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Partial Radiative Muon Capture on ^{12}C

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I report on a calculation of radiative muon capture on ^{12}C(g.s.) to ^{12}B(g.s.), done within the framework of the impulse approximation and a standard shell-model description of the nucleus. It is shown that the photon asymmetry as well as polarization and alignment of the ^{12}B nucleus depends strongly on the magnitude of the induced pseudoscalar coupling. The effects of possible contributions of second-class axial currents are also studied.

The long-standing problem of determining the pseudoscalar form factor and testing its momentum dependence predicted by partial conservation of axial-vector current (PCAC) may be settled by measuring either radiative muon capture on the proton or an exclusive capture process on a suitable nucleus. Both experiments seem to be of similar difficulty. In this Letter I show that exclusive radiative capture on ^{12}C is a good candidate for this matter. I show, specifically, that alignment and polarization of the daughter nucleus ^{12}B determine g_P if second-class currents are assumed to be absent, rather independently of the uncertainties inherent in the theoretical treatment.

In a radiative capture process $\mu^- + (Z,A) \to (Z-1,A) + \nu_\mu + \gamma$ one expects polarizations and asymmetries to depend strongly on the spins and parities of initial and final nuclear states. Therefore it is important to study exclusive radiative capture, i.e., radiative capture into definite final nuclear states, rather than integrated capture rates. I have investigated two typical examples of such partial transitions, ^{12}C(g.s.) with $I^P = 1^+$ to ^{12}B(g.s.) with $I^P = 1^+$, and ^{16}O(g.s.) with $I^P = 0^+$ to ^{16}N(120-keV level) with $I^P = 0^-$. In this Letter I report on some of my results for the first of these processes

$$\mu^- + {}^{12}C(\text{g.s.}) \to {}^{12}B(\text{g.s.}) + \nu_\mu + \gamma \quad (1)$$

and comment briefly on the capture process on oxygen. A more detailed account of these calculations for both cases will be presented elsewhere.

Inclusive radiative capture has been investigated by Rood and Tolhoek for the example of ^{40}Ca.[1] In particular, these authors study the effects of varying the induced pseudoscalar and second-class tensor contributions as well as the dependence of the predicted capture on nuclear-model uncertainties. Their treatment is based, however, on the closure approximation over nuclear final states.[2]

Exclusive radiative capture from ^{12}C(g.s.) to ^{12}B(g.s.) has been studied by Bernstein.[3] He was the first to consider the effects of weak magnetism g_M and induced pseudoscalar g_P. For g_P he uses an effective pointlike γ_5 interaction which neglects the variation due to the pion pole. In his calculation which uses the impulse approximation he also neglects the velocity-dependent nuclear matrix elements. Recently Beder has analyzed partial radiative muon capture on ^{3}He to ^{3}H applying the elementary-particle model.[4]

In this work, my main goal is to study the sensitivity of radiative capture to the induced pseudoscalar coupling and to possible second-class-current contributions as well. The relevant form factors are defined as usual through the nucleonic matrix element of the weak axial current,

$$\langle p'|A^{\nu}|p\rangle = \bar{u}(p')[g_A\gamma^{\nu} + \alpha_P g_P(p-p')^{\nu} + \alpha_T g_T(p-p')_{\rho}(i\sigma^{\rho\nu}/2M)]\gamma^5 u(p). \quad (2)$$

Here I have introduced the scaling factors α_P and α_T in order to study the sensitivity of the various experimental quantities to the pseudoscalar and tensor coupling. In the absence of second-class currents and with the standard PCAC relationship[5] for the pseudoscalar form factor, one has

$$\alpha_T = 0, \quad \alpha_P = 1, \qquad g_P(Q^2) = \sqrt{2} f_{\pi} g_{\pi NN}/(Q^2 - m_{\pi}^2). \quad (3)$$

I arbitrarily take g_T to be equal to the magnetic form factor g_M ("weak electricity" of same magnitude as weak magnetism). In contrast to nonradiative muon capture where these form factors appear at a fixed momentum transfer of about $-m_{\mu}^2$, the leptonic momentum transfer in radiative capture varies between approximately $-m_{\mu}^2$ and $+m_{\mu}^2$, depending on the energy of the outgoing photon. This variation with photon energy can be utilized to test the sensitivity of the radiative capture process to these coupling terms and possibly even to test the functional dependence (3) of $g_P(Q^2)$ on Q^2 (if the tensor term is assumed to be absent).

The capture process is treated in the impulse approximation. The elementary amplitude $\mu^- p \to n\nu_{\mu}\gamma$ is derived from the amplitude for ordinary muon capture using Low's theorem.[6] This construction proceeds along the lines worked out by Adler and Dothan.[7] The Hamiltonian is then deduced from this gauge-invariant relativistic amplitude by an expansion in terms of p/M (p being the nucleon momentum, M being the nucleon mass). Terms up to first order in p/M are kept. They contribute up to 10% in the angular correlations and polarizations. The reduction to the nonrelativistic Hamiltonian is done by means of the algebraic program SCHOONSCHIP.[8] The ground state of ^{12}C is described by a fully closed $p_{3/2}$ subshell of a harmonic oscillator potential with oscillator parameter $b = 1.65$ fm.[9] For the ^{12}B ground state I used the $|p_{3/2}{}^h, p_{1/2}{}^p|$ particle-hole state coupled to spin 1^+ of the same harmonic oscillator potential.[10]

In this Letter I discuss the following observables[11]: (i) the photon spectrum $R(E_{\gamma})$ for unpolarized muons, normalized to the ordinary muon capture rate, (ii) the photon polarization integrated over all directions of the photon with respect to the muon spin, $P_{\gamma}(E_{\gamma})$; (iii) the polarization $p_B(E_{\gamma})$ and alignment $a_B(E_{\gamma})$ of the final nucleus. These quantities are defined through the following expressions:

$$R(E_{\gamma}) = \frac{S(E_{\gamma})}{\Gamma_{\mu}} = \frac{\alpha M_{\text{tot}}}{2\pi M_f E_{\max}} \Big(\int dP_f P_f \tfrac{1}{2} \sum_{\substack{\text{all}\\ \text{spins}}} |M_{fi}|^2\Big) \Big/ \Big(\tfrac{1}{2} \sum_{\substack{\text{all}\\ \text{spins}}} |T_{fi}|^2\Big).$$

Here, $R(E_{\gamma})$ is the branching ratio of the radiative to the normal muon capture rate Γ_{μ}; P_f is the recoil momentum of ^{12}B; M_{fi} the radiative matrix element; M_f is the mass of the final nucleus; M_{tot} is the mass of the (^{12}C, μ) system, $E_{\max}$ (= 91.81 MeV) is the energy of the neutrino in ordinary capture; T_{fi} is the corresponding matrix element.

The integrated photon polarization $P_{\gamma}(E_{\gamma})$ is given by

$$P_{\gamma}(E_{\gamma}) = \Big[\int dP_f P_f \sum_{\substack{\text{nuclear spin,}\\ \text{muon spin}}} (|M^+|^2 - |M^-|^2)\Big] \Big/ \Big[\int dP_f P_f \sum_{\substack{\text{nuclear spin,}\\ \text{muon spin}}} (|M^+|^2 + |M^-|^2)\Big].$$

The +, − signs in $M^{\pm}$ correspond to the respective helicities of the photon.

The polarization $p_B(E_\lambda)$ of the final nucleus ^{12}B along the photon direction is defined in exactly the same way. For this case the +, − signs correspond to the $m_z = \pm 1$ states of ^{12}B in the direction of the photon.

Finally I define the alignment of ^{12}B through

$$a_B(E_\gamma) = W_{+1} + W_{-1} - 2W_0,$$

where W_α is the probability to find the $m_z = \alpha$ state of ^{12}B such that

$$W_{+1} + W_{-1} + W_0 = 1.$$

For the numerical calculation I use the following values for the coupling constants[5]:

$$g_V(0) = 1, \quad g_A(0) = -1.258, \quad g_M(0) = 3.7,$$

$$f_\pi = 131.7 \text{ MeV}, \quad g_{\pi NN}^2/4\pi = 14.6.$$

In Table I the integrated branching ratios for some values of α_P and α_T are given. Column 3 gives the total branching ratio

$$R_{\text{tot}} = \int_{5\text{ MeV}}^{E_{\max}} R\, dE_\gamma.$$

Column 4 gives the branching ratio for photons with energy higher than 60 MeV:

$$R_{\text{high}} = \int_{60\text{ MeV}}^{E_{\max}} R\, dE_\gamma.$$

The last column gives the ordinary capture rates (calculated with the same nuclear model) for the respective values of α_P and α_T normalized to the rate Γ_μ^0 for $\alpha_P = 1$ and $\alpha_T = 0$.

In Fig. 1 the branching ratio $R(E_\gamma)$ is drawn. One sees that the inclusion of an induced tensor of $\alpha_T = \pm 2$ changes the absolute rates but does not change the *shape* of the spectrum. On the other hand, the variation of α_P changes the shape significantly by shifting the central part of the spectrum towards lower energies as α_P decreases.

TABLE I. Branching ratios for photons from 5 to 91.81 MeV (R_{tot}) and from 60 to 91.81 MeV (R_{high}) and nonradiative capture rates $\Gamma_\mu(\alpha_P, \alpha_T)$, normalized to $\Gamma_\mu^0 = \Gamma_\mu(\alpha_P = 1, \alpha_T = 0)$. $\Gamma_\mu^0 = 35.8 \times 10^3$ s^{-1} (cf. Ref. 4).

α_P	α_T	$10^4 \times R_{\text{tot}}$	$10^4 \times R_{\text{high}}$	$\Gamma_\mu(\alpha_P, \alpha_T)/\Gamma_\mu^0$
1	0	3.09	0.452	1.000
0	0	2.55	0.277	1.158
−1	0	2.17	0.208	1.400
1	2	3.59	0.580	0.848
1	−2	2.82	0.369	1.217

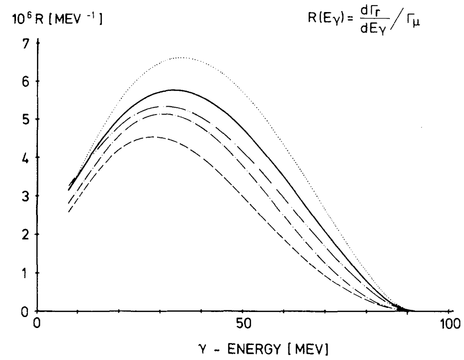

FIG. 1. Branching ratio for the following combinations of scaling factors: ——, $\alpha_P = 1$, $\alpha_T = 0$; −·−·, $\alpha_P = 0$, $\alpha_T = 0$; −−−−, $\alpha_P = -1$, $\alpha_T = 0$; ··−··−, $\alpha_P = 1$, $\alpha_T = -2$; ··········, $\alpha_P = 1$, $\alpha_T = +2$.

(Compare R_{high} with R_{tot} in Table I.)

Figure 2 shows the photon polarization $P_\gamma(E_\gamma)$. Note that the behavior of $P_\gamma(E_\gamma)$ is very similar to the results given by Rood and Tolhoek[1] for ^{40}Ca; this seems to indicate that the main contribution to the ^{40}Ca closure approximation comes from the allowed Gamow-Teller transition to a 1^+ intermediate state. This similarity is specific to the capture process (1). For the case of the radiative $^{16}\text{O}(0^+) \rightarrow {}^{16}\text{N}(0^-)$ transition, I find a qualitatively different behavior of the asymmetries and polarizations.[12] Figures 3 and 4 show $p_B(E_\gamma)$ and $a_B(E_\gamma)$, respectively. A measurement of the these quantities seems feasible.[13] Both observables are very sensitive to the values of α_P and α_T. Positive and negative values of α_P are easily distinguishable by comparing p_B and a_B.

For $P_\gamma(E_\gamma)$ and $a_B(E_\gamma)$ the induced tensor shifts the curves for $\alpha_T = 0$ up and down without changing the shape significantly.

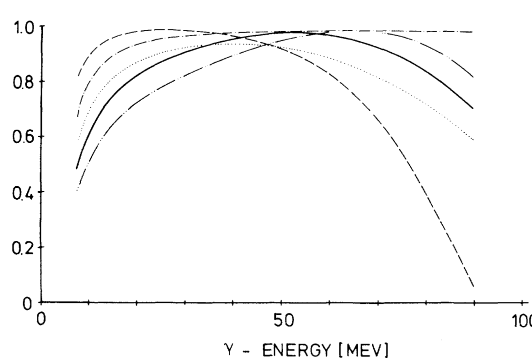

FIG. 2. Photon polarization with the same conventions for the curves as Fig. 1.

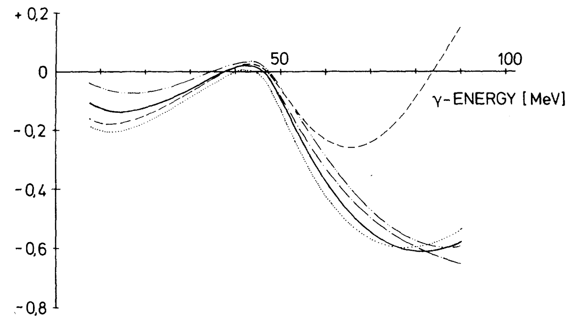

FIG. 3. Polarization $p_B(E_\gamma)$ of ^{12}B, with the same scaling factors as Fig. 1.

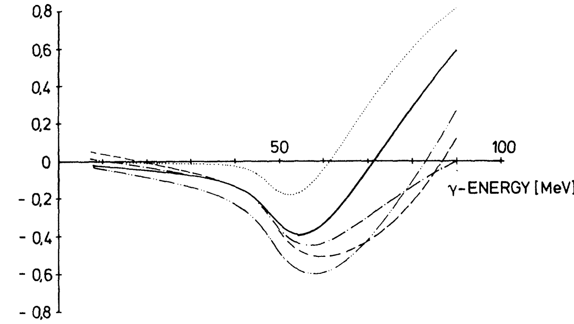

FIG. 4. Alignment $a_B(E_\gamma)$ of ^{12}B, with the same scaling factors as Fig. 1.

From these results I conclude the following: (1) Measurements of p_B and a_B can determine sign and magnitude of α_P if α_T is assumed to be zero. (2) A simultaneous determination of both α_P and α_T, however, does not seem possible in a reliable manner because of the remaining uncertainties in the calculations.

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