measure  $\hat{X}_1$  without perturbing it are relatively straightforward to design. For example, one can couple the oscillator sinusoidally to both a coordinate sensor and a momentum sensor, and add the sensor outputs before sending them through any amplifiers or other noisy or dissipative circuit elements. Alternatively, one can suitably modulate and add the outputs from a coordinate and a momentum sensor which have constant coupling to the oscillator. Unfortunately, most such systems superimpose uncertainty-principle noise on the output signal. The figure shows an idealized system which, in principle, can be noiseless. The oscillator is an  $LC$  circuit; the sensors for charge (generalized coordinate) and current (generalized momentum) produce torques on a torsion pendulum; and a torque balance measures the sum of the torques. We describe this system classically:

The  $LC$  oscillator consists of the two coils (total inductance  $L$ ) near the bottom of the figure, and the four capacitor plates  $A$ ,  $A'$ ,  $B$ , and  $B'$  near the top. The total capacitance between  $A$  and  $A'$  (via  $B$  and  $B'$  and a zero-impedance voltage source connecting them) is  $C$ . The generalized coordinate  $x$  of the oscillator is the charge on plate  $A$ , the eigenfrequency is  $\omega \equiv (1/LC)^{1/2}$ , and the generalized mass is the inductance  $L$ . The coordinate (charge) sensor consists of plates  $B$  and  $B'$  (mechanically attached to the torsion pendulum), to which are applied a sinusoidal volt-

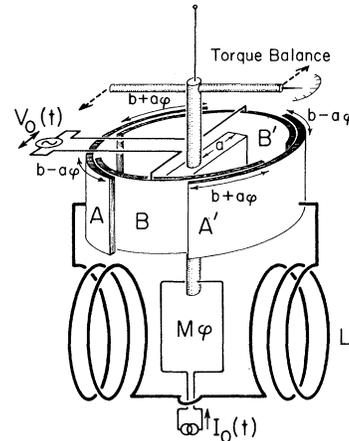


FIG. 1. Schematic of apparatus described in text.

age difference  $V_0 \equiv -(b/a)K \cos \omega t$ . This voltage, together with the oscillator's charge  $x$ , produces a torque  $\Gamma = -Kx \cos \omega t - (K^2C \cos^2 \omega t) \varphi$  on the pendulum. The momentum (current) sensor consists of the thin wire loop at the bottom of the figure, through which a sinusoidal current  $I_0 \equiv (K/M\omega) \sin \omega t$  is driven. The loop is attached to the central rod so that its mutual inductance with the oscillator,  $M\varphi$ , is proportional to the angular displacement  $\varphi$  of the torsion pendulum. Current (momentum) in the oscillator produces a torque  $\Gamma = K(\dot{x}/\omega) \sin \omega t$  on the pendulum. The torsion pendulum consists of the central rod and paraphernalia attached to it, and the torsion fiber that suspends it. (For the moment we ignore the torque balance.) The pendulum has moment of inertia  $I$ , natural frequency  $\Omega$ , and generalized coordinate (equal to angular displacement)  $\varphi$ .

The complete apparatus is described by the classical Lagrangian

$$\mathcal{L} = \frac{1}{2}L(\dot{x}^2 - \omega^2 x^2) + \frac{1}{2}I(\dot{\varphi}^2 - \Omega^2 \varphi^2) - \frac{1}{2}(K^2C \cos^2 \omega t)\varphi^2 - K[x \cos \omega t - (\dot{x}/\omega) \sin \omega t] \varphi. \quad (3)$$

The generalized momenta are  $p = \partial \mathcal{L} / \partial \dot{x} = L\dot{x} + (K/\omega)(\sin \omega t)\varphi$  and  $J = \partial \mathcal{L} / \partial \dot{\varphi} = I\dot{\varphi}$ ; and the Hamiltonian, after quantization, is

$$\hat{H} = \frac{1}{2}\hat{p}^2/L + \frac{1}{2}L\omega^2\hat{x}^2 + K\hat{X}_1\hat{\varphi} + \frac{1}{2}\hat{J}^2/I + \frac{1}{2}I\bar{\Omega}^2\hat{\varphi}^2. \quad (4)$$

Here the eigenfrequency  $\bar{\Omega}$  of the pendulum is shifted from its natural value  $\Omega$  by coupling to the coordinate and momentum sensors,  $\bar{\Omega}^2 = \Omega^2 + K^2C/I$ .

The interaction part of the Hamiltonian (4) has the desired form (2). In the measurement process, the oscillator's amplitude produces a torque  $\hat{\Gamma} = (d\hat{J}/dt)_{\text{signal}} = -K\hat{X}_1$  on the pendulum; and one

monitors this torque and thence  $\hat{X}_1$  with a torque balance that keeps  $\varphi$  as close to zero as possible. For an ideal torque balance with only uncertainty-principle noise,  $\hat{\Gamma}$  can be monitored with precision  $\Delta\Gamma \approx (\hbar/\tau^3)^{1/2}$ , where  $\tau$  is the balance's averaging time for sensing rotations. The corresponding precision on  $\hat{X}_1$  is

$$\Delta X_1 \approx \left(\frac{\hbar}{L\omega}\right)^{1/2} \left(\frac{I\omega^2}{K^2C}\right)^{1/2} \left(\frac{1}{\omega\tau}\right)^{3/2}. \quad (5)$$

In the limit  $K^2C/I\omega^2 \rightarrow \infty$ , the measurements can be arbitrarily accurate and arbitrarily quick.<sup>8</sup>

To achieve better accuracy than the usual quantum limit  $(\hbar/2L\omega)^{1/2}$ , with averaging times less than a cycle, requires  $K^2C/I\omega^2 \gtrsim 1$  ("strong coupling" of oscillator to torsion pendulum) and also strong coupling of the pendulum to the rotation-sensing part of the torque balance. Such strong coupling may be very hard to achieve in practice.

This system is only one of several schemes for coupling to the  $\hat{X}_1$  of an  $LC$  circuit. By running such systems backwards, one could measure the  $\hat{X}_1$  of a mechanical oscillator (torsional or linear), with the readout being a voltage or current in an  $LC$  circuit. The  $\hat{X}_1$  of a microwave cavity could be measured by coupling to "coordinate" and "momentum" with appropriately placed electric and magnetic dipole antennas.

If one is willing to accept a time resolution of half the oscillator period or longer ( $\tau \gtrsim \pi/\omega$ ), then one can avoid the simultaneous use of coordinate and momentum sensors. For example, one could make "pulsed" measurements of  $\hat{X}_1$  by coupling only to  $\hat{x}$  at times  $\omega t = n\pi$  [ $\hat{H}_I$  of the form (2) with  $K \propto \delta(\sin\omega t)$ ]. Physically, one measures  $\hat{x} = \hat{X}_1$  at  $\omega t = 0$ , obtaining a precise value  $\xi_0$  and giving  $\hat{p}$  a huge kick. Although the kick causes  $\hat{x}$  to evolve in an unknown way, it returns to  $(-1)^n \times$ (its original value) at times  $\omega t = n\pi$ . Subsequent precise measurements of  $\hat{x}$  at these times give values  $(-1)^n \xi_0$  (if driving forces are absent), and give unimportant kicks to  $\hat{p}$ . Errors  $\Delta t$  in the timing of such measurements—due to either timing jitter or finite pulse duration—produce a spread  $\Delta x \approx [(\omega\Delta t)(\hbar/m\omega)]^{1/2}$  in the results.

As another example, one could measure  $\hat{X}_1$  with precision  $\Delta X_1 \ll (\hbar/m\omega)^{1/2}$  by suitably coupling a coordinate sensor to  $\hat{x}$  in a continuous but oscillatory manner, and by averaging the output over a time  $\tau \gg 1/\omega$ . For example, one could sinusoidally couple the coordinate  $\hat{x}$  of the primary oscillator to the coordinate  $\hat{\phi}$  of a secondary oscillator in a manner that gives the Hamiltonian

$$\hat{H} = \frac{1}{2}\hat{p}^2/m + \frac{1}{2}m\omega^2\hat{x}^2 + K\hat{\phi}\hat{x}\cos\omega t + \frac{1}{2}\hat{J}^2/I + \frac{1}{2}I\Omega^2\hat{\phi}^2. \quad (6)$$

Then the interaction Hamiltonian,  $\hat{H}_I = K\hat{\phi}\hat{x}\cos\omega t = \frac{1}{2}K\hat{\phi}(\hat{X}_1 + \hat{X}_1\cos 2\omega t + \hat{X}_2\sin 2\omega t)$ , produces a time-averaged output "force"  $\langle -\partial\hat{H}_I/\partial\hat{\phi} \rangle$  on the secondary oscillator proportional to  $\hat{X}_1$  with little contamination from  $\hat{X}_2$ . If this "force" is measured by a device whose back action has spectral density which falls off rapidly above  $f_{\max} \sim 1/\tau$ , then the secondary oscillator will perturb  $\hat{X}_1$  negligibly

while perturbing  $\hat{X}_2$  strongly. This near-nondemolition measurement is analogous to the Unruh<sup>9</sup>-Braginsky<sup>10</sup> scheme for measuring the number of quanta  $\hat{N}$  in an oscillator. Their interaction Hamiltonian has the form  $\hat{H}_I = K\hat{\phi}\hat{x}^2$ ; and they measure  $\hat{N}$  by averaging over a time  $\tau \gg 1/\omega$ . [Note that a precise coupling to  $\hat{N}$  requires summing the outputs of sensors of  $\hat{x}^2$  and  $(\hat{p}/m\omega)^2$ . For example, in an electromagnetic oscillator ( $LC$  circuit or microwave cavity)  $\hat{x}^2 \propto$ (electric field)<sup>2</sup> can be sensed by tension on a capacitor plate or at a suitable point on a cavity wall, or by the force on a dielectric object;  $\hat{p}^2 \propto$ (magnetic field)<sup>2</sup> can be sensed by induced-current forces in a conductor or on a cavity wall, or by the force on an object of high magnetic permeability; and the sum  $\hat{x}^2 + (\hat{p}/m\omega)^2$  can be constructed mechanically.]

Several fundamental principles underlie this Letter: (i) A perfect nondemolition measurement of an observable  $\hat{A}$  at times  $t_0, t_1, \dots, t_n$  is possible only if<sup>11</sup>  $\hat{A}(t_0) = \hat{A}(t_1) = \dots = \hat{A}(t_n)$ . (ii) For continuous measurements, this requires  $d\hat{A}/dt = 0$ —which usually means that  $\hat{A}$  commutes with the interaction Hamiltonian  $\hat{H}_I$ , and that  $\partial\hat{A}/\partial t = (i/\hbar)[\hat{A}, \hat{H} - \hat{H}_I]$ . (iii) Near-nondemolition measurements can be achieved by coupling to an observable  $\hat{B}$  ( $\hat{H}_I \propto \hat{B}$ ) that equals such an  $\hat{A}$  plus an oscillating quantity, by averaging over many oscillation cycles, and by using a measuring system with a spectral density of back-action force that is vanishingly small near the oscillation frequency.

These principles, and the examples discussed in this paper, may have uses in a variety of fields of physics and engineering.

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<sup>(a)</sup>On leave of absence from Department of Natural Philosophy, Glasgow University, Glasgow, Scotland.

<sup>1</sup>See, e.g., V. B. Braginsky and A. B. Manukin, *Measurement of Weak Forces in Physics Experiment* (Univ. of Chicago Press, Chicago, Ill., 1977).

<sup>2</sup>"Experimental Gravitation, Proceedings of 1975 Pavia Conference," edited by B. Bertotti (to be published)

(see chapters by V. B. Braginsky and D. H. Douglass).

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<sup>4</sup>R. Giffard, *Phys. Rev. D* **14**, 2478 (1977).

<sup>5</sup>A simple extension of our argument gives a very general proof of the quantum limit,  $T_n \geq \hbar\omega / (k \ln 2)$ , on the noise temperature of amplifiers. For a less general proof see H. Hefner, *Proc. IRE* **50**, 1604 (1962).

<sup>6</sup>V. B. Braginsky, unpublished lectures at California Institute of Technology, Stanford University, Louisiana State University, Massachusetts Institute of Technolo-

gy, and Princeton University.

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<sup>8</sup>The possibility of precise, arbitrarily quick nondestruction measurements was discussed by J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932), pp. 222-237.

<sup>9</sup>W. G. Unruh, to be published.

<sup>10</sup>V. B. Braginsky, Y. V. I. Vorontsov, and F. I. Hali, to be published.

<sup>11</sup>Actually the condition is  $\hat{A}(t_0) = f_1[\hat{A}(t_1)] = \dots = f_n[\hat{A}(t_n)]$ , where  $f_j$  is an arbitrary real function; but we ignore this trivial generalization.

## Measurement of the $D$ Semileptonic Branching Ratio in $e^+e^-$ Annihilation at the $\psi''(3770)$

W. Bacino, A. Baumgarten, L. Birkwood, C. Buchanan, R. Burns,<sup>(a)</sup> M. Chronoviat, P. Condon, R. Coombes, P. Cowell,<sup>(b)</sup> A. Diamant-Berger,<sup>(c)</sup> M. Faessler,<sup>(d)</sup> T. Ferguson, A. Hall, J. Hauptman, J. Kirkby, J. Kirz,<sup>(e)</sup> J. Liu,<sup>(f)</sup> L. Nodulman, D. Ouimette, D. Porat, C. Rasmussen, M. Schwartz, W. Slater, H. K. Ticho, S. Wojcicki, and C. Zupancic<sup>(g)</sup>

*Stanford Linear Accelerator Center and Physics Department, Stanford University, Stanford, California 94305, and University of California, Los Angeles, California 90024, and University of California, Irvine, California 92664*

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We have observed the  $\psi''$  resonance in the cross section for  $e^+e^- \rightarrow$  hadrons at  $E_{c.m.} = 3770 \pm 6.0$  MeV, of total width  $\Gamma = 24 \pm 5$  MeV and partial width to electron pairs  $\Gamma_{ee} = 180 \pm 60$  eV. The cross section for hadronic events which contain anomalous electron provides both unambiguous evidence of  $D$  semileptonic decays and a branching ratio measurement of  $(11 \pm 2)\%$ .

We report the first results of a SPEAR experiment performed with a new detector DELCO<sup>1</sup> (Fig. 1), designed to identify electrons over 60% of the total solid angle by means of a large atmospheric-pressure Cherenkov counter.

Six concentric cylindrical multiwire proportional chambers (MWPC) extend from the beam pipe to a radius of 30.0 cm. The inner four cylinders subtend 80% of  $4\pi$  steradians. Azimuthal readout is provided by axial anode wires of 2-mm spacing and crude depth measurement by four cylindrical high-voltage foils divided into 1-cm-wide strips inclined at  $\pm 45^\circ$  to the beam axis. The MWPC's are in a 3.5-kG magnetic field provided by two discrete coils wrapped on steel pole pieces 85 cm apart with a return yoke on the outside of the detector. The magnet provides a near-axial field over the MWPC volume, with an average field integral out to the magnetostrictive wire spark chambers of 1.7 kG m.

Immediately beyond the MWPC the particles enter a twelve-module ethane-filled Cherenkov counter<sup>2</sup> sensitive only to electrons ( $\pi$  threshold

$= 3.7$  GeV/c). Particles which count by striking the photocathodes are identified by plastic guard counters. Within each module, the Cherenkov light is focused by a 1.5 m  $\times$  1.5 m ellipsoidal mirror via a flat mirror onto a 5-in. RCA 4522

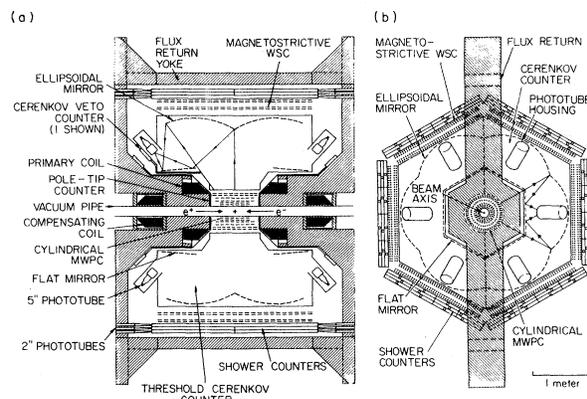


FIG. 1. (a) Polar and (b) azimuthal projections of the apparatus. For illustrative purposes, in (a) the apparatus in the yoke has been rotated by  $30^\circ$ .