PHYSICAL REVIEW
LETTERS

VOLUME 40 | 27 FEBRUARY 1978 | NUMBER 9

# Photonuclear Sum Rule for $^3$He and $^3$H

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(Received 19 December 1977)

The enhancement factor $\kappa$ in the electric dipole sum rule for $A = 3$ nuclei is calculated using the bound-state wave function obtained from a solution of the Faddeev equations with the Reid soft-core two-nucleon potential. We find $\kappa = 0.8$ for $^3$He and $^3$H. The $S$-$D$ interference terms are found to be very important. Our result is consistent with experimental data for $^3$He.

Investigations of bound-state properties of the three-nucleon system are a useful means for studying the electromagnetic structure of the nucleus. Of particular interest are electromagnetic sum rules, such as that of Thomas, Reiche, and Kuhn[1] (TRK) for photonuclear absorption cross sections, which was derived using the long-wavelength approximation and nonrelativistic wave functions. In this Letter, we present our calculation of the enhancement factor, $\kappa$, in the TRK electric dipole sum rule for $^3$He and $^3$H using the trinucleon wave functions[2] obtained from a solution of the Faddeev equations with the Reid soft-core potential.[3] To our knowledge, this is the first consistent calculation of the enhancement factor $\kappa$ for $^3$He and $^3$H which involves realistic nuclear forces and off-diagonal $S$-$D$ contributions.

Until several years ago, both experimental measurements and theoretical estimates indicated that the photonuclear absorption cross section, integrated up to the pion threshold, exceeds the classical limit by about 40% (i.e., $\kappa \approx 0.4$).[4] This result appeared to be on a seemingly firm ground, because the same conclusion had been reached from the TRK approach and from the Gell-Mann–Goldberger–Thirring[5] (GGT) approach which uses unitarity, causality, and analyticity of the scattering amplitude. Therefore, the new experimental results of Ahrens *et al.*,[6] $\kappa \approx 1.0$, reported first at the Sendai conference in 1972, were a surprise and stimulated new theoretical investigations.

Arima *et al.*[7] were the first to point out that two-body correlations, particularly those induced by the tensor force, lead to a substantial increase of the photoabsorption cross section over the previously accepted value. Using a one-pion exchange potential regularized at short distances and tensor-correlated Fermi gas wave functions, they obtained $\kappa = 1.57$. This value decreased to $\kappa = 1.17$ for a shell-model calculation for $^{40}$Ca using the Hamada-Johnston potential. A further calculation was done by Weng, Kuo, and Brown[8] using Brueckner reaction-matrix theory and the linked-diagram expansion of the effective operator. Using the Reid soft-core potential, they obtained enhancement factors of about $\kappa = 1.0 \pm 0.2$.

Subsequently, Fink, Gari, and Hebach[9] improved the wave functions by using the $e^S$ formalism, which gives (in principle) exact solutions

of the many-body system, once the nucleon-nucleon force is fixed. For various potentials and nuclei, Fink, Gari, and Hebach obtain $\kappa = 0.5 \pm 0.1$, roughly half the value given in Refs. 7 and 8. In a further extension of this work[10] the inclusion of three-body correlations was shown to give only a small further increase of the enhancement factor. For the deuteron, a smaller value of $\kappa$ between 0.18 and 0.27 was obtained by them,[10] and 0.34 by Lucas and Rustgi.[11]

Clearly the results obtained by Hebach and co-workers[9,10] and by Brown and co-workers[7,8] are at variance. In view of the rather fundamental importance of the integrated photoabsorption cross section, in particular its relation to general principles, new independent calculations seem appropriate. Recently, Rustgi *et al.*[12] and Arenhövel and Fabian[13] re-examined the case of the deuteron and found, for various potentials, enhancement factors slightly above $\kappa = 0.5$.

In calculating the enhancement factor $\kappa$ for $^3$He and $^3$H, we use the nonrelativistic wave functions[2] calculated by solving the Faddeev equations in momentum space using the Reid soft-core potential,[3] effective in the $^1S_0$ and $^3S_1$-$^3D_1$ partial wave states. The completely antisymmetric trinucleon wave function $\Psi$ is expanded in terms of the $\mathcal{L}$-$\mathcal{S}$ basis state, $\varphi_\alpha(p,q)$ [i.e., $\Psi = \sum_\alpha \varphi_\alpha(p,q)$], which is defined to be an eigenstate of the operators $\vec{p}^2$, $\vec{q}^2$, $\vec{L}^2$, $\vec{\mathcal{L}}^2 = (\vec{L}+\vec{l})^2$, $\vec{S}^2 = (\vec{s}_2+\vec{s}_3)^2$, $\vec{s}_2^{\,2}$, $\vec{s}_3^{\,2}$, $\vec{s}_1^{\,2}$, $\vec{\mathcal{S}}^2 = (\vec{S}+\vec{s}_1)^2$, $\vec{\mathcal{J}}^2 = (\vec{\mathcal{L}}+\vec{\mathcal{S}})^2$, $J_z$, $\vec{t}_2^{\,2}$, $\vec{t}_3^{\,2}$, $\vec{T}^2 = (\vec{t}_2+\vec{t}_3)$, $\vec{t}_1^{\,2}$, $\vec{\mathcal{T}}^2 = (\vec{T}+\vec{t}_1)^2$, and $\tau_{z0}$. $\vec{\mathcal{L}}$ is the relative orbital angular momentum of the (2,3) pair; $\vec{l}$ is the orbital angular momentum of nucleon 1 in the c.m. system; $\vec{s}_i$ and $\vec{t}_i$ are the spin and isospin of the nucleon $i$. For $^3$He and $^3$H, $\mathcal{J} = \frac{1}{2}$, $\tau_z = +\frac{1}{2}$ (for $^3$He) and $\tau_z = -\frac{1}{2}$ (for $^3$H). The momenta $\vec{p}$ and $\vec{q}$ are defined by $\vec{p} = \frac{1}{2}(\vec{k}_2 - \vec{k}_3)$ and $\vec{q} = (\vec{k}_2 + \vec{k}_3 - 2\vec{k}_1)/2\sqrt{3}$, and are conjugate to the coordinate vectors $\vec{r} = \vec{r}_2 - \vec{r}_3$ and $\vec{\rho} = (\vec{r}_2 + \vec{r}_3 - 2\vec{r}_1)/\sqrt{3}$, respectively, in the c.m. system. The components and corresponding probabilities of the trinucleon wave function, used in our calculation of $\kappa$, are listed in Table I.

TABLE I. Components of trinucleon bound-state wave function in the $\mathcal{L}$-$\mathcal{S}$ basis.

| Component label $\alpha$ | $L$ | $l$ | $\mathcal{L}$ | S | $\mathcal{S}$ | $T$ | Probability (%) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | $\frac{1}{2}$ | 0 | 45.18 |
| 2 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | 1 | 44.50 |
| 3 | 2 | 2 | 0 | 1 | $\frac{1}{2}$ | 0 | 0.49 |
| 4 | 2 | 2 | 0 | 0 | $\frac{1}{2}$ | 1 | 0.92 |
| 5 | 1 | 1 | 0 | 0 | $\frac{1}{2}$ | 0 | 0.42 |
| 6 | 1 | 1 | 0 | 1 | $\frac{1}{2}$ | 1 | 0.42 |
| 7 | 2 | 0 | 2 | 1 | $\frac{3}{2}$ | 0 | 3.12 |
| 8 | 0 | 2 | 2 | 1 | $\frac{3}{2}$ | 0 | 1.02 |
| 9 | 1 | 1 | 2 | 1 | $\frac{3}{2}$ | 1 | 2.45 |
| 10 | 3 | 1 | 2 | 1 | $\frac{3}{2}$ | 1 | 0.40 |
| 11 | 1 | 3 | 2 | 1 | $\frac{3}{2}$ | 1 | 1.08 |

In the TRK approximation, the integrated photonuclear absorption cross section[4] is given by

$$\int_0^\infty \sigma_\gamma(E)dE = 60(NZ/A)(1+\kappa)\ \text{MeV mb}, \tag{1}$$

with the enhancement factor

$$\kappa = (A/NZ)(M/\hbar^2)\langle\Psi|[D_z,[V,D_z]]|\Psi\rangle. \tag{2}$$

Here $\Psi$ denotes the ground-state wave function of the nucleus, $M$ is the nucleon mass, $V$ is a sum of the nucleon-nucleon potentials, i.e.,

$$V = \sum_{i>j} V_{ij},$$

and $D_z$ is the $z$ component of the electric dipole operator

$$\vec{D} = \sum_{i=1}^{A} \tfrac{1}{2}[1+\tau_z(i)]\vec{r}_i. \tag{3}$$

It is easy to show that, for $A>2$ in the c.m. system,

$$\kappa = \frac{A}{NZ}\frac{M}{\hbar^2}\frac{A(A-1)}{6}\langle\Psi|[\vec{D}^{(2)}\cdot,[V_{23},\vec{D}^{(2)}]]|\Psi\rangle, \tag{4}$$

where

$$\vec{D}^{(2)} = \tfrac{1}{4}[\tau_z(2)-\tau_z(3)]\vec{r};\quad \vec{r} = \vec{r}_2 - \vec{r}_3.$$

Since our trinucleon bound-state wave function $\Psi$ is given in momentum space, the double commutator in Eq. (4) is Fourier transformed into momentum space. After some algebra, we obtain the final expression for the $A=3$ case as

$$\kappa = \frac{3}{2\pi}\frac{M}{\hbar^2}\sum_{\alpha',\alpha} i^{L-L'}\sum_{J_1,j_1}\hat{J}_1^{\,2}\hat{j}_1^{\,2}\hat{\mathcal{L}}'\hat{\mathcal{L}}\hat{\mathcal{S}}'\hat{\mathcal{S}}\begin{Bmatrix} L' & l_1 & \mathcal{L}' \\ S & s_1 & \mathcal{S}' \\ J_1 & j_1 & \mathcal{J} \end{Bmatrix}\begin{Bmatrix} L & l_1 & \mathcal{L} \\ S & s_1 & \mathcal{S} \\ J_1 & j_1 & \mathcal{J} \end{Bmatrix}$$

$$\times \int p'^2 dp' \int p^2 dp \int q^2 dq\, \varphi_{\alpha'}(p',q) F_{L',L}^{J_1 S T}(p',p)\varphi_\alpha(p,q), \tag{5}$$

with

$$F_{L',L}{}^{J_1ST}(p',p) = \int dr\, r^4 j_{L'}(p'r) j_L(pr) \Big[ -\langle (L'S)J_1 | V^T(r) | (LS)J_1 \rangle + \sum_{L_f,L_i,J'} (\hat{J}')^2 \hat{L}_f \hat{L}_i \hat{L}' \hat{L} \begin{Bmatrix} L_f & J' & S \\ J_1 & L' & 1 \end{Bmatrix} \begin{Bmatrix} L_i & J' & S \\ J_1 & L & 1 \end{Bmatrix} \begin{pmatrix} L_f & 1 & L' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_i & 1 & L \\ 0 & 0 & 0 \end{pmatrix} \times \langle (L_f S)J' | V^{(1-T)}(r) | (L_i S)J' \rangle \Big], \quad (6)$$

where $\hat{a} = (2a+1)^{1/2}$ and the superscript of $V(r)$ denotes the isospin quantum number of the two-nucleon potential. For the radial part $\langle (L'S)J | \times V(r) | (LS)J \rangle$, we use the Reid soft-core potential for $J \leq 2$ partial wave states and the one-pion exchange potential for $J > 2$ states. For potentials of the one-boson exchange type, the relevant radial integrals are linear combinations of the type

$$R_{L',L}{}^{(m)}(p',p,\mu) = \int_0^\infty j_{L'}(p'r) j_L(pr) r^n e^{-\mu r} dr. \quad (7)$$

These integrals may be evaluated analytically using generalizations of previously obtained expressions.[14,15] The integration over $q \leq 8$ F$^{-1}$ is carried out using the Gauss-Legendre quadrature with 24 points and is found to be stable within 1% against variations of the cutoff momentum and the number of the quadrature points. The integrations over $p'$ ($\leq 13$ F$^{-1}$) and $p$ ($\leq 13$ F$^{-1}$) are carried out using the Gauss-Legendre quadrature with 16 points for both cases and are found to be stable within 1% against variations of the number of the quadrature points and the momentum cutoff. Thus, our estimated overall error is less than 2% for the final value of $\kappa$.

In Table II, our results obtained for three-body nuclei have been compared to those of Ref. 13 for the deuteron. In both cases, the importance of tensor correlations is clearly borne out. Also the relative contributions of the triplet states are comparable. The overall increase of the enhancement factor due to the third nucleon may be attributed to the increase in density and in the number of relevant nucleon pairs. For the same reason we should expect a further increase of the enhancement factor $\kappa$ upon adding a fourth nucleon, in agreement with the result of Ref. 8 ($\kappa = 1.0 \pm 0.2$) in the case of $^4$He. Further theoretical support for a larger enhancement factor $\kappa \approx 1.0$ comes from a recent calculation involving the excitation energy of giant dipole resonances as computed in a microscopic theory of self-consistent particle-vibration coupling.[17] For heavier nuclei, the central density remains rather constant and the number of the relevant pairs in relative $S$ states does not increase faster than the factor $NZ/A$ in definition (2) of the enhancement factor. Consequently, the calculated values of $\kappa$ remain roughly constant for $A > 4$.[7-10]

Unfortunately, there are no direct photoabsorption experiments for $A < 7$. For the deuteron Lucas and Rustgi[11] obtain, from an evaluation of all experimental disintegration data, an enhancement factor $\kappa = 0.37 \pm 0.11$. For $^3$He, Fetisov, Gorbunov, and Varfolomeev[18] obtained $\kappa = 0.75 \pm 0.10$ from photodisintegration measurements. For the case of $^4$He a value $\kappa \approx 0.63$ has been derived by adding the experimental contributions of known decay channels.[19] In comparing the experimental values of $\kappa$ with the TRK results, the following considerations must be taken into account.

(1) The experimental measured photoabsorption cross section integrated up to pion threshold is approximately equal to the TRK value. The proof of Gerasimov[20] that higher multipole con-

TABLE II. Calculated values of $\kappa$ for $A = 3$ ($^3$He and $^3$H) nuclei. The $A = 2$ (deuteron) case (Refs. 13 and 16) is also presented for comparison. Probability $P$ of each dominant component of the $A = 2$ and $A = 3$ wave functions is listed. The Reid soft-core potential is used for both $A = 3$ and $A = 2$ cases.

| Final and initial states $(\alpha',\alpha)$ | $A=3$ ($^3$He, $^3$H) $P$ (%) | $\kappa$ | $A=2$ ($^2$H) $P$ (%) | $\kappa$ |
|---|---|---|---|---|
| $^3S_1$-$^3S_1$ (1,1) | 45.2 | 0.11 (14%) | 93.5 | 0.09 (18%) |
| $^3S_1$-$^3D_1$ (1,7) and (7,1) | ... | 0.46 (58%) | ... | 0.42 (84%) |
| $^3D_1$-$^3D_1$ (7,7) | 3.1 | −0.05 (−6%) | 6.5 | −0.01 (−2%) |
| $^1S_0$-$^1S_0$ (2,2) | 44.5 | 0.27 (34%) | ... | ... |
| Others | 7.2 | 0 | ... | ... |
| Total | 100 | 0.79 (100%) | 100 | 0.50 (100%) |

tributions and dipole retardation cancel exactly has been shown to be founded on wrong premises, namely the convergence of the total cross section and the analyticity of the Compton scattering amplitude. As demonstrated by Weise,[21] there is considerable experimental evidence against the former assumption. Even at very high energies a nucleus may not be considered to be an ensemble of free nucleons, but hadronic components in the photon propagator lead to considerable screening effects. As far as the analyticity of the scattering amplitude is concerned, recent calculations have shown that nonrelativistic models give rise to anomalous branch cuts.[22-24] At the same time, however, these calculations have demonstrated that there is indeed considerable, though not complete, cancellation of dipole retardation against higher multipoles.

(2) The evaluation of the double commutator with the conventional nuclear physics includes photoabsorption also above pion threshold.

(3) There are contributions from the subnucleonic level, isobaric and meson exchange currents.

(4) Finally, relativistic corrections have been estimated to increase the integrated cross section by a few percent.[24]

In conclusion, we want to point out that the previous work on the deuteron[13] and the present calculation for $A = 3$ nuclei seem to support the results obtained by Brown and co-workers.[7,8] The enhancement factor $\kappa = 0.8$ for $A = 3$ will probably slightly overestimate the experimental value for absorption up to pion threshold. In view of the fundamental importance of photonuclear sum rules, we strongly advocate experimental measurements of total photoabsorption cross sections for light nuclei.

We are grateful to H. Arenhövel and B. Ziegler for many helpful discussions. The numerical calculations have been performed at the TR 440, at the Rechenzentrum, Kaiserslautern, Germany. One of us (Y.E.K.) acknowledges receipt of a support award from the Alexander von Humboldt Foundation. This work was supported in part by the U. S. National Science Foundation.

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