Electroexcitation of the Ground-State Rotational Band in ^{181}Ta

F. N. Rad, T. Sasanuma, W. Bertozzi, J. Heisenberg, M. V. Hynes, S. Kowalski, H. Miska, B. Norum, C. P. Sargent, W. Turchinetz, and C. F. Williamson
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 28 November 1977)

Elastic and inelastic form factors for electron scattering from the ground-state rotational band of ^{181}Ta have been studied for momentum transfers $0.4\ \text{fm}^{-1} \leq q_{\text{eff}} \leq 2.7\ \text{fm}^{-1}$. The data are shown to provide an independent check of the rotational assumption, thus adding confidence that the experimental form factors are characteristic of a stable intrinsic state associated with the rotational band. The data are also compared to a Hartree-Fock calculation using density-dependent interactions.

The basis of all present models of deformed nuclei is the assumption that there exists a stable deformed intrinsic state, $X_K(\vec{r})$, and hence a deformed intrinsic density $\rho(\vec{r}) = \langle X_K(\vec{r})|\rho|X_K(\vec{r})\rangle$. From this intrinsic state one projects states of good angular momentum $I = K, K+1, K+2, \ldots$, to form the rotational band. An energy spacing of the states that follows $I(I+1)$ is assumed to imply a stable deformation with X_K maintaining its identity while undergoing collective rotations.

The first term of transition densities in an expansion in powers of $1/\langle J_\perp^2\rangle$ is provided by the simple model of Bohr and Mottelson[1] and Rainwater.[2] In this model the wave function is understood to factorize into the product of a wave function describing the intrinsic and static core and a wave function describing the rotation. Assuming axial symmetry, the rotational wave function can be written as[3]

$$\psi_{MK}^{I}(\vec{r},\varphi) = [(2I+1)/16\pi^2]^{1/2}[D_{MK}^{I}(\varphi)X_K(\vec{r}) + (-1)^{I+K}D_{M-K}^{I}(\varphi)X_{-K}(\vec{r})].$$

Using the formalism of the distorted-wave Born approximation (DWBA) to account for Coulomb distortion in electron scattering, one can relate the Legendre projections of the intrinsic densities $\rho_L(r)$ to the form factors $F_L(q,\theta)$ in the expression for the cross section,

$$\sigma_{I_i \to I_f}(q,\theta) = \sigma_{\text{Mott}}(q)\textstyle\sum_L |\langle I_i, K; L, 0|I_f, K\rangle F_L(q,\theta)|^2$$

and

$$\rho_L(r) = (2L+1)\textstyle\int_0^{\pi/2} \rho(r,\theta) P_L(\theta) \sin\theta\, d\theta.$$

In this work we assume the distortion of the electron waves to be caused only by the spherical part of the intrinsic charge density $\rho_0(r)$. Using these distorted waves, the contributions to the elastic and inelastic cross sections of the higher multipole densities, $\rho_L(r)$, are derived as first-order perturbations.

In the region of the deformed nuclei, there has been an extensive study of the even-even nuclei, using electron scattering by various groups.[4-6] From these measurements the parameters of deformed Fermi distributions were derived to characterize the intrinsic density. The resulting deformation parameters and the various $B(EL)$ are in general agreement with values derived from Coulomb excitation and muonic x rays. One conclusion from electron-scattering data is that the Fermi distribution is not adequate to describe the q dependence of the form factors. The results of Ref. 5 also have been compared with microscopic models based on the Hartree-Fock method and using various models for the nucleon-nucleon interaction adjusted to fit the densities of the spherical even-even nuclei ^{208}Pb and ^{40}Ca.[7] These calculations succeed rather well in predicting $|F_L(q,\theta)|^2$ for $L = 0$ and 2 for the rare-earth nuclei and ^{238}U and ^{232}Th.[5] However, when one considers the higher multipoles, $L = 4, 6$, these microscopic models are progressively less able to reproduce the data. A similar systematic difficulty is apparent in the results of Nakada *et al.*[6] where electron-scattering results from ^{152}Sm are compared to the Hartree-Fock calculations of Decharge *et al.*[8] The explanation of these discrepancies is unclear, but they could be the result of a breakdown of the rotational model, of coupled-channel effects, or of a poor description of the intrinsic state. Experiments on the even-even rotators cannot distinguish between these effects.

In contrast to the even-even nuclei where a transition $0^+ \to I^+$ has only one contributing multipole, $\rho_I(r)$, each transition in odd-even nuclei

can have contributions from several multipoles. For the case of ^{181}Ta over the q range investigated, there are more measured cross sections than there are multipoles that make significant contributions. Therefore, the F_L's are overdetermined, and it is possible to predict some of the cross sections in terms of others. This can serve as an independent check of some of the assumptions of the analysis outlined above. In particular, our results add confidence that the method of analysis we have outlined yields a good determination of the intrinsic shape of well-deformed nuclei without the requirement of a polarized target to separate the various multipoles.

The present experiment was performed at the 400-MeV electron scattering facility of the Massachusetts Institute of Technology Bates Linear Accelerator Laboratory. The high-resolution spectrometer, whose performance[9] and design[10] are described in detail elsewhere, was used to obtain these data at laboratory scattering angles of 90° and 45° over the effective momentum transfer range 0.4 fm$^{-1} \leq q_{\text{eff}} \leq$ 2.7 fm^{-1}. Electron-beam currents in the range of 10–60 μA were incident on natural tantalum targets ranging in thickness from 4 to 25 mg/cm^2. The spectrometer energy resolution ($\Delta p/p \approx 1 \times 10^{-4}$ over the entire momentum range studied) allowed a clear separation between the members of the ground-state rotational band up to and including $I = \frac{17}{2}$.

The measured cross sections were normalized to the elastic ^{12}C measurements published by Sick and McCarthy.[11] However, in the region of the minimum of the ^{12}C elastic (0^+) cross section, where good data are not available, we relied on the constancy of the parameters of our apparatus determined by this normalization. Our absolute measured cross sections for the 0^+ are in good agreement with those of Ref. 11.

Our experimental results at 90° are shown in Fig. 1 where the "form factor" $[F = (\sigma/\sigma_{\text{Mott}})^{1/2}]$ has been plotted as a function of effective momentum transfer q_{eff},[12] for each member of the ground-state rotational band up to and including the $\frac{17}{2}^+$ state. The 45° data are also included in Fig. 1 after suitable correction by comparing the DWBA predictions at 45° and 90° for the same q_{eff} using the Hartree-Fock densities described below. The error bars indicated are statistical, and errors due to the ^{12}C scattering cross sections of Ref. 11 have not been included. Where no error bars are shown, the statistical uncertainty is equal to or smaller than the symbols. A comparison of cross sections measured at 160° at

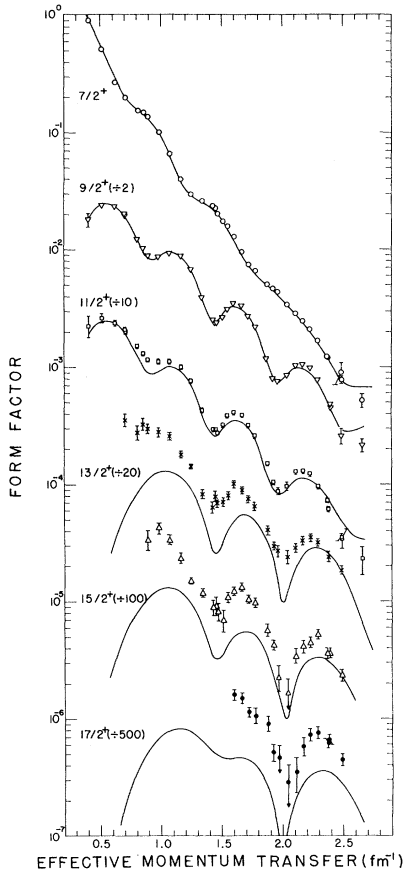

FIG. 1. Experimental "form factor" $[F = (\sigma/\sigma_{\text{Mott}})^{1/2}]$ for the ground-state rotational band of ^{181}Ta as a function of effective momentum transfer. All data points were taken at 90° laboratory angle except those below $q_{\text{eff}} = 0.8$ fm^{-1} which were taken at 45°. The solid curves are the theoretical calculation based on the density-dependent interaction with filling approximation using the Hartree-Fock method.

$q_{\text{eff}} = 1.92$ and 1.72 fm^{-1} shows that the transverse component of the cross section at 90° is small. In addition a calculation[13] shows a broad maximum in the transverse component of the form factors at $q_{\text{eff}} \sim 2$ fm^{-1}. This corresponds to a minimum in the Coulomb form factors and at this value of q the transverse interaction contributes no more than 10% to the cross section. No correction for transverse components is made for the purposes of discussion which follows. The solid curves are the result of a Hartree-Fock calculation based on the density-dependent interaction.[7] Since these codes at present are limited to axially symmetric even-even rotational nuclei, we have made the so-called "filling approximation" to allow application of this code to odd-A nuclei. In this approximation the states of angu-

lar momentum projection on the Z axis (Ω and $-\Omega$) are assumed to be populated equally for the odd orbit. The agreement between the data and the theory is excellent for the $\frac{7}{2}^+$ and the $\frac{9}{2}^+$ states and deteriorates progressively as the angular momentum increases. An examination of the contribution of the separate multipoles shows that the $\frac{7}{2}^+$ and $\frac{9}{2}^+$ states are dominated by the F_0 and F_2 form factors, respectively. For the higher angular momentum states F_4 and F_6 are important. The extracted intrinsic quadrupole moment from the Hartree-Fock calculation is 7.1 e·b which is in agreement with the measured values of $Q_0 = 7.38 \pm 0.13$ e·b of McLaughlin *et al.*[14] and $Q_0 = 6.82 \pm 0.06$ e·b of Powers *et al.*[15]

Only F_0, F_2, F_4, and F_6 contribute significantly to the reported data. This has been verified by obtaining best values for these form factors from the $\sigma_{7/2}$, $\sigma_{9/2}$, $\sigma_{11/2}$, $\sigma_{13/2}$, $\sigma_{15/2}$, and then predicting $\sigma_{17/2}$ and comparing to experiment. $\sigma_{17/2}$ is considerably more sensitive to F_8 and higher multipoles than are the other observed cross sections. Excluding $\sigma_{17/2}$ there are five cross sections and only four significant multipoles. The consistency of the rotational analysis for ^{181}Ta can be checked by using four of the cross sections to determine the squared form factors of the four multipoles and then using these to predict the fifth cross section and comparing it with experiment. The prediction is thus in the form of a linear combination of the four cross sections, the coefficients involving only the Clebsch-Gordon coefficients. Two of the four possible predictions (no prediction is possible for $\sigma_{7/2}$, since F_0 contributes only to $\sigma_{7/2}$) are illustrated in Fig. 2. Figure 2(a) shows the ratio (R) of the measured $\sigma_{11/2}$ to that predicted from $\sigma_{9/2}$, $\sigma_{13/2}$, and $\sigma_{15/2}$. Similarly Fig. 2(b) shows the ratio of the measured $\sigma_{13/2}$ to that predicted from $\sigma_{9/2}$, $\sigma_{11/2}$, and $\sigma_{15/2}$. In these cases and also in the cases of the $\sigma_{9/2}$ and $\sigma_{15/2}$ (not shown) the data are consistent with the fundamental rotational assumption ($R = 1$). One can therefore, with more confidence, interpret the extracted best values of the form factors $F_0, \ldots, F_6$ (or their associated charge distributions) as characteristic of an intrinsic state associated with the rotational band, and the discrepancies between the experiment and the Hartree-Fock predictions are mostly due to an inadequacy of the Hartree-Fock results for the $\rho_4(r)$ and $\rho_6(r)$ densities.

This work was supported in part by the U. S. Department of Energy, Contract No. EY-76-C-02-3069*000, in part by Deutscher Akademischer Austausch Dienst, and in part by the National Research Council of Canada.

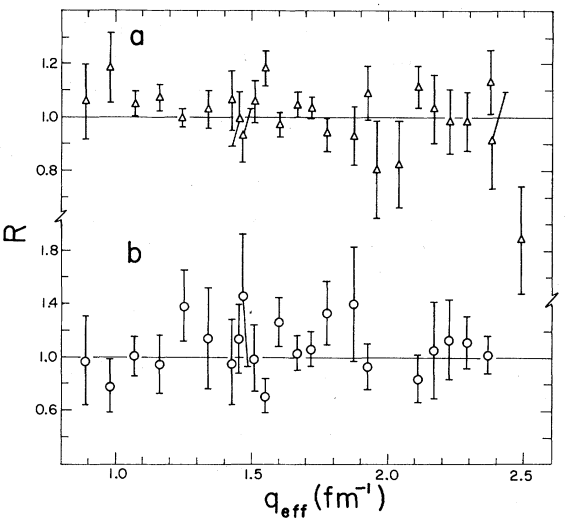

FIG. 2. The ratio (R) of the measured cross section (a) $\sigma_{11/2}$, and (b) $\sigma_{13/2}$ to that predicted by the rotational assumption from (a) $\sigma_{9/2}$, $\sigma_{13/2}$, and $\sigma_{15/2}$, and (b) $\sigma_{9/2}$, $\sigma_{11/2}$, and $\sigma_{15/2}$.

[1]A. Bohr and B. Mottelson, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 27, No. 16 (1953).

[2]J. Rainwater, Phys. Rev. 79, 432 (1950).

[3]John D. Rogers, Annu. Rev. Nucl. Sci. 15, 241 (1965).

[4]W. Bertozzi *et al.*, Phys. Rev. Lett. 28, 1711 (1972); T. Cooper *et al.*, Phys. Rev. C 13, 1083 (1976).

[5]C. W. Creswell, Ph.D. thesis, Department of Physics, Massachusetts Institute of Technology (unpublished); A. S. Hirsch, Ph.D. thesis, Department of Physics, Massachusetts Institute of Technology (unpublished); C. W. Creswell *et al.*, to be published.

[6]A. Nakada *et al.*, Phys. Rev. Lett. 38, 584 (1977).

[7]J. W. Negele and G. Rinker, Phys. Rev. C 15, 1499 (1977).

[8]J. Decharge *et al.*, Phys. Lett. 55B, 361 (1975).

[9]W. Bertozzi *et al.*, Nucl. Instrum. Methods 141, 457 (1977).

[10]S. Kowalski *et al.*, in *Medium Energy Nuclear Physics with Electron Linear Accelerators—MIT 1967 Summer Study*, U. S. Atomic Energy Commission Technical Information Division Report No. TID-24667, edited by W. Bertozzi and S. Kowalski (U. S. Atomic Energy Commission, Washington, D. C., 1967), p. 39.

[11]I. Sick and J. S. McCarthy, Nucl. Phys. A150, 631 (1970).

[12]$q_{\rm eff} = q(1 + 3Ze^2/2E_0R_u)$, where q is the momentum

transfer, E_0 the incident energy, and R_u the equivalent uniform radius: $R_u^2 = \frac{5}{3}\langle r^2 \rangle$.

[13]E. Moya deGuerra and S. Kowalski, to be published, and private communication.

[14]D. McLaughlin *et al.*, Phys. Rev. C 13, 1644 (1976).

[15]R. J. Powers *et al.*, Nucl. Phys. A278, 477 (1977).

Possible Mechanism for the Resonances in the $^{12}C + ^{16}O$ System

D. Shapira

Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627, and Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

and

R. M. DeVries, M. R. Clover, and R. N. Boyd

Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627

and

R. N. Cherry, Jr.

Hamilton College, Clinton, New York 13323

(Received 18 October 1977)

New data on resonances in the $^{12}C + ^{16}O$ system are presented. A systematic pattern is observed suggesting that theoretical models which have recently been formulated for the mechanism of resonances appearing in the $^{12}C + ^{12}C$ system also apply to this system. Furthermore, energies, spins, and widths for entrance-channel resonances predicted by a recent microscopic calculation of $^{12}C + ^{16}O$ scattering agree very well with the data presented here.

Significant progress has been made recently in understanding the mechanism responsible for the appearance of correlated resonant structure in several heavy-ion–induced reactions.[1-4] Most of the theoretical descriptions have several common features. A mechanism is postulated wherein the contribution of a single partial wave in the entrance channel is enhanced over a broad energy range (e.g., shape elastic resonances). These broad entrance-channel resonances then couple to other degrees of freedom of the system which have special states with longer lifetimes. The various models differ in their description of the broad entrance-channel resonances and the intermediate degrees of freedom involved. Such coupling to intermediate states fragments the broad entrance-channel resonances into the intermediate structure which is observed. So far the most extensive body of data on such resonances has been available for the $^{12}C + ^{12}C$ system (see Refs. 1 and 2 and references therein). Resonances of the same spin in that system are grouped in clusters that form broad enhancement regions. This systematic pattern is cited as evidence that coupling of the entrance-channel resonances to the intermediate degrees of freedom is weak.[5]

The $^{12}C + ^{16}O$ and $^{12}C + ^{12}C$ systems are similar in many respects. Description of the elastic scattering can be obtained by using similar optical potentials.[6] Hence, formation of molecular resonances in the entrance channel is equally likely in both systems and their coupling to other degrees of freedom should occur with similar strength. Also, the level densities in the compound nuclei formed and the number of channels open for their decay are very close for both systems (at comparable energies).[7] Therefore, intermediate states, once formed, are expected to have comparable spreading widths in both systems. Many resonances were observed for $^{12}C + ^{16}O$ reactions,[8-13] but a pattern similar to that in $^{12}C + ^{12}C$ has not been reported yet. In the following we present new data on $^{12}C + ^{16}O$ inelastic scattering that, together with other previously published data, demonstrate that the systematic behavior seen so far of resonances in $^{12}C + ^{12}C$ also exists in the $^{12}C + ^{16}O$ system.

We have measured excitation functions for the inelastic scattering of ^{16}O by ^{12}C at several large center-of-mass angles. Natural carbon targets were bombarded with ^{16}O ions and the recoiling ^{12}C ions were detected in the focal plane of an Enge split-pole spectrograph at six angles between 4° and 10.6° in the laboratory (a c.m. angle span of 14°). Figure 1 presents a summary of our excitation-function data. Two clusters of

