havior in t with a slope of 8.0 ± 0.7 $(\text{GeV}/c)^{-2}$ in excellent agreement with the value 7.36 ± 0.14 $(\text{GeV}/c)^{-2}$ found in the literature for $\pi^- p$ elastic scattering at 4.16 GeV/c [11] and 9.0 ± 0.6 $(\text{GeV}/c)^{-2}$ found at 3.9 GeV/c in our data.[10] The solid line has a slope of 8.0 $(\text{GeV}/c)^{-2}$.

In conclusion, from examining the helicity-0, unnatural parity exchange in both the ρ^0 and ω^0 inclusive production, we find that ρ^0 production in the beam fragmentation region seems to proceed by pion exchange, while in the central region other mechanisms are dominant. For the ω^0, all exchange contributions are equally important in all kinematic regions. A triple-Regge analysis of the ρ^0 inclusive production in the appropriate kinematic region confirms that pion exchange is dominant and we extract the π trajectory. We also find that off-mass-shell corrections are small.

We wish to thank Stanford Linear Accelerator Center and the operating crew of the 82-in. bubble chamber for their proficient production of this exposure and the scanning and measuring technicians of the PEPR analysis group for their superior efforts. We would also like to thank Professor Carleton DeTar for enlightening discussions. This work was supported in part through funds provided by the U. S. Energy Research and Development Administration under Contract No. EY-76-C-02-3069*000. One of us (J.E.B.) acknowledges receipt of a fellowship from the Fannie and John Hertz Foundation.

(a)On leave of absence from Tel-Aviv University, Tel-Aviv, Israel.

(b)Present address: Tel-Aviv University, Tel-Aviv, Israel.

(c)Present address: The Weizmann Insitute of Science, Rehovot, Israel.

[1]P. K. Malohtra, CERN Report No. CERN/EP/Phys. 76–46, 1976 (unpublished); P. Schmidt, CERN Report No. CERN/EP/Phys. 76–57, 1976 (unpublished); D. R. O. Morrison, CERN Report No. CERN/EP/Phys. 76–45, 1976 (unpublished).

[2]F. diBianca *et al.*, Phys. Lett. 63B, 461 (1976).

[3]D. H. Brick, Ph.D. thesis, Massachusetts Institute of Technology, 1973 (unpublished); P. A. Miller, S. M. thesis, Massachusetts Institute of Technology, 1974 (unpublished); P. C. Trepagnier, Ph.D. thesis, Massachusetts Institute of Technology, 1976 (unpublished).

[4]M. Aguilar-Benitez *et al.*, Phys. Rev. D 6, 29 (1972).

[5]F. Barreiro *et al.*, to be published.

[6]R. D. Field, California Institute of Technology Report No. CALT 68-466, 1974 (unpublished).

[7]C. DeTar *et al.*, Phys. Rev. Lett. 26, 675 (1971).

[8]H. Nielsen and G. C. Oades, Nucl. Phys. B41, 525 (1972).

[9]J. Pumplin, Phys. Rev. D 8, 2249 (1973).

[10]Unpublished.

[11]R. L. Eisner *et al.*, Phys. Rev. 164, 1699 (1967).

Hadronic Transitions between Quark-Antiquark Bound States

Kurt Gottfried
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 20 December 1977)

A multipole expansion of the color gauge field is shown to yield selection rules and rate estimates for hadronic transitions between nonrelativistic quark-antiquark bound states. The non-Abelian character of the field is of critical importance in determining the selection rules for the leading ("allowed") transition.

One positronium-like hadronic system, the ψ family, is now well established,[1] and another, the Υ family, appears to be in the offing.[2] Transitions within these families occur via photon and/or hadron emission, as in $\psi' \to \psi\gamma\gamma$ and $\psi' \to \psi\pi\pi$. The former can be described with some success by the concepts of atomic spectroscopy,[3] but there is no comparable theory of the hadronic transitions. I address this problem by drawing on an analogy to another spectroscopic phenomenon, the emission of atomic electrons (internal conversion) and e^+e^- pairs by nuclei.[4,5]

In the nuclear process lepton emission is caused by the changing electromagnetic field of the nucleus. As nuclear velocities are low, and as nuclei are small compared to the wavelength of the emitted leptons, a multipole expansion of the nuclear field converges rapidly and thereby provides selection rules and rate estimates. If hadrons are really composed of quarks interacting via a color gauge field, and the positroniumlike families are $Q\bar{Q}$ systems, where Q is a heavy quark of mass m, the analogy to the nuclear process is obvious: The $Q\bar{Q}$ system moves slowly and has dimensions small compared to those of the emitted light-quark (q) system. Therefore $Q\bar{Q}$ can be treated nonrelativistically, and a multipole expansion of the changing gauge field converges rapidly for sufficiently large m.

Despite these analogies, there are essential differences between the hadronic and nuclear phenomena: (i) Only statements true to all orders of perturbation theory can be taken seriously in such "soft" hadronic processes; (ii) $Q\bar{Q}$ transitions of different multipole order emit quite different hadronic systems, e.g., 2π *vis à vis* 3π. Because of these complications this is only a first step towards a complete theory. Nevertheless, I shall show that only a handful of $Q\bar{Q}$ operators govern these processes, that useful selection rules exist, that scaling laws as a function of m hold for all transitions, and that some relative rates within the Υ family can be estimated.

To expose the gist of the argument I first ignore the transverse field $\vec{A}_a$. The instantaneous interaction causing the transition is then

$$H_I{}^0 = g^2 \int [\tfrac{1}{2}\lambda_a D(\vec{r} - \tfrac{1}{2}\vec{\rho}) - \tfrac{1}{2}\lambda_a{}^* D(\vec{r} + \tfrac{1}{2}\vec{\rho})] s_a(r)\, d^3r, \tag{1}$$

where $s_a(r) = \frac{1}{2}\bar{q}(r)\gamma^0\lambda_a q(r)$ is the light-quark density, $\frac{1}{2}(\lambda_a, \vec{\sigma}_1, \vec{\rho}, \vec{v})$ and $\frac{1}{2}(-\lambda_a{}^*, \vec{\sigma}_2, -\vec{\rho}, -\vec{v})$ are the color matrices, spins, positions, and velocities of Q and $\bar{Q}$, respectively, and $D(x) = 1/4\pi x$. The multipole expansion takes the form $H_I{}^0 = H_I{}' + H_I{}''$, where

$$H_I{}' = F_a V_a [1 + O(\rho^2)], \tag{2}$$

$$H_I{}'' = G_a \vec{\rho} \cdot \vec{\mathcal{E}}_a [1 + O(\rho^2)]. \tag{3}$$

$F_a = \frac{1}{2}(\lambda_a - \lambda_a{}^*)$ is a generator of SU(3) in the $Q\bar{Q}$ subspace, while $G_a = \frac{1}{2}(\lambda_a + \lambda_a{}^*)$ has[6] singlet-octet and octet-octet $Q\bar{Q}$ matrix elements. The form of V_a and $\vec{\mathcal{E}}_a$ do not concern us as they act only on q, and the argument rests wholly on the properties of the $Q\bar{Q}$ system.

Consider the process $\Upsilon_1 \to \Upsilon_2 \lambda$, where $\Upsilon_{1,2}$ are *both singlet* $Q\bar{Q}$ states uncontaminated by q pairs, and λ is *any* hadron made of light quarks. The amplitude is

$$M_L = \sum_{k=0}^{\infty} \left\langle \Upsilon_1 \middle| H_I{}^0 \frac{1}{E_1 - H_0} \left(H_I{}^0 \frac{1}{E_1 - H_0} \right)^k H_I{}^0 \middle| \Upsilon_2 \lambda \right\rangle, \tag{4}$$

because color forbids a first-order transition. The $H_I{}^0$ abutting Υ_1 and Υ_2 must be $H_I{}''$ because $H_I{}'$ annihilates $Q\bar{Q}$ singlets, but the other $H_I{}^0$ can be either $H_I{}'$ or $H_I{}''$. As m increases, the $Q\bar{Q}$ separation $\bar{\rho} \to 0$, so all these other $H_I{}^0$ are replaced by $H_I{}'$ for a leading transition, whose amplitude is thus $O(\rho^2)$, and has the form

$$M_L = \langle \Upsilon_1 | G_a \vec{\rho} \cdot \vec{\mathcal{E}}_a\, \mathcal{G}(E_1) \vec{\rho} \cdot \vec{\mathcal{E}}_b G_b | \Upsilon_2 \lambda \rangle. \tag{5}$$

This says that a dipole ($H_I{}''$) is needed to lift the initial singlet into an octet, where monopole interactions between the QQ and q systems are possible, as described by $\mathcal{G}(E_1) = (E_1 - H_0 - H_I{}')^{-1}$. The octet then returns to the final singlet by another dipole transition. The selection rules for such an "allowed" transition are evident from (5): $\Delta S = 0$, $\Delta_L = 0$ or 2, S and L being the total spin and orbital angular momenta of $Q\bar{Q}$. We shall, however, learn that in the complete theory ($\vec{A} \neq 0$) this is not the leading transition.

I now include $\vec{A}$ in the Coulomb gauge.[7] Several interactions then arise: (i) a coupling H_T between $\vec{A}$ and $Q\bar{Q}$ whose form is familiar from electrodynamics; (ii) an $\vec{A}$-dependent instantaneous interaction H_I involving $Q\bar{Q}$; and (iii) couplings H_{qA} that do not involve $Q\bar{Q}$ at all.

I separate H_T into convection and spin parts; to lowest order in $\vec{v}$ and $\vec{\rho}$ these are[8]

$$H_v = \tfrac{1}{2} g G_a \vec{v} \cdot \vec{A}_a, \tag{6}$$

$$H_\sigma = (g/4) m G_a (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot [\nabla \times \vec{A}_a - \tfrac{1}{2} g f_{abc} \vec{A}_b \times \vec{A}_c], \tag{7}$$

where $\vec{A}_a \equiv \vec{A}_a(r = 0)$.

The interaction H_I has two terms. The first couples Q *or* $\bar{Q}$ to $q(r)$ and $\vec{A}_a$, and replaces (1). Its multipole expansion has precisely the form (2) and (3), with V_a and $\vec{\mathcal{E}}_a$ replaced by operators that depend on $\vec{A}$ as well as q. The other part of H_I is the $\vec{A}$-dependent $Q\bar{Q}$ interaction

$$H_{Q\bar{Q}} = -\tfrac{1}{8}g^2 G_c\{if_{abc}\langle\tfrac{1}{2}\vec{\rho}|\Phi_{ab}^-|-\tfrac{1}{2}\vec{\rho}\rangle + d_{abc}\langle\tfrac{1}{2}\vec{\rho}|\Phi_{ab}^+|-\tfrac{1}{2}\vec{\rho}\rangle\} - \tfrac{1}{12}g^2\langle\tfrac{1}{2}\vec{\rho}|\Phi_{aa}^+|-\tfrac{1}{2}\vec{\rho}\rangle. \qquad (8)$$

Here $\Phi = D(1-KD)^{-2}$, which is a matrix equation wherein

$$\langle x|D_{ab}|y\rangle = \delta_{ab}D(x-y) \text{ and } \langle x|K_{ab}|y\rangle = g\delta^3(x-y)f_{abc}\vec{A}_c\cdot\nabla_y,$$

while

$$\langle x|\Phi_{ab}^{\pm}|y\rangle = \langle x|\Phi_{ab}|y\rangle \pm (x \leftrightarrow y).$$

$H_{Q\bar{Q}}$ plays two roles. In combination with H_{qA} it provides a set of graphs which, for large m, supposedly generates a static $Q\bar{Q}$ interaction.[9] These are to be incorporated into the $|\Upsilon_i\rangle$ and the "unperturbed" Hamiltonian H_0 of Eq. (4). The remaining graphs contribute to the transition.

The expansion of $H_{Q\bar{Q}}$ is now obvious: Φ^+ (Φ^-) has even (odd) powers of $\vec{\rho}$, and therefore[8]

$$H_{Q\bar{Q}} \simeq G_a[I_a + \vec{\rho}\cdot\vec{\mathfrak{F}}_a + \rho_i\rho_j T_a^{ij}] + \rho_i\rho_j U^{ij} + O(\rho^3). \qquad (9)$$

It is essential to recognize that though I_a, $\vec{\mathfrak{F}}_a$, T_a^{ij}, and U^{ij} are complicated functions of the operator $\vec{A}_a$, they are merely c-numbers in the $Q\bar{Q}$ subspace.

The terms that contribute to a transition of low multipole order are thus (6), (7), (9), and (3), with $\vec{\mathcal{E}}_a$ generalized in the manner indicated. I now establish their relative magnitude when m is large.[10] To this end I assume that the observation[2] $m(\psi') - m(\psi) \simeq m(\Upsilon') - m(\Upsilon)$ implies that for the mass range of present interest[10] the characteristic $Q\bar{Q}$ period, $\bar{\rho}/\bar{v}$, is roughly m independent, and therefore $\bar{\rho}$ and $\bar{v}$, the characteristic radii and velocities, have the *same* m dependence.[11] Thus $\vec{v}\cdot\vec{A}_a$ and $\vec{\rho}\cdot(\vec{\mathcal{E}}_a + \vec{\mathfrak{F}}_a)$ are both $O(\bar{\rho})$, and for that reason I define the "electric" dipole operator

$$H_d = G_a[\vec{\rho}\cdot(\vec{\mathcal{E}}_a + \vec{\mathfrak{F}}_a) + \tfrac{1}{2}g\vec{v}\cdot\vec{A}_a]. \qquad (10)$$

To build an amplitude, return to (4). Both the first and last H are now one of the terms in (7), (9), (10). The leading ("allowed") amplitude uses G_aI_a to reach the intermediate octet, and H_d to return,[12] or vice versa[13]:

$$M_{\text{allowed}} = \langle\Upsilon_1|G_aI_a\mathcal{G}(E_1)H_d|\Upsilon_2\lambda\rangle. \qquad (11)$$

Here the resolvent $\mathcal{G}$ includes *all* interactions (such as H_{qA}) that couple the intermediate $Q\bar{Q}$ octet to q and $\vec{A}_a$. Equation (11) is depicted in Fig. 1(a). Thus the leading transition is of order $\bar{\rho}$, and satisfies $\Delta L = 1$, $\Delta S = 0$. *Observe that the very existence of (11) depends on the non-Abelian instantaneous interaction,[14] and that the leading transition (5) in the* $\vec{A} \equiv 0$ *exercise was* $O(\bar{\rho}^2)$ *and therefore had different selection rules.*

To illustrate the power and limitations of this approach, consider two superficially similar transitions in the Υ family,[15] $2^3P \to 1^3S + 3\pi$ (allowed), and $3^3S \to 1^1S + 3\pi$ ($\Delta L = 0$, $\Delta S = 1$). The latter requires H_σ to be used once, and[16] either $G_a\rho_i\rho_jT_a^{jj}$ once or H_d twice. Thus

$$\frac{\Gamma(3^3S \to 1^1S + 3\pi)}{\Gamma(2^3P \to 1^3S + 3\pi)} \simeq (k\bar{\rho})^2(k/m)^2 \simeq \tfrac{1}{400}, \qquad (12)$$

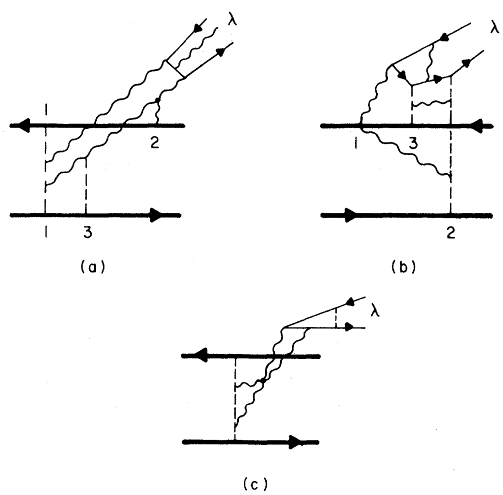

FIG. 1. Time-ordered graphs for $\Upsilon_1 \to \Upsilon_2\lambda$. Heavy and light straight lines represent Q and q, respectively; dashed lines, $D(x)$; and wavy lines, transverse gauge mesons. In (a) I_aG_a acts at t_1, H_v at t_2, yielding a $\Delta L = 1$, $\Delta S = 0$ amplitude. In (b) H_σ acts at t_1, $\vec{\rho}\cdot\vec{\mathcal{E}}$ at t_2, giving a $\Delta L = 1$, $\Delta S = 1$ transition. For $t_1 < t < t_2$, $Q\bar{Q}$ is in an octet, and H_I monopoles proportional to F_a and G_a can act, say at t_3. In (c) U^{ij} produces a direct singlet $\to$ singlet transition, with $\Delta L = 0$ or 2, $\Delta S = 0$.

where k is a typical loop momentum in Fig. 1, which I take as $\frac{1}{2}$ GeV, and[11] $\bar{\rho}\sim 1\ \mathrm{GeV}^{-1}$. Unfortunately the large ratio in (12) is not the whole story: The transitions in (12) lead to different 3π states, and also have different phase-space factors.

Currently the transition of central interest[3] is $\Upsilon'\to\Upsilon\pi\pi$, i.e., $\Delta L=\Delta S=0$. Several operators contribute: $\rho_i\rho_j U^{ij}$ can produce the transition directly [Fig. 1(c)], but so can the combinations $(I_a, \rho_i\rho_j T_b{}^{ij})$, and (H_d, H_d). All of these amplitudes are therefore[17] $O(\bar{\rho}^2)$. The m dependence of the rate also depends on the m dependence of the distance to the Okubo-Zweig-Iizuka threshold because the intermediate states in (5) and (11) are virtual decays into hadrons carrying the flavor of Q. It appears[18] that this distance is a very weak function of m, and I therefore assume the dominant m dependence to stem from the explicit factors of ρ. Thus $\Delta L=\Delta S=0$ transitions vary like[11] m^{-2}, and similar scaling laws hold for all transitions. If the asymptotic m^{-2} law is already valid for ψ's then[19] $\Gamma(\Upsilon'\to\Upsilon\pi\pi)\simeq 10$ keV.

Thus I have shown that the $Q\bar{Q}$ facet of these phenomena is quite tractable, and that the multipole expansion leads to a number of testable predictions.[20] Nevertheless, it is equally clear that a complete theory must explain how a particular multipole field converts into light hadrons. Concerning this I plead total ignorance.

It should be possible to extend the approach presented here to any process where a very small color-neutral object interacts with conventional hadrons, the most obvious examples being the photoproduction and nuclear scattering of massive $Q\bar{Q}$ states.

I am indebted to E. Eichten for innumerable suggestions, and thank T. Kinoshita and F. Wilczek for helpful remarks. This work was supported in part by the National Science Foundation.

---

[1]G. J. Feldman and M. L. Perl, Phys. Rep. 33C, 285 (1977).

[2]S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977); W. R. Innes, Phys. Rev. Lett. 39, 1240 (1977).

[3]For a recent review see K. Gottfried, Cornell University Report No. CLNS-376 (to be published).

[4]A preliminary account of these ideas appears in Ref. 3.

[5]My approach is similar in spirit to the work on $\psi\to\psi\pi\pi$ by H. Goldberg, Phys. Rev. Lett. 35, 605 (1975), but he only kept $\vec{A}$ to second order because he assumed the applicability of asymptotic freedom.

[6]"Singlet" and "octet" always refer to color.

[7]J. Schwinger, Phys. Rev. 127, 324 (1962).

[8]Here I do not show terms which couple only to $Q\bar{Q}$ octets, or which leave the selection rules unaltered to the present order. The "sea gull" $(\vec{A})^2/m$ is not shown either because it too is negligible.

[9]F. L. Feinberg, Phys. Rev. Lett. 39, 316 (1977); T. Appelquist, M. Dine, and I. J. Muzinich, Phys. Lett. 69B, 231 (1977); W. Fischler, Nucl. Phys. B129, 157 (1977).

[10]Here and throughout "large m" does *not* mean $m\to\infty$, for then the system supposedly becomes Coulombic.

[11]For the logarithmic potential [C. Quigg and J. L. Rosner, Phys. Lett. 71B, 153 (1977)], $\bar{\rho}\propto\bar{v}\propto m^{-1/2}$. I use this variation to estimate the m dependence of rates. While $\bar{\rho}$ is a reasonably well-known parameter for the ψ and Υ families, my guess or $\frac{1}{2}$ GeV for the characteristic loop momentum may be too conservative. A more solid value of k—and hence a quantitative understanding of the convergence of the multipole expansion—must await detailed numerical evaluation of typical graphs.

[12]Using I_a twice cannot result in a transition because of the orthogonality of the $Q\bar{Q}$ wave functions.

[13]This and succeeding formulas do not include final-state interactions because they do not alter the selection rules or relative rates.

[14]The leading operator I_a stems from the d_{abc} term of (8), and does not even exist in an SU(2) gauge theory. That such non-Abelian terms might dominate H_d was surmised in footnote 122 of Ref. 3.

[15]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977), especially Fig. 3.

[16]The combination $(G_a I_a, H_\sigma)$ can only produce transitions between hyperfine partners.

[17]This is no accident: All contributing amplitudes are of the same order in $\bar{\rho}$ in all cases.

[18]C. Quigg and J. L. Rosner, Fermilab Report No. Fermilab-Pub 77/101-THY (unpublished).

[19]In Ref. 3 it is shown that this rate is not inconsistent with the data. [See, however, R. N. Cahn and S. D. Ellis, Phys. Rev. D (to be published).] The m^{-2} law for $\Upsilon'\to\Upsilon\pi\pi$ was derived from a very different viewpoint by J. Ellis *et al.*, Nucl. Phys. B131, 285 (1977); for a critique see footnote 111 of Ref. 3.

[20]As another example, consider (Ref. 15) $2^3P_J\to 1^3P_{J'}+\pi\pi$. There are six such d-wave $\pi\pi$ transitions, all involving the same $\Delta L=\Delta S=0$ operators, and therefore constrained by the Wigner-Eckart theorem.