nearly field-free zone, and an anode sheath. The anode sheath functions as an electrostatic barrier to prevent escape of those ions whose energies are lower than the barrier height. Ions reflected from the wall sheath do not necessarily recede to the cathode, because of the relatively small volume it occupies near the core of the discharge, and because of ion-ion deflections in the dense core. The dominant ion loss mechanisms would then be recombination in the volume and at the target, offset by electron impact ionization. From the measurements presented above, we place a lower limit of 100 μsec on the mean lifetime of the laser-produced ions.

This method of plasma production offers a novel means for sustaining long-lived, dense, voluminous, highly stripped, high-Z plasmas. Applications to spectroscopy and ion source development suggest themselves. Such a plasma formed with Li^{+++} ions could serve as a target for slowing down energetic protons in a two-component fusion experiment based on $^6Li(p, \alpha)^3He$.[6]

This work was supported originally in part by the National Science Foundation, and more recently in part by the U. S. Energy Research and Development Administration.

[a]Permanent address: Racah Institute of Physics, Hebrew University, Jerusalem, Israel.

[1]P. Mulser, R. Sigal, and S. Witkowski, Phys. Rep. 6C, 189 (1973).

[2]D. A. Swanson, B. E. Cherrington, and J. T. Verdeyen, Phys. Fluids 16, 1939 (1973).

[3]H. L. Seguin, K. Manes, and J. Tulip, Rev. Sci. Instrum. 43, 1134 (1972).

[4]C. W. Kimblin, in *Eleventh International Conference on Phenomena in Ionized Gases*, edited by I. Stoll *et al.* (Czechoslovak Academy of Sciences, Prague, Czechoslavakia, 1973), p. 73.

[5]This collector was an ion energy analyzer comprised of three closely spaced 500-mesh grids biased to reject all electrons, and a collector biased to accept all ions.

[6]J. L. Hirshfield, in Proceedings of the Electric Power Research Institute (EPRI) Workshop on Advanced Fusion Fuels, Chicago, June 1977 (to be published), and Bull. Am. Phys. Soc. 21, 1186 (1976).

Are Drift-Wave Eigenmodes Unstable?

David W. Ross and Swadesh M. Mahajan
Fusion Research Center, The University of Texas at Austin, Austin, Texas 78712
(Received 8 November 1977)

It is shown that the eigenmodes of the collisionless drift wave in slab geometry are stable. Previous studies yielding instability (the "universal" instability) were based upon an incomplete treatment of the electron dynamics; i.e., the principal part of the plasma dispersion function was ignored.

In the absence of magnetic shear, and with a given $k_\parallel$, the dispersion relation for collisionless drift waves yields an instability driven by the electron wave-particle resonance coupled with the density gradient. Radial eigenmodes of these waves in plasmas with magnetic shear[1,2] were also found to be unstable for sufficiently weak, but reasonable, shear. In these studies, however, the electrons were treated in the adiabatic approximation,

$$\frac{\tilde{n}_e}{n_0} = \frac{e\tilde{\varphi}}{T_e}\left[1 + i\pi^{1/2}\frac{\omega - \omega^*}{|k_\parallel| v_e}\exp\left(\frac{-\omega^2}{k_\parallel{}^2 v_e{}^2}\right)\right]. \tag{1}$$

where $v_e{}^2 = 2T_e/m_e$, $\omega^* = k_y \rho_s c_s/L_n$, $\rho_s = c_s/\Omega_i$, c_s/Ω_i, $c_s{}^2 = T_e/m_i$, and L_n is the density gradient scale length. More generally, however, one should write

$$\frac{\tilde{n}_e}{n_0} = \frac{e\tilde{\varphi}}{T_e}\left[1 + \frac{\omega - \omega_e{}^*}{|k_\parallel| v_e} Z\left(\frac{\omega}{|k_\parallel| v_e}\right)\right], \tag{2}$$

where Z is the plasma dispersion function.[3] Although not explicitly so stated, the form (1) was employed in the numerical work as well as the perturbation theory of Ref. 2.[4] Since $k_\parallel = (r - r_0)k_y/L_s$, where L_s is the shear length, and the mode is centered at the mode-rational surface $r = r_0$, there exists a layer where the difference between (1) and (2), which we shall call the principal part of the Z function, is important. We have found, analytically and numerically, that with the full Z function the modes are strongly stabilized. Numerically, the modes become completely stable, to within the accuracy of our approximation ($\sim 10^{-3}\omega^*$), for all shear (L_n/L_s down to 6×10^{-3}). For $k_y\rho_s$ approaching 1, the magnitude of the growth or damping rate is zero to this accuracy independent of shear.[5] The analytic approximation yields good agreement with this result and indicates that the mode is completely stable.

Our analytical method employs a variational principle analogous to the one derived by Hazeltine for drift-tearing modes[6] and related to the ones we use for finite-β drift waves.[7] Our numerical scheme is identical to that employed by Gladd and Horton,[2] except that the full Z function is used. (Indeed, we use a more recent version of the same code.) It employs Numerov's algorithm with WKB boundary conditions at large x, and the dispersion relation for symmetric modes is $\varphi'(0)/\varphi(0)=0$.

For simplicity, we take the ions to be cold, and omit the electron temperature gradient. Then, from quasineutrality, the hydrodynamic equations for the ions, and Eq. (2), we obtain the radial eigenmode equation

$$\frac{d^2\varphi}{dx^2}+\left[\frac{\omega^*}{\omega}-1-k^2+\frac{\omega_s^{\,2}}{\omega^2}x^2+\left(\frac{\omega^*}{\omega}-1\right)\frac{a\omega}{|x|}Z\left(\frac{a\omega}{|x|}\right)\right]\varphi=0, \tag{3}$$

where $x=(r-r_0)/\rho_s$, $k=k_y\rho_s$, $\omega_s=kc_s/L_s$, and $a^2=(m_e/2m_i)(1/\omega_s)^2$. Defining $\langle\cdots\rangle\equiv\int_{-\infty}^{\infty}dx\cdots$, we note that the functional

$$S\equiv[\langle\varphi'^2\rangle-\langle x^2\varphi^2\rangle\omega_s^{\,2}/\omega^2-(\omega^*-\omega)aI]\langle\varphi^2\rangle^{-1} \tag{4}$$

is variational, in that $\delta S=0$ reproduces Eq. (3). Here,

$$I=\langle\varphi^2Z(a\omega/|x|)/|x|\rangle. \tag{5}$$

The extermal value of S is given by

$$S^*=-k^2-1+\omega^*/\omega, \tag{6}$$

which provides the dispersion relation. Restricting ourselves to the lowest Pearlstein-Berk mode, we choose as a trial function the $n=0$ Hermite function,

$$\varphi=\exp(-i\alpha x^2/2), \tag{7}$$

and look for solutions with α in the neighborhood of the zeroth-order (in the perturbation I) result, $\alpha_0=\omega_s/\omega$. Noting that $Z(z)=i\pi^{1/2}\exp(-z^2)[1+\Phi(iz)]$, where Φ is the error function,[8] and employing Eq. (7) and an integral given by Gradshteyn and Ryzhik,[9] we find, after some manipulation,

$$I=2i\pi^{1/2}[K_0(2a\omega\sqrt{i\alpha})+i(\pi/2)J_0(2ia\omega\sqrt{i\alpha})-(\pi/2)H_0(2ia\omega\sqrt{i\alpha})], \tag{8}$$

where K_0 and J_0 are Bessel functions, and H_0 is a Struve function.[10] The first term yields, with $\alpha=\alpha_0$, the usual growth term that one would obtain from Eq. (1).[2] The remaining two terms follow from the principal part of the Z function. With the usual ordering of parameters, $L_n/L_s\gg m_e/m_i$, the arguments of these functions are small, so that

$$K_0(2a\omega\sqrt{i\alpha})\cong-\ln(a\omega\sqrt{\alpha})-i\pi/4-C, \tag{9}$$

where C (=0.5772...) is Euler's constant, and

$$(i\pi/2)[J_0(2ia\omega\sqrt{i\alpha})+[J_0(2ia\omega\sqrt{i\alpha})+iH_0(2ia\omega\sqrt{i\alpha})]\cong i\pi/2-2ia\omega\sqrt{i\alpha}. \tag{10}$$

The crucial point here is that, while the argument of the logarithm in Eq. (9) is small, the logarithm itself, for typical parameters, is not large compared to the other terms! The term $i\pi/2$ from Eq. (10), then, changes the results completely, and serves to damp the wave. Note that, since the factor $\langle\varphi^2\rangle^{-1}$ in the variational functional, Eq. (4), is $(i\alpha/\pi)^{1/2}$, the real and imaginary parts of I contribute equally to the real and imaginary parts of the frequency. The physical interpretation may be that the principal part of the Z function forms an additional "antiwell" which pushes the wave function away from $x=0$, thus increasing the shear stabilizing effect and weakening the driving term simultaneously.

To complete the calculation, we substitute (7) into the remaining terms of (4) to get

$$S=i\frac{\alpha}{2}-\frac{\omega_s^2}{\omega^2}\frac{1}{2i\alpha}-2i(\omega^*-\omega)a(i\alpha)^{1/2}[-\ln(a\omega\sqrt{\alpha})-C+\frac{i\pi}{4}-2ia\omega\sqrt{i\alpha}], \tag{11}$$

and

$$\frac{dS}{d\alpha}=\frac{i}{2}\left\{1-\frac{\omega_s^2}{\omega^2\alpha^2}-2(\omega^*-\omega)a\left(\frac{i}{\alpha}\right)^{1/2}\left[-\ln(a\omega\sqrt{\alpha})-1-C+\frac{i\pi}{4}-4ia\omega\sqrt{i\alpha}\right]\right\}. \tag{12}$$

The variational value of α and, hence, S^* are found by setting $dS/d\alpha=0$, which, with Eqs. (6) and (11), yields the dispersion relation. We see from (12) that $\alpha_0=\omega_s/\omega$ is indeed a solution in zeroth order, corresponding to the outgoing Pearlstein-Berk wave.[1,2] Using this value in the dispersion relation gives the equivalent of the Gladd-Horton perturbation theory[2] for this problem. A stabilizing effect from the principal part of the Z function is seen, as described above. However, this in itself is not sufficient to stabilize the mode completely. As $k\to 1$, the solution to Eq. (12) departs from α_0 to produce an additional stabilizing effect. We have, unfortunately, had to resort to a numerical solution of Eqs. (6) and (12) in order to obtain a valid comparison with the shooting code.

Results are exhibited in Fig. 1. For the very weak shear, $L_n/L_s=0.02$, we plot the growth rate, normalized to ω^*, vs k_y/ρ_s. Three calculations are compared: (a) the direct numerical integration of Eq. (3); (b) the variational solution, Eqs. (6), (11), and (12), evaluated numerically; and (c) the perturbative approximation to Eq. (12), $\alpha=\alpha_0$. It can be seen that according to (a) and (b) the mode is at marginal stability. (Excellent agreement is also obtained for the real part of ω.) For $k_y\rho_s=1$, we have varied L_n/L_s from 0.04 to 0.006 with no essential change in γ/ω^*. The small residual growth or damping, $\gamma/\omega^*\lesssim 5\times10^{-4}$, seems to be a function of the grid parameters in the code, i.e., unphysical. The variational method, on the other hand, yields $\gamma/\omega^*\to 0$. For comparison, the same case is given in Fig. 2, but with the adiabatic approximation, Eq. (1). The modes are seen to be unstable in all three calculations.

Additional numerical studies with the shooting code have failed to yield growing modes except in the presence of a strong parallel current ($u_d/c_s>1$). Details of this work will be presented in a separate paper. In particular, the modes of higher n, e.g., the $n=1$ antisymmetric mode, are found to be even more stable than the $n=0$ mode. One problem in studying these is that, with the electron dynamics included, the modes are not pure Hermite functions. Superposition of

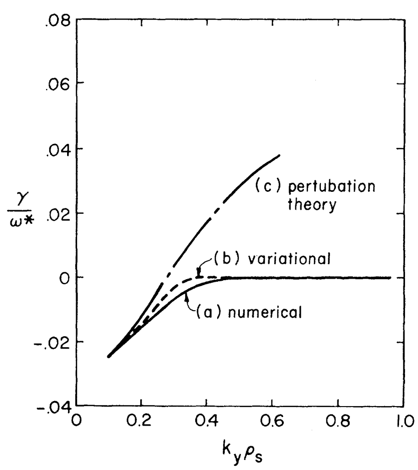

FIG. 1. Growth rate vs wave number with full Z-function electrons, Eq. (2). $L_n/L_s=0.02$, $T_i/T_e=0$, $d\ln T_e/d\ln n_0=0$. See text for description of approximations (a), (b), and (c).

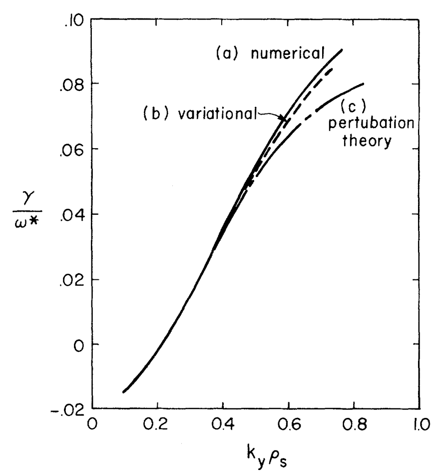

FIG. 2. Growth rate vs wave number with adiabatic approximation for electrons, Eq. (1). Parameters are the same as in Fig. 1. On this sacle, the variational result is indistinguishable from the direct numerical integration for $k_y\rho_s<0.5$.

modes of different n becomes more pronounced as $k_y\rho_s \to 1$, and the eigenmodes become nearly degenerate and difficult to differentiate. We have also carried out numerical studies with finite ion temperature, and electron and ion temperature gradients, employing the complete collisionless slab model of Ref. 2. For $L_n/L_s = 0.02$ and $0 \le T_i/T_e \le 1$, no growing modes were found for $-1 \le \eta_e, \eta_i \le 1$, where $\eta_j = d\ln T_j/d\ln n$. For $k_y\rho_s \to 1$, the damping rates remained near zero over the entire range of parameters. For $k_y\rho_s \ll 1$, the effect of a reversed electron temperature gradient, $\eta_e < 0$, was found to be in the direction of instability, as expected, but unable to overcome the shear stabilization.

Finally, we remark that we have not shown all drift waves to be stable. In a torus, there are a number of effects, such as ballooning or trapped-particle collisions and drifts, which can lead to instability. Further, the existence of convectively growing wave packets should be reexamined in the light of the present work.

We wish to acknowledge helpful discussions concerning variational methods with Dr. Richard Hazeltine. This work was supported by the U. S. Department of Energy, Contract No. EY-77-C-05-4478.

[1] L. D. Pearlstein and H. L. Berk, Phys. Rev. Lett. 23, 220 (1969).

[2] N. T. Gladd and W. Horton, Jr., Phys. Fluids 16, 879 (1973).

[3] B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic, New York, 1961).

[4] N. T. Gladd and W. Horton, Jr., private communication.

[5] We have recently been informed by K. T. Tsang that he and collaborators have obtained similar results. See K. T. Tsang *et al.*, following Letter [Phys. Rev. Lett. 40, 327 (1978)].

[6] R. D. Hazeltine and D. W. Ross, University of Texas Report No. FRCR 154, October 1977 (unpublished).

[7] S. M. Mahajan and D. W. Ross, to be published.

[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1965), p. 930.

[9] Ref. 8, p. 651.

[10] Ref. 8, p. 982.

"Absolute Universal Instability" Is Not Universal

K. T. Tsang, P. J. Catto,[a] J. C. Whitson, and Julius Smith
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 22 November 1977)

The roots of an improved analytic eigenvalue equation for the absolute universal or collisionless drift instability in a sheared magnetic field are found numerically and compared with the eigenvalues obtained from a numerical solution of the exact differential equation. The startling result is that both techniques predict stability, no matter how weak the shear or how large the transverse wave number, in contradiction to all previous work. Stability is due primarily to the stabilizing influence of the nonresonant electrons.

Since the pioneering work of Pearlstein and Berk,[1] the instability of the collisionless drift wave in a sheared magnetic field has been the subject of numerous investigations.[2-4] This instability, which is driven by the the wave-particle interaction between the drift wave and the electrons, is also termed the "universal instability" because its existence requires only a density gradient which is a necessary feature of a confined plasma. Previous investigations have employed perturbation theory[2] or approximate numerical solutions[3] of the perturbation-theory solution near marginal stability.[4] In Ref. 4 the same differential equation is solved by breaking up the spatial domain into inner and outer regions. In the outer region, the resonant electron term is subdominant and the equation can be solved iteratively. This outer solution is then matched to a jump condition derived in the inner region.

Recently, Catto and Tsang[5] extended the work of Rosenbluth and Catto[4] to obtain an improved eigenvalue equation for all even and odd radial eigenmodes. More importantly, they were able to discover the limit in which the perturbation-theory results could be recovered from the more exact expressions valid for arbitrary growth rates.[5,6] As a result, there emerged the possibility that the perturbation-theory form of the dispersion relation is inadequate because it can only be recovered in a limit in which small corrections can be important. In particular, the per-

