not included in the BEM calculations.

Our results show that heavy-ion–induced two-nucleon–transfer reactions provide rather sensitive tests of nuclear structure theories. The multistep processes involving excited 0^+ states make important contributions to the transfer reactions. The striking shape differences between 2_1^+ angular distributions in stripping and pickup reactions on transitional Sm nuclei are not explained satisfactorily by current theoretical models.

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[1] B. Sørensen, Nucl. Phys. A142, 411 (1970); T. Kishimoto and T. Tamura, Nucl. Phys. A270, 317 (1976).

[2] K. Kumar, Nucl. Phys. A231, 189 (1974).

[3] R. J. Ascuitto, N. K. Glendenning, and B. Sørensen, Phys. Lett. 34B, 17 (1971), and Nucl. Phys. A103, 60 (1972).

[4] B. Sørensen, Phys. Lett. 66B, 119 (1977).

[5] R. J. Ascuitto and J. S. Vaagen, in *Proceedings of the International Conference on Reactions Between Complex Nuclei, Nashville, Tennessee,* edited by R. L. Robinson *et al.* (North-Holland, Amsterdam, 1974), p. 257; N. K. Glendenning, *ibid.*, p. 137; T. Tamura, K. S. Low, and T. Udagawa, Phys. Lett. 51B, 116 (1974); K. Yati, D. L. Hendrie, L. Kraus, C. F. Maguire, J. Mahoney, D. K. Scott, Y. Terrien, T. Udagawa, K. S. Low, and T. Tamura, Phys. Rev. Lett. 34, 96 (1975).

[6] D. K. Scott, B. G. Harvey, D. L. Hendrie, U. Jahnke, L. Kraus, C. F. Maguire, J. Mahoney, Y. Terrien, K. Yagi, and N. K. Glendenning, Phys. Rev. Lett. 34, 895 (1975).

[7] B. G. Harvey, J. Mahoney, F. G. Pühlhofer, F. S. Goulding, D. A. Landis, J. C. Faivre, D. G. Kovar, M. S. Zisman, J. R. Merriwether, S. W. Cosper, and D. L. Hendrie, Nucl. Instrum. Methods 104, 21 (1972).

[8] The shift in the 0_g^+ peak position in the two inverse oxygen reactions is mainly due to the somewhat different relative energy with respect to the Coulomb barrier, the pickup reaction being the more energetic one. The relative kinetic energy is in both cases roughly 1.5 times the barrier energy.

[9] R. J. Ascuitto, J. S. Vaagen, K. Erb, D. L. Hanson, D. A. Bromley, and J. J. Kolata, Phys. Lett. 55B, 289 (1975).

[10] K. A. Erb, D. L. Hanson, R. J. Ascuitto, B. Sørensen, J. S. Vaagen, and J. J. Kolatar, Phys. Rev. Lett. 33, 1102 (1974); J. S. Vaagen and R. J. Ascuitto, Nucl. Phys. A260, 317 (1976).

[11] J. S. Vaagen, R. J. Ascuitto, and K. Kumar, to be published.

[12] J. H. Bjerregaard, O. Hansen, O. Nathan, and S. Hinds, Nucl. Phys. 86, 145 (1966); W. Oelert, G. Lindström, and V. Riech, Nucl. Phys. A233, 237 (1974); P. Bebenham and N. M. Hintz, Nucl. Phys. A195, 385 (1972).

[13] K. Kumar, B. Remaud, P. Aguer, J. S. Vaagen, A. C. Rester, R. Foucher, and J. H. Hamilton, Phys. Rev. C 16, 1235 (1977).

Quest for Triaxial Nuclei: Some Hartree-Bogoliubov Predictions

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The existence of static triaxial shapes is investigated in the framework of the Hartree-Bogoliubov method. Although some statically triaxial nuclei are found the main result of the calculation is the γ softness of nuclei. This fact implies appreciable dynamic triaxiality for most nuclei.

The interest on triaxial nuclei has been steadily increasing in the last years. The importance of the triaxiality on the dynamics of the nuclear collective motion is a long known fact.[1] However, only recently has the quality of experimental data reached the point where one can hope to investigate this elusive nuclear property.[2]

In this Letter we examine first the question of the existence of static triaxial shapes. The actual theoretical situation of the problem seems rather confused. There exist mainly two different approaches which give totally different results. The first is the usual microscopic or macroscopic-microscopic structure calculation. Based on a variational principle for the binding energy, this method calculates the ground-state deformation of the nucleus. In the context of the Hartree-Fock (HF) method there exist several calculations[3-6] on light nuclei, which allow for the triaxial degree of freedom. Most of these calculations predict triaxial shapes for ^{24}Mg, the excited 4p-4h (four-particle, four-hole) state of ^{16}O, and eventually ^{32}S. Outside the *s-d* shell there exist some "restricted" Hartree-Bogoliubov calculations on *f-p* shell nuclei,[7] or HF + BCS calculations with a phenomenological treatment of pairing correlations.[8] These calculations do actually predict triaxial shapes, but the validity of their results is questionable because of either the restricted variational space used or the unrealistic pairing force. Recently some Strutinsky-type calculations have been performed in the same region of the periodic table,[9] but a complete neglect of pairing correlations might invalidate the results obtained.

As far as heavier nuclei are concerned, few systematic studies exist. Kumar and Baranger[10] have investigated the rare-earth region in the framework of a Hartree-Bogoliubov model which included only valence nucleons. A study of some eighty nuclei produced only three triaxial shapes. More recently a Strutinsky-type calculation has been performed by Götz *et al.*[11] resulting in mostly axial shapes for the nuclei studied. One would tend thus to conclude that triaxial shapes, at least for heavy nuclei, are quite rare if they exist at all.

The second approach to the problem is one developed in recent years for the study of even-odd nuclei first by Meyer-Ter-Vehn[12] and subsequently by Toki and Faessler.[13] The main assumption in this method is that the odd nucleon is coupled to a triaxial rigid-rotor core, upon which a determination of the core deformation is necessary. The latter is extracted from the spectrum of the even-even core nucleus and the existence of a low-lying second 2^+ state is sufficient in order to impart to the even-even core a triaxial deformation. The validity of the model is subsequently tested on the even-odd nucleus spectrum, and actually the quoted authors have obtained very satisfactory results.

We are thus faced with two theories, fairly successful as far as the rest of their predictions is concerned, which are quite contradictory in the triaxiality results they yield. According to the static calculations triaxial shapes are scant, while the dynamic rotor model predicts an abundance of triaxial nuclei.

In order to answer the question of the existence of triaxial nuclei in the framework of a detailed microscopic model we have performed a Hartree-Fock-Bogoliubov (HFB) calculation with Gogny's D1 interaction.[14] The Bogoliubov transformation is given as an expansion on a triaxial Cartesian harmonic oscillator basis:

$$\eta_\lambda^\dagger = \sum_\alpha U_\alpha^\lambda a_\alpha^\dagger + V_\alpha^\lambda a_\alpha,$$
$$\alpha = (n_x, n_y, n_z; \sigma).$$

The deformation of the basis is characterized by the ratios $q = \omega_x/\omega_z$ and $p = \omega_x/\omega_y$ of its oscillator lengths. In the present calculation we have included in the basis all states satisfying the condition

$$n_x + n_y + n_z \leq N$$

with $N \leq 8$. The conservation of the volume is ensured by $\omega_0^3 = \omega_x\omega_y\omega_z$. The parameters ω_0, p, q are determined through a minimization of the HFB energy. The same interaction D1 was used to generate both the Hartree-Fock and pairing fields, without any inert core assumption, as has been traditionally done until now.

For the light *s-d* shell nuclei, where a Hartree-Fock calculation is sufficient, the only triaxial shape obtained was the one corresponding to a 4p-4h state of ^{16}O, in accordance with the previous Skyrme interaction results.[6]

For the heavier nuclei, a systematic study was ruled out, because of the considerable amount of computer time necessary for every calculation. We have thus preferred to select six nuclei for which triaxial shapes have been predicted. A complete HFB calculation has been performed for each one of these nuclei. Our results are displayed in Table I, together with the existing predictions.

We remark that some triaxial nuclei are indeed present: three out of the six nuclei studied. This fact, corroborated by more extensive calculations that we have performed in various regions of the periodic table, would support the conclusion that the existence of a triaxial shape is exceptional. It may occur in a family of transitional nuclei, as the minimum-energy trajectory crosses the γ plane, that one isotope corresponds to a triaxial ground state.

Independently of the ground-state deformations,

TABLE I. Static deformation parameters of the six studied nuclei. Under the heading "Previous work" we display the results of previous calculations for which we give the corresponding reference. Our results appear in the columns labeled HFB.

	Previous work			HFB	
	β	γ	Ref.	β	γ
^{58}Fe	0.25	25°	9	0.21	0°
^{62}Zn	0.18	47°	7	0.22	30°
^{74}Ge	0.20	30°	8	0	0°
^{132}Ba	0.20	24°	12	0.15	0°
^{134}Ce	0.21	23°	12	0.17	18°
^{186}W	0.23	16°	13	0.21	31°

in every nucleus studied we have remarked a great softness in the γ direction. This is most easily seen in the case of ^{62}Zn for which we present the complete potential energy surface (Fig. 1) in the β-γ plane, obtained through a constrained HFB calculation with β and γ as constraints. Less than 300 keV separate the triaxial minimum from the prolate one. Because of this γ softness the existence of a rigid (axial or triaxial) rotor appears doubtful. With no appreciable barrier in the γ direction the zero-point collective motion may well wash out the static deformations. Actually it is through this remark that one hopes to reconcile the two different schools on the question of triaxiality.

It is shown,[15] in the case of even-even nuclei, that the vibrator rotator model for the nucleus, materialized by the Bohr's Hamiltonian, gives results quite similar to the triaxial rotor ones. Recently Yamazaki has extended this result to the case of even-odd nuclei.[16] The main conclusion of this work is that the β-γ deformation entering the rigid-order model is the dynamical mean value of β and γ in the rotator-vibrator model. The interpretation of the multitude of triaxial shapes predicted by Meyer-Ter-Vehn's model is then clear. The great majority of nuclei, being γ soft, have nonzero dynamical mean values of γ. This is in fact borne out by the calculations of Kumar and Baranger, who, starting from axial static equilibrium shapes, found triaxial ones after the full dynamical calculation. In the case of ^{186}W, for example, for which they predict a static shape $\beta = 0.20$, $\gamma = 0°$, they found $\bar{\beta} = 0.24$, $\bar{\gamma} = 23.4°$ as dynamical mean values. We are currently performing a full dynamic calculation in some rare-earth nuclei.[17] Our preliminary results also corroborate this fact. In the case of ^{150}Sm, for which we have obtained an axial static shape, the solution of Bohr's Hamiltonian gives $\bar{\gamma} \sim 30°$.

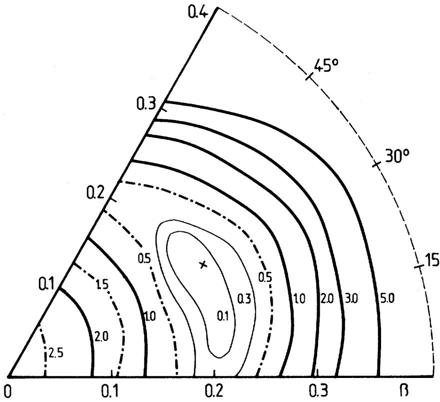

FIG. 1. Potential energy surface, i.e., total energy as a function of deformation, for the nucleus ^{62}Zn, in the β-γ plane. The energy scale (relative to the static ground state) is in MeV.

The results presented here allow us to draw the following conclusions. Static triaxial shapes in nuclei are rather rare. Moreover, their occurrence in a specific isotope may depend on the details of the effective interaction. On the contrary, γ softness is a common feature in nuclei and leads systematically to dynamic triaxiality. In order to distinguish between the two models proposed for the description of transitional nuclei, rigid triaxial rotor or rotator-vibrator, complete microscopic dynamical calculations will be needed for even-even as well as for even-odd nuclei, which is a formidable task. We believe, however, that the Hartree-Bogoliubov results presented here demonstrate that such an approach lies within the reach of present day capabilities.

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[1] A. Bohr, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 27, No. 14 (1952).

[2] F. S. Stephens, in *Proceedings of the International Conference on Nuclear Physics, Munich, 1973*, edited by J. de Boer and J. Mang (North-Holland, Amsterdam, 1973), Vol. 2, p. 367.

[3] J. Zofka and G. Ripka, Nucl. Phys. A168, 65 (1971).

[4] P. Curry and D. W. L. Sprung, Nucl. Phys. A216, 125 (1973).

[5] K. Lassey, M. Manning, and A. Volkov, Can. J. Phys. 51, 2522 (1973).

[6] B. Grammaticos, thèse 3ème cycle, Université de Paris, 1974 (unpublished).

[7] T. S. Sandhu and M. L. Rustgi, Phys. Rev. C 14, 675 (1976).

[8] D. Ardouin *et al.*, Phys. Rev. C 12, 1745 (1975).

[9] S. E. Larsonn *et al.*, Nucl. Phys. A261, 77 (1976).

[10] K. Kumar and M. Baranger, Nucl. Phys. A110, 529 (1968).

[11] M. Götz, H. C. Pauli, K. Adler, and K. Junker, Nucl. Phys. A192, 1 (1972).

[12] J. Mayer-Ter-Vehn, Nucl. Phys. A249, 111, 141 (1975).

[13] H. Toki and A. Faessler, Nucl. Phys. A253, 231 (1975), and Z. Phys. A276, 35 (1976).

[14] D. Gogny, *Nuclear Self Consistent Fields*, edited by G. Ripka and M. Porneuf (North-Holland, Amsterdam, 1975), p. 333.

[15] T. Yamazaki, Nucl. Phys. 49, 1 (1963).

[16] T. Yamazaki, in *Proceedings of the "Colloque Franco-Japonais de Spectroscopie et Réactions Nucléaires" and INS Symposium in Nuclear Collectivity* (Institute for Nuclear Study, Tokyo, 1976), p. 480.

[17] P. Aguer, M. Girod, B. Grammaticos, and K. Kumar, to be published.

Evidence for a Nonrotational Interpretation of $\langle 0^+||M(E4)||4^+\rangle$ in ^{180}Hf

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Angular distributions for the reaction ^{180}Hf(α,α') were measured for laboratory α-particle energies of 21 and 24 MeV. Data were obtained for 0^+, 2^+, 4^+, and 6^+ members of the ground-state rotational band. Data for the 4^+ state show unambiguously that $\langle 0^+| \times |M(E4)\|4^+\rangle$ is large and negative. Data for the 6^+ state are not well-fitted by rotational-model coupled-channels calculations. These results may indicate that the interpretation of the $E4$ matrix element as arising from a static β_4 deformation in the framework of the simple rotational model is incorrect.

Hexadecapole, as well as quadrupole, components in the static nuclear shape can have profound effects on nuclear properties. Coulomb-excitation studies of the $J^\pi=2^+,4^+$ members of the ground-state rotational band of many nuclei in both the rare-earth and actinide regions have provided measurements of the reduced transition matrix elements $M_{0\lambda}=\langle 0^+\| i^\lambda M(E\lambda)\| J^\pi=\lambda^+\rangle$. These, under the assumption of a rotational model, have been used to deduce static quadrupole and hexadecapole deformation parameters (β_2,β_4) for the deformed charge distribution.

A region where the β_4 degree of freedom plays an important role in the collective nuclear properties is that of the heavy transitional nuclei $(180 \lesssim A \lesssim 200)$. Calculations[1,2] of nuclear shapes in this region show that inclusion of a negative β_4 deformation lowers the potential-energy minimum more for prolate than for oblate shapes, thus keeping the calculated prolate-oblate transition from occurring sooner than observed experimentally (between Os and Pt). Many experi-