New Symmetry in the *sd* Boson Model of Nuclei: The Group O(6)

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We suggest that, within the framework of the interacting-boson model, the group O(6) of orthogonal transformations in six dimensions may be useful in classifying nuclear spectra at the end of major shells. We derive analytic expressions for the energy levels and electromagnetic transition rates in this limit.

Recently we have proposed[1] a description of even-even nuclei in terms of a system of interacting bosons, able to occupy two levels, one with angular momentum $L=0$ (called s) and another with angular momentum $L=2$ (called d). These bosons generate a SU(6) symmetry spanned by the $1+5$ components of the one-boson states $s^\dagger|0\rangle$ and $d_\mu{}^\dagger|0\rangle$. We have also indicated that whenever the Hamiltonian H can be written in terms of the generators of subgroups $G \subset \mathrm{SU}(6)$ alone, analytic solutions to the eigenvalue problem for H can be found and a dynamical symmetry in the sense of Gell-Mann[2] arises. We have so far considered two cases, (i) $G \equiv \mathrm{SU}(5)$ and (ii) $G \equiv \mathrm{SU}(3)$, and have shown that these correspond approximately to the anharmonic vibrator and axial rotor limits of the classical geometrical description. Several cases of both dynamical symmetries are known,[3,4] the first kind, SU(5), being found in nuclei at the beginning of major shells and the second kind, SU(3), being found in nuclei in the middle of major shells.

In this Letter we want to point out that a third subgroup, $G \equiv \mathrm{O}(6)$, may be useful in describing nuclei at the end of major shells. Possible experimental evidence for this symmetry at the end of the neutron shell 82–126 is given in Cizewski *et al.*,[5] while more complete evidence, including the 50–82 neutron shell (Ba and Xe isotopes), will be given in a forthcoming longer paper. Here we briefly discuss the properties of this group.[6] We begin by listing the fifteen generators of O(6), $(d^\dagger d)^{(1)}$, $(d^\dagger d)^{(3)}$, $(d^\dagger s + s^\dagger d)^{(2)}$. We then note that five extra labels are needed to classify the totally symmetric irreducible representations $[N]$ of SU(6) when the group chain $\mathrm{SU}(6) \supset \mathrm{O}(6) \supset \mathrm{O}(5) \supset \mathrm{O}(3)$ is used. These are as follows: a quantum number σ which characterizes the totally symmetric irreducible representations of O(6), where

$$\sigma = N, N-2, \ldots, 0 \text{ or } 1 \text{ for } N = \text{even or } N = \text{odd}; \tag{1}$$

a quantum number τ which characterizes the totally symmetric irreducible representations of O(5), where

$$\tau = \sigma, \sigma-1, \ldots, 0; \tag{2}$$

a quantum number ν_Δ which counts boson triplets coupled to zero angular momentum; and finally the total angular momentum L and its Z component. The values of L contained in each representation (τ) of O(5) are obtained by partitioning τ as

$$\tau = 3\nu_\Delta + \lambda, \quad \nu_\Delta = 0, 1, \ldots, \tag{3}$$

and taking

$$L = 2\lambda, 2\lambda-2, \ldots, \lambda+1, \lambda. \tag{4}$$

An expression for the expectation value of the Hamiltonian H in the state labeled by $|[N]\sigma\tau\nu_\Delta LM\rangle$ can be obtained in the following way. We first consider the pairing operator in O(6) which we denote by P_6. This operator can be written in terms of three operators,

$$\begin{aligned} S_+ &= \tfrac{1}{2}\textstyle\sum_m (-)^m d_m{}^\dagger d_{-m}{}^\dagger - \tfrac{1}{2} s^\dagger s^\dagger, \\ S_- &= \tfrac{1}{2}\textstyle\sum_m (-)^m d_m d_{-m} - \tfrac{1}{2} s s, \\ S_0 &= \tfrac{1}{4}\textstyle\sum_m (d_m{}^\dagger d_m + d_m d_m{}^\dagger) + \tfrac{1}{4}(s^\dagger s + s s^\dagger), \end{aligned} \tag{5}$$

as $P_6 = S_+ S_-$. The expectation value of the product S_+S_- is in turn given by

$$\langle S_+ S_- \rangle = S_0(S_0 - 1) - S(S-1), \tag{6}$$

where $S(S-1)$ is the eigenvalue of the Casimir operator of the group SU(1, 1) defined by (5). Noting that

$$S_0 = \tfrac{1}{2}N + \tfrac{6}{4}, \quad S = \tfrac{1}{2}\sigma + \tfrac{6}{4}, \tag{7}$$

we can write

$$\langle P_6 \rangle = \tfrac{1}{4}(N - \sigma)(N + \sigma + 4). \tag{8}$$

Next we consider the quadratic Casimir operator of O(5) which we denote by C_5. The expectation value of C_5 in the representation (τ) of O(5) is given by[7]

$$\langle C_5 \rangle = \tfrac{1}{6}\tau(\tau + 3). \tag{9}$$

Finally we consider the quadratic Casimir operator of O(3), C_3. The expectation value of C_3 in the representation L of O(3) is trivially given by

$$\langle C_3 \rangle = L(L+1). \tag{10}$$

Thus the Hamiltonian

$$H = AP_6 + BC_5 + CC_3 \tag{11}$$

is diagonal with eigenvalues

$$E([N]\sigma\tau\nu_\Delta LM) = A \times \tfrac{1}{4}(N - \sigma)(N + \sigma + 4) + B \times \tfrac{1}{6}\tau(\tau + 3) + CL(L+1). \tag{12}$$

The spectrum of Eq. (12) for positive A, B, and C is shown in Fig. 1. It consists of repeating patterns 0^+; 2^+; 4^+, 2^+; ... , corresponding to the various values of $\sigma = N, N-2, \ldots$. Within each pattern there are several levels corresponding to the values of τ, ν_Δ, and L. The effect of a positive A is that of placing the representation with $\sigma = \sigma_{\max} = N$ lowest in energy (thus giving maximum population of s bosons to the ground state), while a positive B gives the ordering $0_1^+, 2_1^+, 4_1^+, \ldots$, and it is related to the positive value of $\epsilon = \epsilon_d - \epsilon_s$ in Ref. 1; finally $C > 0$ places the 2_2^+ state below the 4_1^+, etc.

We have also succeeded in constructing closed expressions for $E2$ transition rates. The derivation is rather elaborate and we will present it in a forthcoming paper. Here we only quote some results, which may be useful in analyzing the experimental data. If we insist that the $E2$ operator be a generator of O(6), then the most general form of it is

$$T^{(E2)} = \alpha_2 (d^\dagger s + s^\dagger d)^{(2)}. \tag{13}$$

This operator satisfies the selection rules $\Delta\sigma = 0$ and $\Delta\tau = \pm 1$, the former being a consequence of the fact that $T^{(E2)}$ is a generator of O(6) and thus cannot couple the different O(6) representations, and the latter being a consequence of the fact that $T^{(E2)}$ can only change one d boson into s or vice versa. Using (13) we obtain, for example,

$$B(E2; [N], \sigma = N, \tau+1, L = 2\tau + 2 \to [N], \sigma = N, \tau, L = 2\tau) = \alpha_2^2 \frac{\tau + 1}{2\tau + 5}(N - \tau)(N + \tau + 4) \tag{14}$$

and

$$B(E2; [N], \sigma = N, \tau + 1, L = 2\tau \to [N], \sigma = N, \tau, L = 2\tau) = \alpha_2^2 \frac{4\tau + 2}{(2\tau + 5)(4\tau - 1)}(N - \tau)(N + \tau + 4). \tag{15}$$

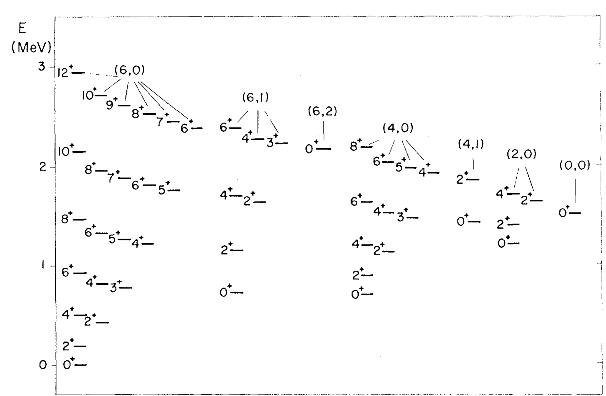

FIG. 1. A typical O(6) spectrum for $N = 6$. The energy levels are given by Eq. (12) with $A = 100$ keV, $B = 240$ keV, and $C = 5$ keV. The values in parentheses are (σ, ν_Δ).

It is interesting to compare the ratios

$$\frac{B(E2; 4_1^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)} = \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \xrightarrow[N\to\infty]{} \frac{10}{7},$$
$$\frac{B(E2; 2_2^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)} = \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \xrightarrow[N\to\infty]{} \frac{10}{7}, \tag{16}$$

with those obtained in the classical geometrical description of a rigid triaxial rotor[8] with $\gamma = 30°$. Because the values (16) (as well as others not discussed here) are identical to those of the rigid triaxial rotor with $\gamma = 30°$ one may say that the O(6) limit corresponds in some sense to that case. There are, however, several differences between these two descriptions. For example, in the O(6) limit excited 0^+ states occur in a natural way, while in the triaxial limit they do not. Moreover, the electromagnetic transition rates deviate considerably from those of a triaxial rotor when N is small.

In conclusion, we have suggested a third dynamical symmetry, in addition to SU(5) and SU(3), which may be useful in describing properties of nuclei at the end of major shells. We point out, however, that microscopic calculations in which both proton and neutron bosons are introduced explicitly[9] indicate that the Hamiltonian for the combined system may not be invariant under proton-neutron transformations (the variable called F spin in Ref. 8) at the end of major shells. The O(6) symmetry must then be viewed only as an approximate symmetry describing the main features of the spectra observed at the end of the major shells, and a detailed comparison with experiment may require the explicit introduction of proton and neutron degrees of freedom.

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New Predictions for Rayleigh Scattering: 10 keV–10 MeV

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New theoretical predictions for the contribution to elastic photon scattering from atoms due to the bound atomic electrons are compared with recent experiments and previous theory. At 1.33 MeV, we resolve the large-angle disagreement for experiments on lead. For 2.75-MeV photons scattered by lead, we confirm the theoretical Rayleigh scattering amplitudes of Cornille and Chapdelaine. At 6.84 MeV, we estimate that the form-factor approximation yielded predictions for the L-shell Rayleigh amplitudes which were too large by 15%. For experiments below 100 keV, the form-factor approximation is poor.

We wish to report resolution of discrepancies between theory and several recent experiments[1-7] for high-energy elastic photon scattering, achieved with a new theoretical calculation of the amplitudes for scattering off bound electrons (Rayleigh scattering). At the same time we are able to indicate under what circumstances the form-factor approximation, most commonly used to predict the Rayleigh-scattering amplitudes, is adequate. Subsequently we will present a more systematic discussion of the Rayleigh-scattering amplitudes for all atomic electrons in the keV and MeV range for all atomic numbers. Interest in these Rayleigh amplitudes, important for the determination of absorption coefficients, has also recently arisen in attempts to observe experimentally the Delbrück-scattering amplitudes,[4] from its proposed use[8] as a diagnostic tool for spatial resolution of densities and temperatures of neutrals in plasmas, and because it is a serious background which cannot be distinguished by energy discrimination in nuclear fluorescence experiments.[9]

Our numerical method, expected to be valid for energies from 1 keV to 10 MeV, follows that of Brown and co-workers,[10] which also gives the details of the basic formalism. We assume that the atom is represented by noninteracting electrons in a screened central potential V resulting from the charge distribution of the nucleus and the atomic electrons. Starting with the second-order S-matrix element of the quantum electrodynamic interaction of electrons in an external potential V with radiation, we expand the photon wave func-