coupling during resonant decay of electron plasma waves. These results tend to corroborate recent theoretical predictions made for lower hybrid heating of tokamaks,[4,6] namely, that parametric instabilities driven by $\vec{E}\times\vec{B}$ coupling due to the finite-extent slow wave will be effectively stabilized in the region $\omega_0 \geq 2\omega_{pi}$.

One of the authors (K.L.W.) wishes to thank Dr. F. J. Paoloni for his assistance on the design of the rf transformer used in this experiment. Helpful discussions with Dr. P. K. Kaw and Mr. M. Ono are gratefully appreciated. This work was supported by the U. S. Department of Energy, Contract No. EY-76-C-02-3073.

[a]Present address: California Institute of Technology, Pasadena, Calif. 91109.

[1]T. H. Stix, Phys. Rev. Lett. 15, 878 (1965); V. E. Golant, Zh. Tekh. Fiz. 41, 2492 (1971) [Sov. Phys. Tech. Phys. 16, 1980 (1973)].

[2]M. Porkolab *et al.*, Phys. Rev. Lett. 38, 230 (1977).

[3]M. Porkolab, Phys. Fluids 17, 1432 (1974).

[4]M. Porkolab, to be published.

[5]R. L. Berger and F. W. Perkins, Phys. Fluids 19, 406 (1976).

[6]R. L. Berger, L. Chen, and F. W. Perkins, Princeton Plasma Physics Laboratory Report No. PPPL-1307, 1976 (unpublished).

[7]M. Porkolab and R. P. H. Chang, Phys. Fluids 13, 2054 (1970), see Eq. (16); D. Pesme *et al.*, Phys. Rev. Lett. 31, 203 (1973), see Eq. (8).

[8]S. Bernabei *et al.*, Princeton Plasma Physics Laboratory Report No. PPPL-1288, 1976 (unpublished), and Phys. Rev. Lett. 34, 866 (1975).

[9]W. M. Hooke and S. Bernabei, Phys. Rev. Lett. 29, 1218 (1972).

[10]R. P. H. Chang and M. Porkolab, Phys. Rev. Lett. 31, 1241 (1973), and 32, 1227 (1974).

[11]T. K. Chu, S. Bernabei, and R. W. Motley, Phys. Rev. Lett. 31, 211 (1973).

[12]M. Porkolab, V. Arunasalam, and N. C. Luhman, Plasma Phys. 17, 405 (1975).

[13]R. W. Boswell, in *Proceedings of the Seventh European Conference on Controlled Fusion and Plasma Physics, Lausanne* (Centre de Recherches en Physique des Plasma, 1975), p. 175.

[14]R. R. Weynants, P. Javel, and G. Muller, in *Proceedings of the Topical International Conference on Theoretical and Experimental Aspects of Heating of Toroidal Plasmas, Grenoble* (Commissariat à l'Energie Atomique, Paris, 1976), Vol. II, p. 421.

[15]W. Gekelman and R. L. Stenzel, Rev. Sci. Instrum. 46, 1386 (1975).

[16]P. Bellan and M. Porkolab, Phys. Rev. Lett. 34, 124 (1975).

Evidence for Electronic Ferromagnetism in Superfluid ^{3}He-A

D. N. Paulson and J. C. Wheatley
Department of Physics, University of California at San Diego, La Jolla, California 92093
(Received 19 December 1977)

Experiments on ^{3}He-A show that the orientation of the orbital field $\hat{l}$ depends on the *sign* of the applied magnetic field, as if there were a ferromagnetic magnetization along $\hat{l}$ estimated by $|(10\text{ mG})(1-T/T_c)\chi_N|$, in agreement with a recent prediction by Leggett.

The orbital-ordering vector $\hat{l}$ in superfluid liquid[1,2] ^{3}He-A can be conceived either as that direction along which no correlated pairs have their relative linear momentum or possibly as the direction of the relative angular momentum of the correlated pairs. Changes in the direction of $\hat{l}$ can be measured with great sensitivity[3] using the anisotropy of zero-sound attenuation with respect to the angle between $\hat{l}$ and $\vec{q}$, the sound propagation vector. In the presence of very weak superfluid flow fields $\vec{v}_s$ and for open geometries which diminish the effect of boundaries on the orientation of $\hat{l}$, the direction of $\hat{l}$ can be controlled efectively by weak magnetic fields via (at least) the nuclear magnetic susceptibility anisotropy energy (coupling the magnetic field to the spin-ordering vector $\hat{d}$ which is maintained parallel to $\hat{l}$ by the strong coherent nuclear magnetic dipolar energy). Under such rather ideal conditions, necessary because of the smallness of the present effect, we have discovered that the orientation of the average field of $\hat{l}$ for ^{3}He-A in a zero-sound cell depends on the *sign* of the applied magnetic field $\vec{H}_{ext}$: A change from $\vec{H}_{ext}$ to $-\vec{H}_{ext}$ changes the average orientation of $\hat{l}$ in a way which depends both on temperature and on the texture (field of $\hat{l}$) and which cannot be explained as a simple consequence of field rotation due to the residual trapped and therefore nonreversing magnetic fields in our apparatus. The magnitude of the effect is very close to the recent calculations of Leggett,[4] who proposed that liquid ^{3}He-A is "quite literally ferromagnetic" in an electronic sense. The existence of something like orbital ferromagnetism in superfluid ^{3}He, though not in the sense shown in these experiments nor calculated by Leggett, was foreseen already in 1960 by Anderson and Morel,[5] who, commenting on the total angular momentum of the superfluid liquid, wrote, "The only analogy ... we can draw is to the alignment of electron spins in a ferromagnet; we seem here to have ferromagnetically aligned orbital angular momentum." It is likely that in the present work we are sensing indirectly the relative angular momentum of the pairs, rather than the total angular momentum of the liquid which includes that due to an inhomogeneous $\hat{l}$ field.

The heart of our apparatus is a fused quartz cube 12.7 mm on a side with three face-centered and mutually perpendicular cylindrical holes 8 mm in diameter ground through it. One of the three mutually perpendicular axes (Z) is the same as the cryostat axis and is presumably the principal axis for heat flow. The other two are closed off by 5-MHz fundamental quartz-crystal ultrasonic transducer pairs, only one pair (X) of which is needed in these experiments. Coils are provided to make a field $\vec{H}_{ext}$ in the XZ plane; the sound is propagated along $\hat{X}$. Extensive demagnetization procedures were followed before cooling the apparatus, and the entire sound region was heavily shielded magnetically using superconducting shields. The whole apparatus was shielded against Earth's magnetic field during cooldown and additional coils allowed reduction of the residual trapped field to a few milligauss. Cooling was provided by a nuclear cooling cryostat[6] which could be manipulated to stabilize the ^{3}He temperature for long time periods, and temperature was measured magnetically by a 5% Ce lanthanum-magnesium-nitrate thermometer[7] whose reading for our temperature range appears to be as accurate as any of the provisional absolute scales.[2] The zero-sound system was operated at 25 MHz to provide a high sensitivity of sound level to average $\hat{l}$-field orientation. The measurements extended in temperature from roughly $1-T/T_c \simeq 0.06$, where the ubiquitous persistent motions[8] became too large, to $1-T/T_c \gtrsim 0.2$, where the sensitivity of sound level to $\hat{l}$ orientation became too small.

The qualitative idea of the experiment is to investigate whether a definite direction in space can be preferred by the $\hat{l}$ field of ^{3}He-A, using a magnetic field $\vec{H}_{ext}$ and ultrasound as probes; and especially using Leggett's suggestion that there is a ferromagnetic moment along the axis of $\hat{l}$. If bending energies are neglected, the free energy density affecting the local orientation of $\hat{l}$ is

$$\Delta F = -\tfrac{1}{2}\lambda_0(\hat{l}_0\cdot\hat{l})^2 + \tfrac{1}{2}\lambda_A H^2(\hat{H}\cdot\hat{l})^2 - \lambda_m H(\hat{H}\cdot l), \tag{1}$$

where λ_0 is a parameter that, for example, might reflect flow, $\vec{H}$ is the sum of residual and externally applied magnetic fields, $\hat{l}_0$ is the preferred local direction of $\hat{l}$ in $\vec{H}=0$, λ_A is the susceptibility anisotropy $\chi_N-\chi_{\hat{d}}$ measured by Paulson, Krusius, and Wheatley,[9] and λ_m is a possible ferromagnetic magnetization along $\hat{l}$. In an *idealized* geometry and $\hat{l}$ field in which $\hat{l}$ is uniform and lies in the XZ plane (the plane in which we can rotate $\vec{H}$), λ_m can be measured in terms of $\lambda_A H\Delta\theta$ in a measurement in which the sound-signal level change on rotating $\vec{H}$ through angle $\Delta\theta$ is compared with the corresponding signal change when $\vec{H}_{ext}\to-\vec{H}_{ext}$. Then from Eq. (1) one finds

$$\Delta H_{eff} \equiv \left[\frac{\text{signal }(\vec{H}_{ext}) - \text{signal }(-\vec{H}_{ext})}{\text{signal }(|H_{ext}|,\theta) - \text{signal}(|H_{ext}|,\theta+\Delta\theta)}\right] H\Delta\theta = \pm\frac{2\lambda_m}{\lambda_A}, \tag{2}$$

where $\Delta\theta$ is small and it is further supposed that the residual trapped magnetic field is zero.

In a realistic case Eq. (2) will be useful to *estimate* the size of λ_m, but an accurate measurement in our apparatus is unlikely. This can be understood by referring to the sketches in Fig. 1 which show, in a section containing $\vec{H}$, what a particularly simple $\hat{l}$ field ($\to$) might look like (no nonsingular vortices[10] in the immediate vicinity of the sound) (a) in the absence of external field, and (b) in the presence of the typical measuring field of 300 mG. The vector $\hat{l}$ is required to be normal to the bounding surfaces and should be more rigid against bending near the transmitting crystal, because of heat flow from the sound pulses, than near the receiving crystal. As a result, in the presence of $\vec{H}$, all the $\hat{l}$ field in the sound field is not directed perpendicular to $\vec{H}$; both bending energy and boundary conditions in this low field will lead to significant fractions of the fluid with $\hat{l}$ field bending toward and away

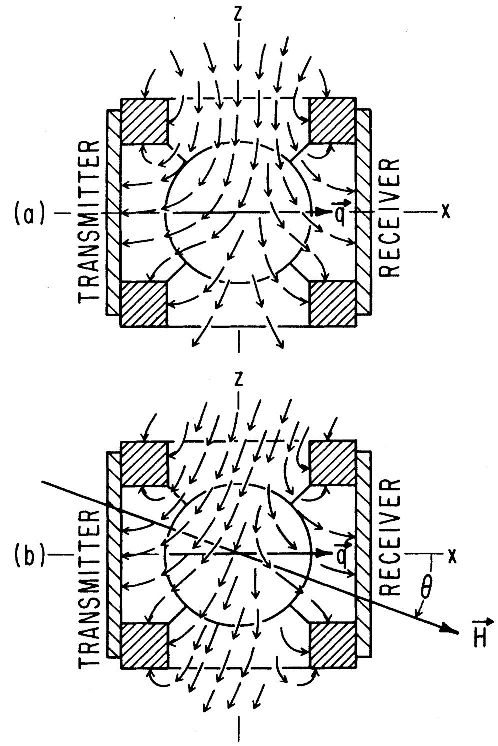

FIG. 1. Section through the sound cell illustrating possible textures of the $\hat{l}$ field (→) both (a) in zero field and (b) in external magnetic field of 300 mG. The sound propagation vector is $q\hat{X}$.

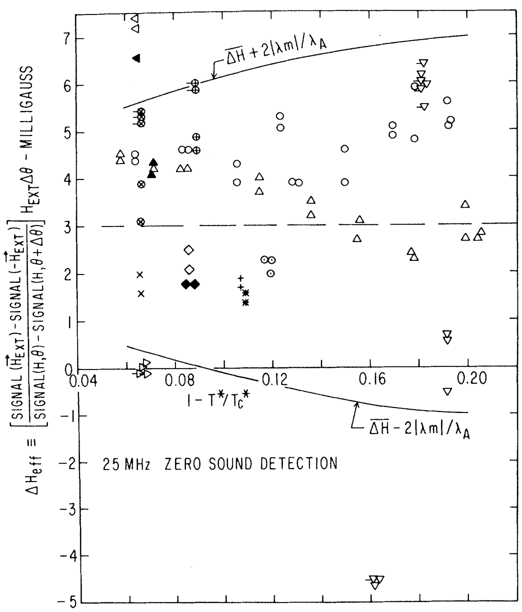

FIG. 2. Measurements in ^{3}He-A at 29.4 bars of $\Delta H_{\rm eff}$ [Eq. (2)] as it depends on reduced temperature difference, texture, and measuring field. The solid lines labeled $\overline{\Delta H}\pm 2\lambda_m/\lambda_A$ show the expected theoretical temperature dependence of $\Delta H_{\rm eff}$ for an ideal $\hat{l}$ field and reasonable choices of $|\lambda_m|$ at one temperature and of $\overline{\Delta H}\equiv\langle\Delta H_{\rm eff}\rangle_{\rm av}$ (assumed to be 3 mG). A different symbol is used for each new texture obtained by cycling the ^{3}He above T_c. Filled in symbols represent 600-mG data; all others are 300-mG data. A flag to the left of a symbol (as -∇) denotes that θ [Fig. 1(b)] has been changed in sign for the same magnitude of field, temperature, and texture as the same symbol without the flag.

from $\vec{H}$ and thereby decreasing the *average* effect of the term in λ_m in Eq. (1). Additionally, Eq. (2) correctly gives the size of λ_m from experimental data only if $\hat{l}$ lies in the same plane as the one in which $\vec{H}$ is rotated for calibration purposes. A more detailed examination of the problem for $\hat{l}$ lying outside the XZ plane, but still with neglect of bending energy, shows that in this case analysis of the data using Eq. (2) tends to overestimate λ_m. Hence, depending on the nature of the departure of the $\hat{l}$ field from ideal, values of λ_m calculated from Eq. (2) can either underestimate or overestimate the true value of λ_m.

Our first concern was to decide whether any effect observed resulted from a ferromagnetic moment along $\hat{l}$ or from a small trapped (and therfore nonreversible) magnetic field. The latter imperfection is equivalent to a field rotation on changing $\vec{H}_{\rm ext}\to-\vec{H}_{\rm ext}$. This would lead to an effective field calculated from Eq. (2) which is temperature and texture independent. From measurements of persistent $\hat{l}$-field motions in this cell[11] we know that, once formed, the $\hat{l}$ field seems to have considerable stability at our very low flow levels. But if the liquid is warmed above T_c and the texture re-formed then we guessed that simple $\hat{l}$ fields, as in Fig. 1, might sometimes have $\hat{l}$ generally up, and sometimes generally down, thereby reversing under *ideal* conditions the effect of $\vec{H}_{\rm ext}\to-\vec{H}_{\rm ext}$. Therefore we performed a series of experiments in which we cooled from above T_c to the same $1-T/T_c\simeq 0.13$ a number of times, finding that the effective fields calculated from Eq. (2) fell into two well-defined groups. This qualitative result encouraged us to try to quantify the effect.

The actual field $\vec{H}_{\rm ext}$ used in the experiments should be *large* enough both to allow texture control by field rotation and to give a reasonably short (say several minutes) response time of $\hat{l}$ whose motion is resisted by orbital viscosity.[8] But it should be *small* enough to allow small effects to be measured. The measurements, some of which are shown in Fig. 2, used primarily a

300-mG field; measurements at 600 mG showed no qualitative differences. All were obtained using 25-MHz zero sound at a pressure of 29.4 bars, where a large supercooling enabled us to cover all the useful temperature range. Measurements were performed as described in connection with Eq. (2) by comparing signal changes on external field reversal with those using a 2° field rotation. The angle θ (Fig. 1) was set so that the signal level in field $\vec{H}_{ext}$ was the same as that for $\vec{H}_{ext}=0$. Examination of data for the same texture at the same temperature and the same θ shows that ΔH_{eff} could be measured to a precision of a few tenths milligauss, except at the lowest temperatures where sensitivity is poor because of the smaller sound-attenuation anisotropy. Once a texture has been formed on cooling the liquid below T_c, results for ΔH_{eff} from it were usually quite consistent, even on cycling the temperature. Examples of such results are shown as (○○) and (ΔΔ) on Fig. 2 which represent two extensive series of measurements for two textures with similar θ (as defined above) and rotational sensitivity but with $T>T_c$ in between. (Changing the temperature, and hence probably the details of the texture, introduces "scatter" which is beyond experimental imprecision.) These two series of measurements, which constituted some of our earlier observations, show a substantial separation in ΔH_{eff} at large $1-T^*/T_c^*$, but for $1-T^*/T_c^*\lesssim 0.1$ there is essentially no difference between them. This would be very disturbing if each is thought to represent a uniform $\hat{l}$ field but with the direction of $\hat{l}$ reversed for the (○○) with respect to the (ΔΔ) data, for on the basis of Leggett's theory[4] for λ_m the quantity $2\lambda_m/\lambda_A$ is *not* small at $1-T/T_c\approx 0.06$ compared with its value at $1-T/T_c\simeq 0.2$. We concluded that the $\hat{l}$ field was far from uniform. Indeed, subsequent measurements of *many* re-formed textures, only some of which are shown on Fig. 2 to avoid congestion, show that both high and low values of ΔH_{eff}, relative to an average near 3 to 4 mG, can be observed even at higher temperatures. We believe that the smaller values of $|\Delta H_{eff} - \langle\Delta H_{eff}\rangle_{av}|$, where presumably $\langle\Delta H_{eff}\rangle_{av}\approx 3$ to 4 mG is twice the trapped nonreversible field, can be attributed to the presence in the sound field of significant portions of the $\hat{l}$ field both along and opposite $\vec{H}_{ext}$. This suggestion is supported by the observation that when we change $\vec{H}_{ext}$ from making angle θ [Fig. 1(b)] with $\vec{q}$ to angle $-\theta$, a substantial change in ΔH_{eff} can ensue. This is to be expected since there is *known* to be a heat flow out of the transmitting sound crystal which will tend to "rigidify" the $\hat{l}$ field there and make the texture shown more unidirectional with θ as shown than with $-\theta$. We also draw attention to the series of observations of a large *negative* ΔH_{eff} at $1-T/T_c\simeq 0.16$ for a texture which in other respects appeared similar (e.g., regarding both θ and rotational sensitivity) to most of the others shown. These observations are perhaps our strongest qualitative evidence that the observed effects are not just spurious ones due to trapped fields.

Although the measurements in Fig. 2 do *not* constitute a *measurement* of the ferromagnetic orbital magnetization of ^{3}He-A, they can be used to *estimate* the parameters of Leggett's theory. Assuming intuitively from Fig. 1 that the measured ΔH_{eff} probably underestimates λ_m while insensitivity to field rotation could give some anomalously large values, we have drawn on the figure lines which have the correct temperature dependence ($\lambda_m\propto\Omega_A^2$, where Ω_A is the longitudinal ringing frequency) of the theory[4] and which should approximate $\pm 2\lambda_m/\lambda_A$. These lines correspond to $\lambda_m/\chi_N\approx 0.8$ mG at $1-T/T_c=0.1$, very close to Leggett's numerical estimate. Although our estimate could easily be wrong by a factor 2 or so, even this type of agreement—entirely aside from the remarkable qualitative features of the experiment itself—suggests that at least a part of Leggett's recent suggestion[12] for using superfluid ^{3}He to study a macroscopic parity nonconservation due to neutral currents is soundly based.

We are grateful to Tony Leggett for discussions which led to these experiments, to David Mermin for a discussion of their significance, and to Robert Kleinberg for his very helpful comments on the manuscript. This work has been supported by the U. S. Department of Energy under Contract No. EY-76-S-03-0034, P.A. 143.

[1]For a theoretical review see A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).

[2]For an experimental review see J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).

[3]D. N. Paulson, M. Krusius, and J. C. Wheatley, J. Low Temp. Phys. 26, 73 (1976).

[4]A. J. Leggett, Nature (London) 270, 585 (1977).

[5]P. W. Anderson and P. Morel, Phys. Rev. Lett. 5, 136 (1960).

[6]M. Krusius, D. N. Paulson, and J. C. Wheatley, to be published.

[7]D. N. Paulson, M. Krusius, R. S. Safrata, and J. C. Wheatley, to be published.

[8]D. N. Paulson, M. Krusius, and J. C. Wheatley, Phys. Rev. Lett. 37, 599 (1976).

[9]D. N. Paulson, M. Krusius, and J. C. Wheatley, Phys. Rev. Lett. 36, 1322 (1976).

[10]N. D. Mermin, in *Quantum Fluids and Solids*, edited by S. B. Trickey, E. D. Adams, and J. W. Dufty (Plenum, New York, 1977), p. 3.

[11]M. Krusius, D. N. Paulson, and J. C. Wheatley, to be published.

[12]A. J. Leggett, Phys. Rev. Lett. 39, 587 (1977).

Phase Transitions in Two-Dimensional Systems

Eytan Domany and Eberhard K. Riedel
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 13 September 1977)

Two recently studied experimental systems are shown to constitute realizations of two-dimensional anisotropic N-vector models. Along with a renormalization-group treatment of a general Hamiltonian encompassing these models, we discuss new kinds of multicritical points, nonuniversal critical behavior, and experiments for further study of these systems.

The study of phase transitions in adsorbed thin films is a rapidly developing field of high current experimental and theoretical activity. The experimental methods (such as thermal measurements and diffraction techniques) are approaching the point where reliable determinations of phase diagrams and critical exponents are becoming possible.[1] Theoretical models for overlayer systems can be derived by symmetry arguments[2] and analyzed by renormalization-group (RG) or other techniques. This comprehensive approach is applied here to the magnetic phase transition in molecular oxygen adsorbed on graphite[3,4] and to the order-disorder transition in atomic oxygen on tungsten.[5] These systems represent the first examples of two-dimensional (2D) anisotropic N-vector models (Heisenberg and XY with cubic anisotropy). Realizations of Ising and Potts models were discussed previously.[6] Anisotropic N-vector models in 2D[7] are of special interest since they exhibit features such as nonuniversal critical exponents,[8,9] new kinds of multicritical points, and new universality classes.[10] In 2D the anisotropic perturbations are relevant[7] and grow under RG iterations. This allows the mapping of the Landau-Ginzburg-Wilson (LGW) continuous-spin models, which are derived on grounds of symmetry,[2] onto discrete spin models, which can be conveniently analyzed by position-space RG methods. The two anisotropic N-vector models are shown to be related to special cases of a general, discrete N-state model termed the (N_α, N_β) model. The model is introduced and discussed. In this work, the hypothesis is made that if a discrete spin model and an experimental system are described by the same LGW Hamiltonian, all three belong to the same universality class. This hypothesis has not been extensively tested in 2D. Specifically, the phase diagram for the general six-state model is investigated using duality transformations[11] and Migdal's approximate RG recursion relations.[12] Experiments to test the applicability of the models and to investigate their properties are also discussed.

First, consider overlayers of molecular oxygen physisorbed on graphite. The system exhibits a phase transition into an antiferromagnetically ordered state [Fig. 1(a)] at $T_c = 11.3$ K, which is apparently continuous.[3,4] Neutron scattering experiments[3] indicate that the lattice structures of the disordered and ordered O_2 phases are incommensurate with the substrate and al-

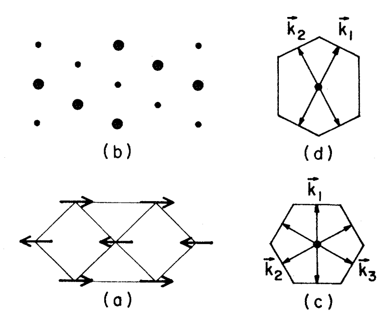

FIG. 1. Structures of the ordered phases of (a) O_2 adsorbed on graphite (the arrows denote the spin orientations on the distorted triangular lattice), and (c) oxygen adsorbed on tungsten (the heavy dots denote occupied sites). The Brillouin zones and wave vectors, $\vec{k}$, that define the different ordered states, are shown in (b) and (d), respectively.