susceptibility for a series of ternary *Pd*FeMn alloys with varying Mn concentration. Three distinct concentration regimes are determined which correspond to a giant-moment ferromagnet, a double or mixed transition (paramagnetic → ferromagnetic → spin-glass), and a spin-glass. Both the susceptibility experiments and the phase diagram are interpretable in terms of the model of Sherrington and Kirkpatrick. An external magnetic field enhances the ferromagnetism while hindering the formation of the spin-glass phase. This opposite shifting of $T_c(H)$ and $T_f(H)$ leads to an interesting variety of critical and multicritical phenomena.

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# Time-Dependent Ginzburg-Landau Model of the Spin-Glass Phase

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The Edwards-Anderson spin-glass phase is studied via time-dependent Ginzburg-Landau models with quenched impurities. A perturbation expansion is used to study the dynamics and statics without the use of the replica method. It is shown that the spin correlation function has a $t^{-1/2}$ long-time tail in the spin-glass phase.

The Edwards-Anderson spin-glass phase, characterized by a frozen magnetization with zero spatial average, has received much recent attention.[1-7] Most of the analytical studies have been on the static properties using the replica ($n \to 0$) method. In this Letter we report results on both the dynamics and statics of certain time-dependent Ginzburg-Landau (TDGL) models which exhibit a spin-glass phase. We make no use of the replica method. In fact, it is our purpose to avoid the replica method, which is in several ways artificial and conceals the basic physics. This simple analysis, using TDGL models, is complementary to the numerical Monte Carlo work and to other methods reported recently.[4,5]

Our models are defined by the following equation of motion for an $n$-component vector spin density

$\vec{\sigma}(x,t)$ in a $d$-dimensional space:

$$\frac{1}{\gamma}\frac{\partial\vec{\sigma}}{\partial t}=(-\nabla^2+r_0+\varphi+\tfrac{1}{2}u\sigma^2)\vec{\sigma}+\vec{h}+\vec{a}(\vec{a}\cdot\vec{\sigma})-\frac{1}{n}a^2\vec{\sigma}+\frac{1}{\gamma}\vec{\xi}, \tag{1}$$

$$\langle\varphi(x)\varphi(x')\rangle=\Delta\delta(x-x'), \tag{2}$$

$$\langle a_i(x)a_j(x')\rangle=\Delta'\delta(x-x')\delta_{ij}, \tag{3}$$

$$\langle\xi_i(x,t)\xi_j(x',t')\rangle=2\gamma\delta(x-x')\delta(t-t')\delta_{ij}. \tag{4}$$

The external field $\vec{h}$, the random easy axis $\vec{a}$, and the noise $\vec{\xi}$ are $n$-component vectors. A random local temperature is effected by $\varphi$. We assume that $\varphi$ and $\vec{a}$ are time-independent Gaussian random variables due to the quenched impurities and $\vec{\xi}$ is a Gaussian noise. For $n=1$, the terms involving $\vec{a}$ drop out. The parameters $r_0$, $u$, $\Delta$, $\Delta'$, and $\gamma$ are constants in standard notation. For $\Delta$ and $\Delta'$ very small, analyses have been carried out studying the effect of weak impurities on the ferromagnetic critical point.[8-11]

Our main results are as follows: (a) A mean-field analysis shows that there is a range of values of $r_0$ and $\Delta$ where a spin-glass phase appears, characterized by a lack of ferromagnetic order, but where the order parameter $q$, defined by

$$\delta_{ij}q\equiv\langle\langle\sigma_i\rangle_t\langle\sigma_j\rangle_t\rangle, \tag{5}$$

is non-zero. Here the inner angular brackets $\langle\rangle_t$ stand for time average and the outer ones for space average. For $r_0$ above this range the system is in a paramagnetic phase. (b) When $n\geq 2$, $d\leq 4$, $\Delta'\neq 0$, there can be no ferromagnetic phase. The low-$r_0$ phase is a spin-glass phase.[12] (c) In the limit $n\to\infty$, with $u$, $\Delta$, $\Delta'\propto 1/n$, the mean-field theory becomes exact. The expansion in powers of $1/n$ is well defined except near the critical point. (d) The spin relaxation time is infinite in the spin-glass phase. More precisely, the correlation function

$$\delta_{ij}C(k,\omega)\equiv\int dt\,d^dx\,e^{i\omega t-ik\cdot x}\langle\sigma_i(x,t)\sigma_j(0,0)\rangle \tag{6}$$

blows up as $\omega^{-1/2}$ as $\omega\to 0$. This means a $t^{-1/2}$ tail of the Fourier transform of $C(k,\omega)$.

We proceed to outline the derivation of these results, and to comment on them.

(a) For simplicity, set $n=1$. The time average $\langle\sigma_k\rangle_t$ is, to first order in the external static field $h$, at a given $\varphi$,

$$\langle\sigma_k\rangle_t\equiv\int d^dx\,e^{-ik\cdot x}\langle\sigma(x,t)\rangle_t=G_0(k,0)h_k+G_0(k,0)\int d^dk'\,\varphi_{k-k'}G_0(k',0)h_{k'}+\dots, \tag{7}$$

where $G_0(k,\omega)\equiv(-i\omega+r+k^2)^{-1}$ is the zeroth-order spin-response function. Figure 1(a) shows the series (7).[13] Now, square (7) and average over $\varphi$. If we keep only terms shown in Fig. 1(b), we obtain

$$q=\langle\langle\sigma\rangle_t{}^2\rangle=(\Pi+\Pi\Delta\Pi+\dots)h^2 \propto\Pi(1-\Pi\Delta)^{-1}h^2, \tag{8}$$

$$\Pi=\int d^dp\,G_0(p,0)^2. \tag{9}$$

Therefore, we can have $q\neq 0$ even if $h\to 0$ provided that

$$1=\Pi\Delta. \tag{10}$$

This defines the spin-glass phase. [In $G_0$, we defined $r=G^{-1}(0,0)$, where $G(k,\omega)$ is the full spin-response function.] If we keep only the graphs in Fig. 1(c) for $G^{-1}$, we have

$$r=r_0-(\Delta-\tfrac{3}{2}u)\int d^dp\,G_0(p,0)+\tfrac{3}{2}uq. \tag{11}$$

Equation (10) determines $r$ and Eq. (11) determines the order parameter $q$ as a function of $r_0$. The spin-glass critical point $r_{0g}$ is given when $q\to 0$. This is our mean-field theory. In fewer than four dimensions, the spin-glass phase always overrides the ferromagnetic transition at $r_0=r_{0f}$. Note that $r>0$ at $r_0=r_{0g}$ and $r=0$ at $r_0=r_{0f}$; in (10), $\Pi$ is a decreasing function of $r$. Hence $r_{0g}-r_{0f}>0$. For $n>1$, the above ladder approximation is easily generalized to include $\Delta'$ and to give qualitatively the same results. If $\Delta'=0$ and $\Delta$ is sufficiently weak, however, previous analysis beyond the mean-field approximation[8] indicates that the ferromagnetic transition wins out.

The spin-glass ordering that occurs here is surprising if we accept the widely held view that the spin-glass state is the consequence of bond competition. The model we have just considered

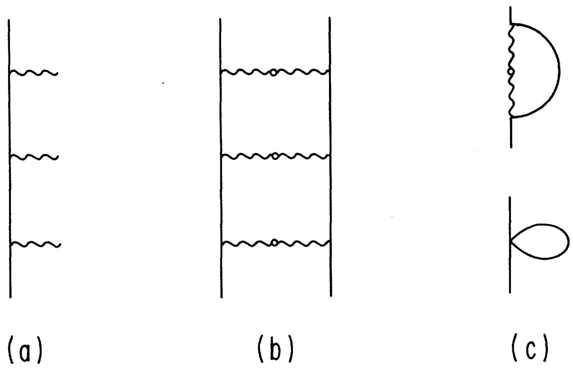


FIG. 1. (a) Graphs for the spin-response function at a given impurity configuration $\varphi$. (b) Ladder approximation for the square of (a) averaged over $\varphi$. Small circles indicate Gaussian averaging. (c) Self-energy graphs in the ladder approximation. The $\frac{3}{2}uq$ term of (11) is not shown.

is commonly described in terms of a ferromagnet with a spatially random transition temperature due to bond dilution, and there is no explicit evidence in the model for antiferromagnetic bonds, which would appear as $\nabla^2$ terms with positive coefficients in the first set of parentheses in (1). One possible explanation for this apparent contradiction is that the continuum model considered here is the result of taking a fixed-length spin system with bond competition, transforming to continuous spin variables using perhaps the dual transformation utilized by Hubbard,[14] and then thinning out degrees of freedom. The random transition temperature might then be the important signature of the competition. Another possibility is that bond competition is not necessary to produce spin-glass behavior. This latter conjecture seems far-fetched to us in the light of our current understanding of spin-glasses. There is, however, intriguing evidence based on a real-space renormalization-group analysis that in a very dilute system—one with few effective bonds—a very small number of antiferromagnetic bonds suffice to turn a ferromagnet into a spin-glass.[15] The possibility that bond dilution alone can induce spin-glass behavior merits at least passing consideration. It should be emphasized, however, that the result we have just been discussing follows from a mean-field analysis. There always remains the possiblity that a more refined treatment will invalidate it. It would certainly be useful to analyze the renormalization-group runaways in random systems[11] some of which, we believe, are signaling a spin-glass transition.

(b) The exclusion of long-range spin order for $n \geq 2$, $d \leq 4$, and $\Delta' \neq 0$ follows from an argument along the line of Imry and Ma's that a quenched random magnetic field destroys long-range order.[12,16] Let us assume $m = \langle\langle \sigma_1 \rangle_t \rangle \neq 0$. We then have what looks like a random magnetic field with zero mean, $m a_1 \vec{a} - n^{-1} a^2 \vec{m}$, according to (1), which would destroy $m$. Therefore $m = 0$.[17] The field is not isotropically distributed, but it can still be shown that transverse fluctuations are massless and the same kind of expression that diverged in Imry and Ma's argument diverges here.

Both Harris and Zobin[18] and Chen and Lubensky[2] have considered the possibility that a random easy axis can induce spin-glass–type behavior.

(c) Naturally, one might ask under what conditions the above mean-field, or ladder, approximation becomes exact. The answer is that it does when $u, \Delta, \Delta' \propto 1/n$ and $n \to \infty$, as a simple counting of graphs reveals. Although this provides an ordering of perturbation terms, it unfortunately does not allow a convenient study of the critical point. As Chen and Lubensky[2] have pointed out, the critical properties of the spin-glass–to-paramagnetic transition when the spin-glass state is due to random uniaxial anisotropy are those of an $n = 1$, or Ising-type, system with bond disorder, regardless of the number of components of the original spin system.

(d) Let us define $\nu$ as the frequency-dependent part of the spin-response function $G(k, \omega)$,

$$G^{-1}(k, \omega) = G^{-}(k, 0) + \nu(k, \omega). \tag{12}$$

Then by the identity (the fluctuation-dissipation theorem) $C(k, \omega) = \mathrm{Im} G(k, \omega)/\omega$, we have

$$C(k, \omega) \propto \nu/\omega. \tag{13}$$

The general analysis is a simple extension of the following mean-field approximation. Keeping only Fig. 1(c) for the self-energy graphs, we have, for small $\omega$,

$$\begin{aligned} \nu &= -i\omega - \Delta \int d^d p \, [G(p, \omega) - G(p, 0)] \\ &= -i\omega + \Delta \int d^d p \, \frac{\nu}{(r + p^2)^2} + O(\nu^2) \\ &= -i\omega + \Delta \Pi \nu + O(\nu^2). \end{aligned} \tag{14}$$

Since in the spin-glass phase $1 = \Delta\Pi$ by (10), Eq.

(14) gives $\nu \propto \omega^{1/2}$. To include all graphs, write

$$\nu(k,\omega) = -i\omega + \Sigma_d(k,\omega) - \Sigma_d(k,0) + \Sigma_c(k,\omega) - \Sigma_c(k,0), \tag{15}$$

where $\Sigma_c$ are self-energy graphs which would remain self-energy graphs when impurity lines are cut (i.e., before averaging over the impurity distribution), and $\Sigma_d$ would not. For small $\omega$, (15) gives

$$\nu(k,\omega) = \int d^d p\, \nu(k,\omega)\left[-\frac{\delta\Sigma_d(k,\omega)}{\delta G(p,\omega)} G^2(p,\omega)\right]_{\omega\to 0} + O(\nu^2) - O(\omega) - i\omega, \tag{16}$$

where the $O(\omega)$ terms come from the dependence on $\Sigma_c$. Now we note that the generalization of (10) is that the equation for $\lambda(p)$,

$$\lambda(p) = \int d^d p\, \lambda(p) G(p,0)^2 \Gamma(p,k), \tag{17}$$

has solutions. Here $\Gamma$ is the "rung" in the ladder sum. Graph by graph, one verifies that

$$\Gamma(p,k) = -[\delta\Sigma_d(k,\omega)/\delta G(p,\omega)]_{\omega\to 0}. \tag{18}$$

From (17), (18), and (16), we again conclude that $\nu \propto \omega^{1/2}$, i.e., $C(k,\omega) \propto \omega^{-1/2}$.[19] This $t^{-1/2}$ behavior also appears in Monte Carlo simulations.[4] Equation (18) is an important identity peculiar to systems with quenched random impurities. It shows that a static quantity $\Gamma$ can be viewed as having a dynamic origin.

With regard to spin-glass critical statics, our mean-field approximation yields the same mean-field exponents for the paramagnetic-to-spin-glass transition as have been obtained elsewhere.[1,2] It is also possible to extend our analysis beyond the mean-field approximation. We obtain exactly the same $\epsilon$-expansion results for critical statics as Lubensky and co-workers[2] obtained using the standard approach based on the replica method. Thus our method is, we believe, the correct generalization to dynamics of the static models widely believed to describe the Edwards-Anderson spin-glass transition. The calculations of the critical statics and further details of the above analysis will be given elsewhere.[20]

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[19] Note that $\omega$ in the functional derivative in (16) is

treated as a fixed parameter. The reader should convince himself that $\delta\Sigma_c(k,\omega)/\delta G(p,\omega)=0$. The divergence of $C(k,\omega)$ for $\omega\rightarrow 0$ indicates the existence of "gapless modes," and should be related to the not-too-well-understood "degeneracy" in the spin-glass phase.

[20]J. Rudnick and S. Ma, to be published.

# ERRATA

$Cr^{5+}$ COUPLING TO LATERAL PROTON CONFIGURATIONS IN $KH_2AsO_4$. J. Gaillard, P. Gloux, and K. A. Müller [Phys. Rev. Lett. 38, 1216 (1977)].

On page 1217, column 1, the sentence beginning on line 20 should read as follows: "We found that, in the $+45°$ direction in the $c$-$a$, $b$-$c$, and $a$-$b$ planes of a right-handed axis system $\{\vec{a},\vec{b},\vec{c}\}$, the ENDOR lines of one of the eight sites were $C$, $A$, and $B'$, respectively."

STUDY OF DENSITY FLUCTUATIONS IN THE ALCATOR TOKAMAK USING $CO_2$ LASER SCATTERING. R. E. Slusher and C. M. Surko [Phys. Rev. Lett. 40, 400 (1978)].

The title should be as given above.

Two numerical corrections are necessary in the first column on page 402: In line 15, 5 should be 6 and in line 20, 12 should be 14.