tum, k_T, of the annihilating quarks[14]; in fact,

$$\langle k_T^2 \rangle = \tfrac{1}{2} \langle p_T^2 \rangle$$

so that the rms k_T of the quarks is of the order of ~1.0 GeV/c.

In summary, we find in the mass range 5-15 GeV a remarkable consistency between the quark-parton model interpretation of the dilepton data and deep inelastic scattering results. In the context of this model we measure the sea-quark distribution and the initial-state quark transverse momenta. Finally we note that the last row of Table I sets a significant upper limit to the cross section for dilepton production from 15 to 27 GeV.

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Three-Body Calculation of πd Elastic Scattering at 142 MeV

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We calculate the pion-deuteron elastic scattering at $E_\pi = 142$ MeV using nonrelativistic three-body equations in which relativistic kinematics are retained only for the pion. The NN tensor force is taken into account, and, for the first time, all S and P πN partial waves are included in an exact way. The coupling between $l = J \pm 1$ πd channels is found to be not negligible, and we establish the large interference effects of all other than P_{33} πN channels leading to a rather good description of the experimental data at $E_\pi = 142$ MeV.

The elastic scattering of medium energy pions off deuterium has recently been given a considerable attention. The properties of the pion nucleon interaction and its richness with interesting physical phenomena (such as absorption, production, resonance structure, etc.) make the pion-deuteron system the most interesting three-body entity. With the advent of mesons factories on the one hand, and complete three-body calculations on the other hand, the pion-deuteron system probably will receive even more attention.

Let us briefly discuss the most relevant calculations performed up till now, together with the motivation for the present work. Two types of

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approach have been used. The first one proposed by Thomas[1] is based on nonrelativistic Faddeev-Lovelace equations with relativistic kinematics only for the pion (RPK), and gives a good description of the low energy π^+d data at 47.7 MeV. The second approach uses a fully relativistic treatment (FR) and has been applied successfully by many authors[2,3] in the (3, 3) resonance region, namely at 142, 180, and 256 MeV. The FR calculations need large computing time and memory for matrix inversion. Therefore, they include only a restricted number of two-body channels which are supposed to be dominant in this energy range. For example, Rinat and Thomas[3] take into account the ${}^3S_1$-${}^3D_1$ NN and only the P_{33} πN channels. Moreover, they neglect the coupling between $l=J\pm 1$ πd channels. Their calculations provide a good fit of experimental data at 142 and 180 MeV. A very large sensitivity to the relativistic effects and to the D-state probability (P_D) of the deuteron is found, and the effects of interference from other than P_{33} πN channels have been calculated recently in perturbation theory.[4]

Besides the apparent need for other independent calculations, there are some disturbing questions which need further attention: (i) Is the coupling between $l=J\pm 1$ πd channels negligible? (ii) What is the effect of other than P_{33} πN channels included in an exact way? (iii) How does the tensor force in the NN channel affect the elastic cross section?

In order to investigate these questions, we report here the results of our complete three-body calculations in the RPK approach which for the first time include all S and P pion-nucleon partial waves in an exact way. Our study is concentrated at 142 MeV in order to be far enough from the (3, 3) resonance. At this energy, it is a good approximation to consider the nucleons as nonrelativistic particles ($v/c\sim 0.1$) and it seems to us reasonable to use the RPK theory. A further step will be to include more correctly the relativistic effects by using a first-order approximation (denoted hereafter RPK2) of the FR equations of Rinat and Thomas.[3] In all our calculations, we choose the same units and the same NN and πN separable interactions as Thomas.[1]

First, it is important to check our program by comparing with the RPK results of Thomas at 142 MeV.[3,5] We therefore include in our calculation the NN tensor force ($P_D=4\%$) and only the P_{33} πN channel [such a calculation will be denoted hereafter RPK(Δ)]. We solve the system of coupled integral equations with the Padé approximant technique, while Thomas uses a Gaussian elimination routine, and we choose the same quadratures as Thomas for the $(0,+\infty)$ and $(-1,+1)$ ranges. With a [3/3] Padé, we obtain uncoupled scattering amplitudes $T_{ll}{}^J(l=J)$ which agree within better than 0.5% with those of Thomas.

Then, we evaluate the effect of the coupling between the $l=J\pm 1$ πd channels on the scattering amplitudes and on the differential cross section. We find that the diagonal amplitudes with $l=l'=J\pm 1$ are very close to those obtained without coupling, and that the nondiagonal terms $T_{J+1,J-1}{}^J=T_{J-1,J+1}{}^J$ are not negligible but of same order of magnitude as the $T_{J+1,J+1}{}^J$. For example, for $J=2^+$, we found $T_{11}=-0.4017-i\,0.5461$, $T_{33}=-0.0105-i\,0.0090$, and $T_{13}=T_{31}=0.0089+i0.0118$, while Thomas obtains $T_{11}=-0.4065-i\,0.5375$ and $T_{33}=-0.0106-i\,0.0089$. In order to evaluate the differential cross section $d\sigma/d\Omega$, we have to calculate all partial wave amplitudes $T_{ll'}{}^J$ with $l,l'\leq l_{\max}$ and $J\leq J_{\max}=l_{\max}+1$, where $(l_{\max}, J_{\max})$ is a reasonable cutoff. We solve with [3/3] Padé for $J=0$ to 3, and we use the single-scattering approximation for $J\geq 4$. The resulting $d\sigma/d\Omega$ calculated with $l_{\max}=5$ is identical to that of Thomas in the forward part, but becomes up to 7% higher at backward angles (Fig. 1). At this energy, the coupling is therefore not negligible, and this effect is of course more apparent when considering the polarization parameters. We have also noted that stabilization was achieved for $l_{\max}=7$.

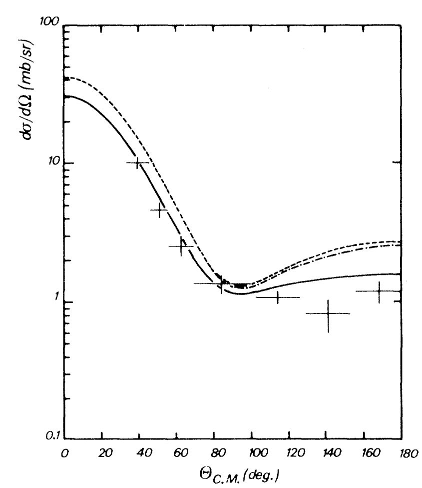

FIG. 1. Elastic differential cross section calculated at $E_\pi=142$ MeV in the c.m. system using RPK theory and the same NN tensor force ($P_D=4\%$). Our RPK(Δ) (---) and RPK(SP) (——) results are compared with the RPK results of Rinat and Thomas (Ref. 3) (-·-·-). Experimental data are from Ref. 6.

In the next step, we study the effect of interferences from other than P_{33} πN channels. We now include the NN tensor force ($P_D = 4\%$) and all S and P πN partial waves in an exact way [this calculation will be denoted RPK(SP)]. For the πN channels, we use the same parametrizations as Thomas.[1] Compared with the RPK(Δ) results, all the RPK(SP) scattering amplitudes are strongly modified, except $T_{ll'}{}^J$ with $l \neq l'$. For instance, the above $J = 2^+$ amplitudes are now $T_{11} = -0.3627 - i0.4403$, $T_{33} = -0.0039 - i0.0097$, and $T_{13} = T_{31} = 0.0076 + i0.0118$. The resulting differential cross section becomes considerably lower throughout the angular range (Table I). Referring to the experimental data, the improvement of RPK(SP) calculation in comparison with RPK(Δ) is obvious (Fig. 1). This large effect is in contradiction with recent results of Thomas *et al.*[4] who find that other than P_{33} channels lower the differential cross section at 142 MeV only in the forward part. Nevertheless, it should be noted that these results have been obtained in the FR approach with a perturbative treatment.

Coming now to the influence of the D-state probability, we find that increasing P_D from 4% to 7% lowers slightly the differential cross section throughout the angular range in RPK(Δ) and in RPK(SP) calculations. However, this effect is small (at most 5% at 90°) by comparison with the FR results[3] where the backward cross section is reduced by 20% as P_D goes from 4% to 7%. We note that McMillan and Landau[7] find an opposite effect when using the single-scattering factorized approximation; the value of $d\sigma/d\Omega$ (180°) increases with increasing P_D. However, the effect deduced from our results and those of Ref. 3 is not a genuine effect of P_D, because the NN tensor forces corresponding to each P_D value are not phase equivalent. Therefore, we need further study with more realistic tensor forces.

At last, we look at the relativistic effects and their relation to the inclusion of all πN channels. In order to include more correctly the relativistic effects in RPK, we start from the FR equations of Rinat and Thomas[3] and we take the first-order approximation in terms of the nucleon kinetic energy. We thus obtain RPK2 equations which are identical to RPK with two modifications: (i) The reduced mass appearing in the πN t matrix written in three-body Hilbert space[1] is $\mu_{N,\pi N}{}^{-1} = m_N{}^{-1} + (m_N + m_\pi + E)^{-1}$ while in RPK the total kinetic energy E was dropped. (ii) The ρ, ρ' functions which are used in the relative momenta at each vertex of the Born term [Ref. 3, Eq. (2.21)] are given by their nonrelativistic limits as in RPK, except m_π is replaced by the invariant mass.

In Table I, we give the results of our RPK2(Δ) and RPK2(SP) calculations at 142 MeV using a 4% tensor force, and also the RPK(Δ) and RPK(SP) calculations. Compared with RPK(Δ), the RPK2(Δ) differential cross section is lower throughout the angular range, especially at backward angles (~11% at 180°). A similar but stronger effect has been found by Rinat and Thomas[3] when using the RPK1 approximation which is identical to RPK except that the relative momenta are defined as in the FR approach.

If we now compare the results with all πN channels included, the RPK2(SP) differential cross section is quite similar to RPK(SP) and becomes lower by about 6% at 180°. It seems therefore that relativistic effects become smaller when all πN channels are included (in an exact way).

Let us point out the main conclusions of our study: (i) The coupling between $l = J \pm 1$ πd channels is not negligible, the differential cross section increasing up to 7% at backward angles. (ii) Including in an exact way all other than P_{33} πN channels has a very large effect on the semirelativistic calculations and leads to a rather good fit of the experimental data. (iii) The effect of D-state probability is small and changing the P_D value from 4% to 7% decreases the differential cross section up to 5%. (iv) Relativistic effects become smaller when all πN channels are taken into account.

Points (ii) and (iii) are in contradiction with the fully relativistic results. Concerning (ii), it will be interesting to introduce exactly (and not perturbatively) other than P_{33} channels in the FR treatment. On the other hand, it is also important to examine in more detail the πN input used in the RPK approach, especially the P_{11} channel

TABLE I. Elastic differential cross section calculated at $E_\pi = 142$ MeV in the c.m. system with RPK and RPK2 equations. The calculations include the same NN tensor force ($P_D = 4\%$), and only P_{33} (label Δ) or all S and P (label SP) πN channels.

Name of the calculation	Elastic differential cross section $\theta_{\text{c.m.}} = 0°$	$\theta_{\text{c.m.}} = 90°$	$\theta_{\text{c.m.}} = 180°$
RPK(Δ)	42.15	1.32	2.71
RPK(SP)	30.58	1.15	1.59
RPK2(Δ)	41.48	1.25	2.41
RPK2(SP)	30.63	1.19	1.50

in which the pion absorption is certainly not negligible. Finally the effect of P_L in our calculation and in FR should be further investigated.

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Determination of Angular-Momentum Transfers for (d, α) Reactions Using Polarized Beams

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The angular distribution patterns of vector analyzing powers for (d,α) reactions on s-d shell nuclei at energies near 16.5 MeV depend distinctly on the orbital angular momentum L and total angular momentum J transferred. Examples of $L = 0$, $L = 2$, and $L = 4$ transfers have been obtained for ^{28}Si, ^{32}S, and ^{40}Ca targets, and these compare well with distorted-wave calculations. These measurements provide information useful in establishing the spins and parities of the levels populated in (d,α) reactions.

The (d,α) reaction involves the pickup of two nucleons (sometimes considered as a deuteron cluster) from a nucleus. It is useful in spectroscopic studies of the residual nuclei, which are frequently odd-odd, and for studying the neutron-proton correlations in the ground state of the target nucleus. The analysis of these reactions is considerably more complicated than single-nucleon transfer reactions, since most final states can be populated by several orbital angular momentum transfers (L). Each L and J value transferred, where J is the total angular momentum of the pair, can also involve a combination of two-particle configurations. Since differential cross sections are often inadequate to unravel the relative contributions of different angular momenta to the (d,α) reaction, it is important to investigate the extent to which vector-analyzing-power (VAP) measurements using polarized beams can improve understanding of the reaction and the final states involved. Several VAP measurements for the (d,α) reaction have been reported recently.[1-3] These measurements, however, do not comprise as complete a study of a given mass region as is contained in the present work.

It is by now well-established that VAP measurements for single-nucleon pickup and transfer reactions distinguish between $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ transfers[4] (where j and l are total and orbital angular momenta of the transferred nucleon). The present Letter shows that analogous effects exist in VAP distributions for a two-nucleon transfer reaction depending on the J and L of the transferred pair of nucleons. For a spin-zero target nucleus, the L and J transferred from the nucleus determine the spin and parity of the final nuclear state and for each L involved in populating this state, J may equal either $L+1$, L, or $|L-1|$. Earlier theoretical work[5] stated that, in the absence of spin-dependent distortions of the incoming and outgoing waves and for a unique L and J, the vector analyzing powers will be proportional to $-L$, 1, and $L+1$, respectively, for the above J values.

In order to consider cases involving only one important angular-momentum component to the reaction, measurements were made with spin-

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