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*M*3 Suppression in Hartree-Fock Theory

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(Received 2 December 1977)

With a zero-range interaction a valence spin-up neutron will not interact with core spin-up neutrons and hence will not deform them; it will deform spin-down neutrons. This introduces a spin-quadrupole correlation. An ideal operator for probing this correlation is the $M3$ operator, since this operator is crudely $E2 \times M1$. The suppression of the $M3$ moment in ^{17}O due to this effect, as well as due to the spin dependence of the interaction, is considered.

It has been found by Bertozzi and others[1] that the $M3$ part of magnetic scattering in ^{17}O is strongly suppressed in the region of momentum transfer where it is expected to be strong when a single-particle picture, of a closed ^{16}O core plus a valence $d_{5/2}$ neutron, is used. A similar suppression in ^{51}V has been reported by Enomoto.[2]

It is not the intent of this work to reproduce the momentum transfer dependence of $M3$ scattering. Rather, only the zero momentum-transfer limit will be considered. The intention is to show that by keeping in mind a Hartree-Fock picture to describe a valence particle and a polarized core, the physical reason for $M3$ spin suppression becomes transparent.

The spin part of the $M3$ operator is basically a product of an $E2$ operator and an $M1$ operator. This ties in nicely with a correlation between quadrupole deformation and spin which had been noted by Zamick, Golin, and Moszkowski (ZGM).[3]

Let us recall the argument which was concerned with the quadrupole deformation of the core due to the presence of a valence nucleon, e.g., ^{16}O core and $d_{5/2}$ nucleon. A simple model of the deformed ^{16}O is one in which all orbits have the same deformation. For example, if one uses harmonic oscillator wave functions, then all orbits in the core would have the same oscillator length parameters $b_x = b_y \neq b_z$.

Such a model had indeed been considered by Mottelson.[4] By also assuming that the potential followed the density he was able to show that the quadrupole moment of the core protons is $(Z/A)Q_{\text{valence}}$. One could also express this result in terms of an effective charge

$$e = Q_{\text{core}}/Q_V = Z/A.$$

But ZGM[3] pointed out that if the interaction between the valence particle and the core is a zero-range interaction, then the above "trial solution" must be modified. Let the valence particle be a spin-up neutron. With a zero-range interaction this valence neutron cannot interact with spin-up neutrons in the core. Hence the spin-up neutrons in the core should not be deformed. On the other hand, the valence neutron *can* interact with spin-down neutrons and so we expect the spin-down neutrons to be deformed.

The magnetic multipole moment is

$$\mathfrak{M}(ML,\mu)=\frac{e\hbar}{2mc}[L(2L+1)]^{1/2}\sum_i r_i^{L-1}\left[\mu_i(Y_{L-1}\sigma)_\mu{}^L+\frac{2g_l}{L+1}(Y_{L-1}l)_\mu{}^L\right]_i,$$

where

$$\mu_i=\begin{cases}2.793 \text{ for a proton,}\\ -1.913 \text{ for a neutron;}\end{cases}$$

and

$$g_l=\begin{cases}1 \text{ for a proton,}\\ 0 \text{ for a neutron.}\end{cases}$$

In the Hartree-Fock approximation the expectation value of the one-body operator $\mathfrak{M}(ML,0)$ is

$$\sum_i(i|\mathfrak{M}(ML,0)|i)$$

summed over occupied states i. We note that the $M3$ operator is basically a quadrupole operator coupled with the magnetic moment to $L=3$, plus a term with the quadrupole operator coupled with the orbital angular momentum to $L=3$.

In this work we will consider mainly the suppression due to spin, not to orbital angular momentum. We therefore consider the $(Y_{L-1}\sigma)_\mu{}^L$ term. If we use l-s wave functions then the only part of the operator which contributes is $Y_{z,0}\sigma_z$. Therefore we will work with the operator

$$M_s(3)=(16\pi/5)^{1/2}\sum_i r_i^2 Y_{2,0}{}^{(i)}\sigma_z(i)\mu_i.$$

The value of this operator for the $d_{5/2}$ valence particle in the $M=\frac{5}{2}$ state (this is a pure spin-up state) is $-1.91Q_{\text{valence}}$ $(Q_V=-[(2j-1)/(2j+2)]\langle r^2\rangle)$.

We now consider the core. Since the spin-up neutrons are not deformed the expectation value of the $M_s(3)$ operator is zero. I introduce the notation $Q_{\pi\uparrow}$, $Q_{\pi\downarrow}$ and $Q_{\nu\downarrow}$ for the quadrupole moments of the core particles—protons spin up, protons spin down, and neutrons spin down. The value of the $M_s(3)$ operator in the core is then

$$(Q_{\pi\uparrow}-Q_{\pi\downarrow})2.79+1.91Q_{\nu\downarrow}.$$

Now assume the interaction of the valence spin-up neutron with the core is a δ interaction $-A[1+(-1)^Tx]\delta(\vec{r}_1-\vec{r}_2)$. A not uncommon choice is $x=\frac{1}{3}$, for which the strength in a $T=0$, $S=1$ state is twice that for a $T=1$, $S=0$ state.

Assume the quadrupole moment of the spin-up and spin-down protons in the core is proportional to the strength of the interaction. I introduce Q_0 such that

$$Q_{\pi\uparrow}=\tfrac{1}{2}Q_0(1+x),$$

$$Q_{\pi\downarrow}=\tfrac{1}{2}Q_0.$$

I arrived at these results by noting that the $\pi\downarrow$-valence-neutron interaction is in a pure $s=1$ state while the $\pi\downarrow$-valence interaction is half singlet and half triplet. The proton contribution to the $M_s(3)$ expectation value is then

$$\tfrac{1}{2}Q_0(2.79)x.$$

Rather than calculate Q_0 we can make an association with an $E2$ effective charge. The effective $E2$ charge for a valence $(d_{5/2})$ neutron is defined as

$$e_\nu=Q_{\text{core proton}}/Q_V=(Q_{\pi\uparrow}+Q_{\pi\downarrow})/Q_V$$
$$=Q_0(1+x/2)/Q_V.$$

Thus the proton contribution to $M_s(3)$ is

$$[x/2(1+\tfrac{1}{2}x)]e_\nu(2.79)Q_V.$$

This is clearly the opposite sign from the value of $M_s(3)$ for the $d_{5/2}$ neutron $(-1.91Q_V)$. We recall that x is about $\frac{1}{3}$ and e_ν is often taken to be $\frac{1}{2}$. Since the proton contribution is proportional to x, the result is due to the spin dependence of the interaction.

We now come to the neutron core contribution $1.91Q_{\nu\downarrow}$. The interaction with the valence neutron is in a pure $s=0$ state, and so if we assume that the deformation is proportional to the interaction (which is equivalent to first-order perturbation theory) we get

$$Q_{\nu\downarrow}=\frac{Q_0}{2}(1-x)(1.91)=\frac{(1-x)e_\nu}{2(1+x/2)}(1.91)Q_V.$$

This is also the opposite sign from the value for the $d_{5/2}$ neutron. The reason is clear: Only spin-down neutrons contribute, plus the fact that a quadrupole distortion of the core has the same sign as that of the particle.

Up to now, the result for the value of $M_s(3)$ is

$$Q_V\left(-1.91+\frac{xe_\nu}{2(1+x/2)}\times2.79+\frac{(1-x)e_\nu}{2(1+x/2)}\times1.91\right).$$

We next consider a modification of the above result due to RPA correlations. We will do this in a simplified way. We first note that the neutron core contribution can be written in terms of the $E2$ effective charge correction for a valence proton. The point is that if charge symmetry holds the valence-proton–core-proton interaction is the same as the valence-neutron–core-neutron interaction.

I define the effective charge for a valence proton (^{17}F) as

$$e_\pi = \frac{Q_0}{2}\frac{1-x}{Q_V}.$$

Hence the result for the value of $M_s(3)$ can be written as

$$Q_V\left(-1.91 + \frac{x}{2(1+x/2)}e_\nu \times 2.79 + \frac{e_\pi \times 1.91}{1+x/2}\right).$$

(Remember that the term with e_ν is due to the protons and the term with e_π is due to the neutron.)

When one calculates e_ν and e_π in first-order perturbation theory, e_π comes out to be much less than e_ν. However, when one does a random-phase-approximation (RPA) calculation they come out to be much closer to each other. Hence the last term, due to the core neutrons, might be very sensitive to RPA correlations.

I now illustrate the effect of the RPA by doing a simplified calculation. I call the first-order charges $e_{F\nu}$ and $e_{F\pi}$ and the RPA values $e_{R\nu}$ and $e_{R\pi}$. I introduce isoscalar and isovector charges as follows:

$$e_{F\nu} = (|e_F{}^0| + |e_F{}^1|)/2,$$
$$e_{F\pi} = (|e_F{}^0| - |e_F{}^1|)/2.$$

The RPA results can be obtained by changing the energy denominators. If one uses zero-range effective interactions, e.g., of the Skyrme type, then the effective mass is one which means that the unperturbed single-particle, single-hole splitting for the quadrupole state is very close to $2\hbar\omega$. An approximate way of simulating the RPA is to change the energy denominators from $2\hbar\omega$ to the energies of the isoscalar and isovector giant quadrupole states. These are approximately $\sqrt{2}\,\hbar\omega$ and $4\hbar\omega$, respectively.[5] We then obtain

$$e_{R\nu} = (\sqrt{2}\,|e_F{}^0| + \tfrac{1}{2}|e_F{}^1|)/2,$$
$$e_{R\pi} = (\sqrt{2}\,|e_F{}^0| - \tfrac{1}{2}|e_F{}^1|)/2$$

or

$$e_{R\nu} = \tfrac{1}{2}(\sqrt{2} + \tfrac{1}{2})e_{F\nu} + \tfrac{1}{2}(\sqrt{2} - \tfrac{1}{2})e_{F\pi},$$
$$e_{R\pi} = \tfrac{1}{2}(\sqrt{2} - \tfrac{1}{2})e_{F\nu} + \tfrac{1}{2}(\sqrt{2} + \tfrac{1}{2})e_{F\pi}.$$

We shall see that it is not so easy to get the effective charges, especially e_π, unambiguously, either from theory or experiment. I quote two recent analyses. Brown, Arima, and McGrory[6] give for $d_{5/2}$-$d_{5/2}$ in mass 17 $e_\nu = 0.33 \pm 0.01$, and $e_\pi = 0.14 \pm 0.23$ according to one analysis and 0.24 $\pm$ 0.27 according to another. In analyzing mass 18 they get $e_\pi = -0.07 \pm 0.03$. Durrell, Harter, and Phillips[7] determine the radial integrals $\langle d_{5/2}|r^2|d_{5/2}\rangle$ from sub-Coulomb heavy-ion transfer and deduce for $d_{5/2}$-$d_{5/2}$ $e_\nu = 0.43 \pm 0.02$, $e_\pi = 0.48 \pm 0.33$.

The range in variation of e_π is too wide for our purposes. Let us keep in mind that by e_π we here really mean the *neutron*-core deformation due to a valence *neutron*, not the proton-core deformation due to a valence proton. One can have large breakdown of charge symmetry due to loose binding effects. Basically what happens is that if a proton is loosely bound it is far away from the core and therefore cannot polarize the core very well. In that case e_π would be very small. But if the corresponding neutron is more tightly bound it may well be able to polarize the core neutrons more strongly.

Despite the difficulties mentioned above, I feel that meaningful limits on e_ν and e_π can be obtained from theory. Let us consider several cases which are in order of increasing believability (in the author's opinion).

(1) Take $e_{F\nu} = \frac{1}{2}$ and $x = \frac{1}{3}$. Use first-order perturbation theory. Then $e_{F\pi} = \frac{2}{7}e_{F\nu} = \frac{1}{7}$. The value of $M_s(3)$ is $Q_V(-1.91 + 0.2 + 0.23) = -1.48Q_V$ (corresponding to valence, core proton, and core neutron contribution). The suppression factor is $1.48/1.91 = 0.77$.

(2) Use the RPA. Assume $e_{F\pi} = \frac{2}{7}e_{F\nu}$ as above and arrange for $e_{R\nu}$ to be $\frac{1}{2}$. We then find $e_{R\pi} = 0.34$. Using the RPA result we find

$$M_s(3) = Q_V(-1.91 + 0.20 + 0.56) = -1.15Q_V.$$

The suppression factor is $1.15/1.91 = 0.6$.

(3) We now change x from $\frac{1}{3}$ to 1. This looks like a drastic step, but it can be justified. The value $x = \frac{1}{3}$ is chosen for spectroscopy of a few valence nucleons. But with such an interaction the symmetry energy is much too low. If we change to $x = 1$ we get a much better symmetry energy. Since core polarization is a one-body

field effect one should choose an interaction which yields the best one-body potential, including the symmetry energy.

In this case $e_{F\pi}=0$. However, the proton charge becomes finite in the RPA. Again we take $e_{R\nu}=0.5$. We find

$$e_{R\pi}=\frac{\sqrt{2}-\frac{1}{2}}{\sqrt{2}+\frac{1}{2}}e_{R\nu}=0.24.$$

Now

$$M_s(3)=Q_V(-1.91+0.46+0.31)=-1.14Q_V.$$

The suppression factor is $1.14/1.91=0.6$, the same as in case (2).

It is our feeling that the RPA calculation is more sound than the first-order one. Theoretical support for this argument comes from the work of Brown,[8] who notes that when one linearizes the Hartree-Fock equations for a core plus one particle, the effective charges satisfy the RPA equations of motion.

We now consider the $[Y_2\vec{1}]_0^3$ term. This can be written as $ar^2Y_{2,0}L_z+b[r^2Y_{2,1}L_{-1}+r^2Y_{2,-1}L_1]$ where a and b are constants. We can further reduce this to the form $C_1z^2L_z+C_2(x^2+y^2)L_z$. We now can easily see that this will vanish if we choose the usual form for the deformation of the core. It is most convenient to discuss this in terms of the asymptotic Nilsson quantum numbers N, n_3, and Λ.

For an axially symmetric deformation the values of z^2 and x^2+y^2 are the same in the states $+\Lambda$ and $-\Lambda$. Hence z^2L_z and $(x^2+y^2)L_z$ will be equal and opposite, and will cancel. Also the matter distribution is the same for $+\Lambda$ and $-\Lambda$, so that it is hard to see why these two states should have different deformations.

In conclusion, I hope I have convinced the reader that there is indeed a spin-quadrupole correlation present in a nucleus and that it has observable consequences. When a valence neutron is added to the core, it is the Pauli principle which causes this correlation in the core neutrons, and it is the spin dependence of the interaction which causes this correlation in the core protons. These two effects add coherently in causing a suppression of the $M3$ moment.

A list of M moments by Migdal[10] indicates that there is a suppression, relative to the single-particle model, in a large number of nuclei.

This work was supported by the National Science Foundation.

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