High-Energy Collisions of Heavy Nuclei

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The profile function for the collision of two heavy nuclei is calculated. In principle, the method used sums all contributing diagrams. The single-particle inclusive distribution is predicted for very-high-energy collisions.

The main objective of this Letter is the summation of the Glauber rescattering series for the collision of a heavy nucleus projectile A_2 on a heavy target A_1. I have in mind lab energies upwards of a few GeV, i.e., present or foreseeable experiments. The rescattering series comes from expansion in powers of Γ of the expression

$$S_{A_1A_2}(\vec{b}) = \int d^2x_1\,\rho_1(\vec{x}_1)\cdots\int d^2x_{A_1}\,\rho_1(\vec{x}_{A_1})\int d^2y_1\,\rho_2(\vec{y}_1)\cdots\int d^2y_{A_2}\,\rho_2(\vec{y}_{A_2})\prod_{i=1}^{A_1}\prod_{j=1}^{A_2}[1-\Gamma_{ij}(\vec{x}_i-\vec{y}_j-\vec{b})], \qquad (1)$$

where ρ_i is the single-particle density of nucleus A_i [normalized such that $\int d^2x\,\rho(\vec{x})=1$]. Γ_{ij} is the profile function for collision of nucleons i, j. The nuclear centers are transversely separated by $\vec{b}$.

This series is considerably more complicated than that for a hadron-nucleus collision. Terms of the series may be represented by diagrams, examples of which are illustrated in Fig. 1. Vertices on the upper/lower levels are nucleons in nuclei A_1, A_2, respectively. Each line joining nucleons i, j involves a factor $-\Gamma_{ij}$. Line (a) consists of diagrams contributing to a hadron-nucleus collision. Line (b) is the set of diagrams building the familiar Chou-Yang (CY) model.[1] More complicated collisions are shown in line (c). On each line of diagrams in lines (a)–(c) there is in momentum space only a momentum transfer $O(R^{-1})$, where R is a nuclear radius. However, in line (d) we encounter closed loops. The nuclear form factors do not cut off the momentum flowing in a closed loop. Such momentum is therefore of order GeV/c.

If the number, N, of lines occurring in a diagram is fixed and $A_1, A_2 \to \infty$ then, as noted by Czyz and Maximon,[2] the dominant diagram at each N is in the set (b). It was argued[2] that in the limit $A_1, A_2 \to \infty$ with σ_{NN} fixed, the CY model is correct.

There are at least two features that make this argument incomplete. Firstly, in the rescattering series, diagrams of varying N are added together with alternating signs, so that the full sum may not be determined by the sum of the dominant terms at each N. Secondly, unless $\sigma_{NN} \to 0$, the dominant values of N in the series are surely dependent on A_1, A_2 and increase as $A_1, A_2 \to \infty$. Now each diagram contains combinational factors involving N and not estimated in Ref. 2. Since $N_{\text{eff}} = N_{\text{eff}}(A_1, A_2)$ these factors effectively acquire A dependence. It is therefore unclear which diagrams really dominate if $A_1, A_2 \to \infty$ at fixed σ_{NN}.

I will show that the diagrams of line (b) are dominant only if $\sigma_{NN} \to 0$. In the physical limit $A_1, A_2 \to \infty$ at fixed σ_{NN} one obtains a completely different answer.

The task of adding all the terms in the rescattering series can be performed using Reggeon field theory[3] (RFT). Suppose that each nucleon in the collision can radiate eikonally an arbitrary number of Pomerons. The reader unfamiliar with RFT may prefer to think of the Pomerons as pure imaginary Gaussian absorptive potentials.

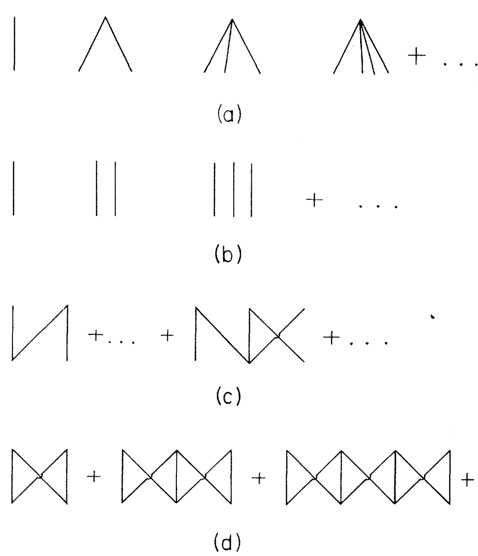

FIG. 1. Some diagrams contributing to the scattering of two heavy nuclei.

The Lagrangian for RFT is

$$\mathcal{L}_0 = \tfrac{1}{2}\bar{\psi}\frac{\overleftrightarrow{\partial}}{\partial y}\psi + \Delta\bar{\psi}\psi + \alpha'\partial_\perp\bar{\psi}\partial_\perp\psi + \tfrac{1}{2}ir_0(\bar{\psi}^2\psi + \bar{\psi}\psi^2). \tag{2}$$

The source term for emission of Pomerons by one nucleon of A_1 is

$$\rho_i(\vec{x}_\perp)\delta(y)(e^{ig\bar{\psi}} - 1). \tag{3}$$

The absorption of Pomerons by a nucleon in A_2 is accomplished by

$$\rho_2(\vec{y}_\perp)\delta(Y-y)(e^{ig\psi} - 1). \tag{4}$$

The S matrix at fixed impact parameter can now be written down.

$$S_{A_1A_2} = \sum_{M_1=0}^{A_1}\binom{A_1}{M_1}\int d^2x_1\cdots d^2x_{M_1}\prod_{i=1}^{M_1}\{\rho_1(\vec{x}_i)[\exp(ig\bar{\psi}(\vec{x}_i,0))-1]\}$$
$$\times\sum_{M_2=0}^{A_2}\binom{A_2}{M_2}\int d^2y_1\cdots d^2y_{M_2}\prod_{j=1}^{M_2}\{\rho_2(\vec{y}_j)[\exp(ig\psi(\vec{y}_j,y))-1]\}\int\delta_{\bar{\psi}}\delta_\psi e^{-F_0}, \tag{5}$$

where F_0 is the action due to $\mathcal{L}_0$. My notation is standard: Y is the lab rapidity of nucleons of A_2. $g^2 \simeq \sigma_{NN}$ as $Y\to\infty$. The path integral is to be performed with boundary conditions

$$\psi = 0 \text{ for } y < 0, \tag{6}$$

$$\bar{\psi} = 0 \text{ for } y > Y. \tag{7}$$

Thus

$$S_{A_1A_2} = \int\delta_{\bar{\psi}}\delta_\psi e^{-F}, \tag{8}$$

where

$$F = F_0 - A_1\ln\int d^2x_\perp e^{ig\bar{\psi}}\rho_1(x_\perp) - A_2\ln\int d^2x_\perp e^{ig\psi}\rho_2(x_\perp). \tag{9}$$

It is well known for similar eikonal problems[4] that the S matrix is a certain generating functional. The approximation

$$\binom{A_i}{m_i}\approx\frac{A_i^{m_i}}{m_i!} \tag{10}$$

is not valid for our problem; so we cannot further simplify the action. The path integral (8) is normalized so that $S_{A_1A_2}\to 1$ as $\rho_1, \rho_2 \to 0$.

Since I am mainly interested in near future experiments I concentrate on lab energies of a few GeV. Therefore set $\Delta, r_0 = 0$. Inclusion of nonzero Δ is trivial. If r_0 is nonzero the method used can still be applied, but the algebra will be a lot more complicated.

In principle $S_{A_1A_2}$ can be estimated in the limit $A_i\rho_i\to\infty$ by application of the saddlepoint method. Generally one will obtain

$$S \simeq e^{-F_{\rm cl}}/N_0, \tag{11}$$

where the action $F_{\rm cl}$ is calculated using the equations of motion

$$\delta F/\delta\bar{\psi} = \delta F/\delta\psi = 0 \tag{12}$$

and N_0 is determined by considering small oscillations about the classical minimum of the action.

In the differential equations we solve to determine $A_{\rm cl}, N_0$, for energies of a few GeV, $\alpha'Y$ is a small parameter compared with R_1^2, R_2^2, g^2. I therefore neglect the third term of $\mathcal{L}_0$. The equations of motion become

$$\dot{\psi} - \frac{igA_1\rho_1\delta(y)e^{ig\bar{\psi}}}{\int d^2x_\perp\rho_1(x_\perp)e^{ig\bar{\psi}}} = 0, \tag{13}$$

$$\dot{\bar{\psi}} + \frac{igA_2\rho_2\delta(Y-y)e^{ig\psi}}{\int d^2x_\perp\rho_2(x_\perp)e^{ig\psi}} = 0. \tag{14}$$

Setting $-ig\psi = s\theta(y)$, $-ig\bar{\psi} = t\theta(Y-y)$, and $g^2\rho_iA_i = \theta_i$, I obtain

$$s - \theta_1e^{-t}/\int d^2x_\perp\rho_1(x_\perp)e^{-t(x_\perp)} = 0, \tag{15}$$

$$t - \theta_2e^{-s}/\int d^2x_\perp\rho_2(x_\perp)e^{-s(x_\perp)} = 0. \tag{16}$$

I now illustrate the properties of these equations in some simple situations. First consider the case of one-dimensional space so there are no transverse integrations in Eqs. (15) and (16). The solution is simply $s = A_1g^2$, $t = A_2g^2$, and we have

$$S_{A_1A_2} = \exp(-g^2 A_1 A_2). \quad (17)$$

This is the CY result. That it is exact in one-dimensional space is obvious from Eq. (1).

Now, consider physical (three-dimensional) space but weak coupling ($g^2 \to 0$). The approximate solution of Eqs. (15) and (16) is now, as $\theta_i \to 0$,

$$s \simeq \theta_1, \quad t \simeq \theta_2, \quad (18)$$

and again the CY result is good.

At finite g^2 the solution of Eqs. (15) and (16) is rather complicated. In order to illustrate the general properties of the solution we shall ignore "edge effects" and treat each nucleus as a cylinder aligned along the z axis (the direction of motion).

For typical $b \sim R_1 \sim R_2$ the denominators of Eqs. (15) and (16) are $O(1)$. At small g the solution is (18). As g increases to a finite value, at large $A_i \rho_i$, two additional real solutions of Eqs. (15) and (16) appear. They are

$$s_0 \simeq \theta_1, \quad t_0 \simeq 0; \quad (19)$$

$$t_0 \simeq \theta_2, \quad s_0 \simeq 0. \quad (20)$$

The corresponding actions are

$$F_1 = -A_1 \ln(1 - \beta/\pi R_2^2), \quad (21)$$

$$F_2 = -A_2 \ln(1 - \beta/\pi R_1^2), \quad (22)$$

respectively. $\beta(\vec{b})$ is the area of overlap of the nuclei.

On the other hand, the original minimum of the action evolves at finite g to the region $s \sim \ln\theta_1$, $t \sim \ln\theta_2$ leading to an action $F \simeq F_1 + F_2$. This contribution is therefore negligible.

It can easily be checked that there are no oscillations about the solutions. We conclude that

$$S_{A_1A_2}(\vec{b}) = e^{-F_1} + e^{-F_2}. \quad (23)$$

We now turn to a different question. As long as the approximation of neglecting Pomeron interaction is valid we can at sufficiently high energy predict the form of $d\sigma/dy$ for $A_1 + A_2 \to \pi + X$. Using the Abramovskii-Kancheli-Gribov rules,[5] the only graph to contribute is the single-Pomeron-exchange graph so that

$$d\sigma(A_1 + A_2)/dy = A_1 A_2\, d\sigma(N+N)/dy. \quad (24)$$

As has been discussed before[6-8] the fragmentation region for nucleus dissociation by a projectile is roughly

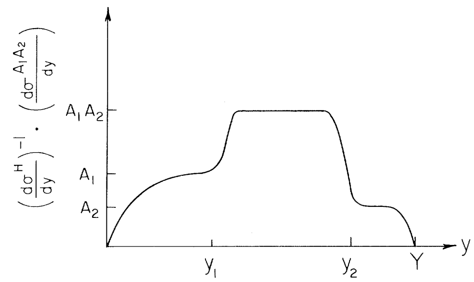

FIG. 2. Predicted form of single-particle inclusive production from two heavy nuclei at very high energies (TeV per nucleon in the lab).

$$y \lesssim \ln(2R\bar{m}), \quad (25)$$

where $\bar{m}$ is a mass parameter, roughly the inverse radius of a nucleon. So we expect for $y \lesssim \ln(2R_1\bar{m}) = y_1$

$$d\sigma(A_1 + A_2)/dy \simeq A_1\, d\sigma(N+N)/dy \quad (26)$$

and for $Y > y > Y - \ln(2R_2\bar{m}) = Y - y_2$

$$d\sigma(A_1 + A_1)/dy \simeq A_2\, d\sigma(N+N)/dy. \quad (27)$$

Consequently, for

$$Y > \ln(4R_1R_2\bar{m}^2) \quad (28)$$

we expect $d\sigma(A_1 + A_2)/dy$ to have the form sketched in Fig. 2. The lab energy per nucleon of the projectile is of order thousands of GeV in formula (28) for reasonable-sized nuclei. So the validity of Fig. 2 cannot be tested for some time yet.

Our predictions for total and inclusive cross sections differ from those of Kancheli[6] and Amati, Coneschi, and Jengo.[9] Their models include part of the effect of Pomeron interactions but allow each nucleon to be struck at most once. In view of the weakness[10,11] of the triple-Pomeron coupling these assumptions would not be expected to apply at foreseeable energies.

For lower energy than allowed by (28) the central hump of Fig. 2 should disappear.

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Evidence for Angular Momentum Fractionation in ^{86}Kr-Induced Reactions on ^{107,109}Ag, ^{165}Ho, and ^{197}Au

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γ-ray multiplicities have been measured as a function of the light-fragment atomic number Z_3 for the above reactions. The failure of the measured γ multiplicities for deep-inelastic collisions to rise with decreasing Z_3, according to the rigid-rotation limit, appears more likely to be associated with a selective population of the low-Z fragments by the lower-l waves rather than with an incomplete relaxation of the rotational energy.

Heavy-ion reaction studies have shown the existence of a rotating "intermediate complex" consisting of a targetlike and a projectilelike fragment which undergoes equilibration in its various degrees of freedom.[1] These equilibration processes, like the relaxation of the relative motion, the neutron-to-proton ratio, and the mass asymmetry, have been extensively investigated.[1] The angular momentum transfer from orbital to intrinsic rotation, leading to the equilibration of rotational degrees of freedom, has been investigated to a lesser degree.[2-5]

Measurements of γ-ray multiplicities M_γ have proven to be a good technique for determining the intrinsic angular momentum in compound nuclei.[6,7] This technique can be applied to deep-inelastic (DI) and quasielastic collisions in order to determine the angular momentum transfer as a function of mass asymmetry as determined from the light-fragment atomic number Z_3. From this dependence it is possible to obtain information on the extent to which rigid rotation has been attained.

In a previous study[2] of the reaction Ag + 175-MeV ^{20}Ne we have shown that (a) for quasielastic products, very close in Z to the projectile, M_γ increases linearly with the mass transfer; (b) for the deep-inelastic components at backward angles, which show nearly *complete* kinetic-energy relaxation, the rigid-rotation limit has been essentially attained; (c) for intermediate degrees of kinetic-energy relaxation, and at somewhat forward angles, γ-ray multiplicities smaller than those expected for rigid rotation are observed. The limited data available for heavier systems[4,5] do not show immediately recognizable patterns, and are difficult to interpret because of the lack of systematics and because of experimental limitations. To bridge the gap from relatively light to heavy systems, we have studied Kr-induced reactions for targets spanning a large mass range (^{107,109}Ag, ^{165}Ho, and ^{197}Au).

In contrast to the Ne + Ag system[2] where the deep-inelastic products cover only a narrow angular momentum window, the present systems give rise to DI products over most if not all of the angular momentum range, with small complete-fusion components. The much larger angular momentum range of these reactions $[l=(0-300)\hbar]$ opens the interesting possibility of having different mass distributions or, in other words, angular momentum fractionation may occur among products of different Z. On the other hand, the possible large transfers of energy and angular momentum to the heavy fragment can also favor decay modes which can efficiently dispose of the angular momentum, like fission and ^{4}He emission. In order to eliminate such a difficulty, we

