colliding coherent tubes[5,10] of nucleons, one obtains two dramatic effects. First, there is a substantial cross section for "subthreshold" production—that is, production when the energy per nucleon alone would be insufficient to produce particles by independent N-N collisions. Second, and of direct relevance to the situation here, at energies per nucleon well above threshold (where the mean multiplicity in an N-N collision is relatively flat) there is a dramatic reduction in the cross sections for particle production from the MCM result.[5] The exact reduction depends on the amount of coherent production one introduces. In the limit that the whole projectile acts coherently on the whole target the mean multiplicity at 3 GeV/c for ^{40}Ar + ^{208}Pb will be reduced by *more than one order of magnitude.* This indicates that there is ample flexibility to describe the π multiplicity data with an admixture of a coherent production mechanism into the MCM.

I conclude that there are distinctive features of the total pion spectrum in the simple multiple-collision model of relativistic A-B collisions. On the other hand, these features are somewhat smeared in the negative-pion spectrum. For ^{40}Ar incident on Pb_3O_4 at 3 GeV/c per nucleon, the introduction of a resonance production model in an approximate fashion to inhibit high-multiplicity events does not dramatically affect the N^- spectrum below $N^- = 25$. I propose, therefore, that the significant deviation in the data from these simple spectra for high-multiplicity events is indicative of nonsimple behavior in central A-B collisions. In particular I argue that this deviation is evidence of the presence of a coherent production mechanism.

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[1]See, for example, BNL Report No. 50445, 1974 (unpublished).

[2]N. Czyz and L. C. Maximon, Ann. Phys. (N.Y.) 52, 59 (1969).

[3]University of California Radiation Laboratory Report No. UCRL-20000 NN, 1970 (unpublished), and references therein.

[4]S. Y. Fung, W. Gorn, G. P. Kiernan, F. F. Liu, J. J. Lu, Y. T. Oh, J. Ozawa, R. T. Poe, L. Schroeder, and H. Steiner, preceding Letter [Phys. Rev. Lett. 40, 292 (1978)].

[5]J. Vary and K. Lassila, to be published.

[6]A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972); K. Gottfried, Phys. Rev. Lett. 32, 957 (1974).

[7]W. Busza, in *High-Energy Physics and Nuclear Structure—1975*, AIP Conference Proceedings No. 26, edited by R. Mischke, C. Hargrove, and C. Hoffman (American Institute of Physics, New York, 1975), p. 211.

[8]G. D. Harp, K. Chen, G. Friedlander, Z. Fraenkel, and J. M. Miller, Phys. Rev. C 8, 581 (1973).

[9]R. M. Edelstein *et al.*, Phys. Rev. Lett. 38, 185 (1977).

[10]G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D 13, 161 (1976).

Pion Multiplicity Distributions in Heavy-Ion Collisions

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We calculate the π^- multiplicity distribution for Ar + Pb_3O_4 at 1.8 GeV/nucleon. Good agreement with data is found. Effects of multipion correlations are investigated.

Recently, the negative-pion multiplicity distribution $P(n)$ has been measured[1] for a variety of heavy-ion targets and projectiles at beam energies of ~2 GeV/nucleon. [$P(n)$ is the probability of producing n negative pions in a given heavy-ion collision.] In order to assess the implications of the data and to help in planning future experimetns, we present here the results of model calculations of $P(n)$. Our study is motivated by the following questions: (1) Can a simple model account for the measured $P(n)$? (2) How sensitive is $P(n)$ to the details of heavy-ion dynamics? (3) What information does $P(n)$ contain that is not available from single-pion inclusive cross sections[2]? And (4) how are multipion correlations reflected in $P(n)$?

We show below that a thermodynamic "fireball" model[3,4] gives $P(n)$ in good agreement with data.[1] Then through a dynamical calculation of $P(n,t)$ based on a master equation, we show that there exists a large class of models that lead to the same result. In particular, we prove that for a fixed impact parameter b, $P(n,t;b)$ is a Poisson distribution *independent* of the dynamical model as long as multipion correlations can be neglected. The *average* multiplicity, $\langle n(b)\rangle$, is model dependent. However, in the absence of correlations, $P(n)$ contains less information than the single-pion inclusive cross section. Finally, we show that correlations due to Bose statistics and coherent pion production mechanisms lead to convoluted multiple Poisson distributions.

To compute $P(n)$ in the fireball model,[3] the pions are assumed[4] to come to chemical as well as thermal equilibrium during heavy-ion collisions. We will relax the assumption of equilibrium later, but for now we consider an isospin-saturated ($N=Z$) nuclear system of volume V and temperature T with pion states $|i\rangle$ having corresponding frequencies ω_i. If we treat pions as an ideal boson gas, the probability of finding n negative pions in the system is

$$P(n)=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\left\{\prod_{i=1}^{\infty}x_i^{n_i}(1-x_i)\right\}\delta(n-\sum_{j=1}^{\infty}n_j), \tag{1}$$

with $x_i=\exp(-\omega_i/T)$, in units $\hbar=c=k_B=1$, and δ is a Kronecker-delta constraint. The $P(n)$ are most easily computed via the generating function

$$F(\lambda)=\sum_{n=0}^{\infty}\lambda^n P(n) \tag{2a}$$

$$=\exp\{\textstyle\sum_i \ln[(1-x_i)/(1-\lambda x_i)]\} \tag{2b}$$

$$=\exp[\sum_{k=1}^{\infty}N(k)(\lambda^k-1)], \tag{2c}$$

where

$$N(k)=\frac{1}{k}\left(\frac{Vm_\pi^3}{2\pi^2}\right)\left(\frac{T}{km_\pi}\right)K_2\left(\frac{km_\pi}{T}\right) \tag{3}$$

follows from Eq. (2b) with K_2 being the modified Bessel function.

By expansion of the exponential in Eq. (2c) and use of the multinomial theorem, $P(n)$ is found to be the following *finite* sum of terms, each having a *finite* number of factors:

$$P(n)=\exp[-\sum_{k=1}^{\infty}N(k)]\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\left\{\prod_{k=1}^{\infty}\frac{[N(k)]^{n_k}}{n_k!}\right\}\delta(n-\sum_{j=1}^{\infty}jn_j). \tag{4}$$

In fact, we have shown that any discrete distribution on the nonnegative integers having $P(0)>0$—a category including every possible multiplicity distribution of collision-*created* particles—can be expressed as a *convoluted multiple Poisson distribution,* as in Eq. (4), with the $N(k)$ uniquely determined by the $P(n)$, $n=0,1,\ldots,k$. In the fireball model, $N(k)$ is given by Eq. (3) and decreases rapidly with k. In fact, for $T<m_\pi$, $N(2)/N(1)<0.04\ll 1$, and a good approximation to Eq. (4) is obtained by setting $N(k)=0$ for $k\geq 2$. Therefore, for $T<m_\pi$, $P(n)$ appears to reduce to a simple Poisson distribution[5]:

$$P(n)\approx e^{-\langle n\rangle}\langle n\rangle^n/n!, \tag{5}$$

where

$$\langle n\rangle=\left.\frac{dF}{d\lambda}\right|_{\lambda=1}=\sum_{k=1}^{\infty}kN(k)\approx N(1)$$

is the *average* number of negative pions. More precisely, Eq. (4) shows that corrections to Eq. (5) arise to orders $N(2)/N(1)$ and $N(2)/[N(1)]^2$. The latter correction depends on the volume V and becomes important only if V is small compared to the pion thermal quantum volume $\propto(m_\pi T)^{-3/2}$.

To understand the conditions leading to Eq. (5), a physical interpretation of $N(k)$ is needed. We only quote here the results of a calculation that shows that $kN(k)$ is the average number of states $|i\rangle$ with occupation number $n_i\geq k$. We refer to a group of k pions in the same state $|i\rangle$ as *statistically* correlated by Bose statistics. The condition $N(1)\gg N(2)$ then means that only a very small fraction of the pions are statistically correlated. Thus, $P(n)$ is a simple Poisson distribution when such correlations can be neglected.

In heavy-ion collisions temperatures are typically[3] $T\lesssim 100$ MeV, and therefore Eq. (5) should apply if equilibrium is established. To compare to experiment, $P(n)$ must be averaged over impact parameters:

$$P(n)=\int_0^{b_{\max}}\frac{2b\,db}{b_{\max}^2}e^{-\langle n(b)\rangle}\frac{\langle n(b)\rangle^n}{n!}. \tag{6}$$

In the fireball model,[3,4] sharp sphere geometry and relativistic kinematics give the total number of interacting nucleons $N_{\rm nuc}(b)$ and the excitation energy per nucleon $E^*(b)$ as functions of b. The relation between $T(b)$ and $E^*(b)$ then follows[4] from the assumption of equilibrium between nucleons and pions and depends on the volume $V(b)$. In this model, $V(b)$ is taken to be the volume of the expanding fireball at the point where the nucleon density reaches a critical "freeze-out" density ρ_c. Typically, $\rho_c \approx (\frac{1}{4}-\frac{1}{2})\rho_0$, $\rho_0 = 0.17$ fm^{-3}, and ρ_c corresponds to a density below which no further pion production or absorption is assumed to take place. The key assumption is that experimentally only the properties of the fireball at this freeze-out density are observed. Thus $V(b) = N_{\rm nuc}(b)/\rho_c$. For $\rho_c = \rho_0/3$, $T(b)$ was found to be related to $E^*(b)$ approximately as[4]

$$T(b) \approx T_0\{1-\exp[-2E^*(b)/3T_0]\}, \tag{7}$$

where $T_0 \approx 92$ MeV. We take $\rho_c = \rho_0/3$ from Ref. 4, where ρ_c was adjusted to fit the low-energy pion inclusive cross section. Since $d^2\sigma_\pi/d\Omega\,dE$ is proportional to the average pion multiplicity, adjusting ρ_c to obtain the correct normalization of $d^2\sigma_\pi$ ensures that the $\langle n(b)\rangle$ entering Eq. (6) are also of the correct magnitude.

The comparison of $P(n)$ from Eq. (6) to experiment[1] is given in Fig. 1(a) for the reaction Ar incident on Pb_3O_4 at 1.8 GeV/nucleon. For this mixed target $P(n)$ is appropriately averaged over Pb and O targets. Curve 1 is obtained by integrating Eq. (6) over all impact parameters up to $R_P + R_T$ = sum of projectile and target radii. Curve 1 should be compared to *untriggered* data. However, in Ref. 1 data were selected in an "inelastic" trigger mode, which, according to their measured total charge multiplicity distribution, corresponds to ~5 or more charged tracks per event. This trigger can be taken into account by an impact-parameter cutoff, $b_{\rm max} < R_P + R_T$. In curve 2 we took $b_{\rm max} \approx R_P + R_T - 1$ fm, which placed a lower bound on the number of interacting nucleons in the fireball to be $N_{\rm nuc}(b_{\rm max}) \approx 8$. As seen in Fig. 1(a), curve 2 fits all features of the data well including the bump at $n=1$ and the rapid falloff for $n > 10$. The contribution of the Pb and O targets to curve 2 are indicated by dashed lines.

To see in detail why the bump at $n=1$ is due to a trigger bias, we plot in Fig. 1(b) $P(n;b)$ as a function of impact parameter for Ar + Pb. Larger b corresponds to smaller interaction volume $V(b)$ and hence smaller $\langle n(b)\rangle$. Thus, $P(n;b)$ is shifted to lower n as b increases. In Eq. (6), $b \approx b_{\rm max}$ is weighted the most, and hence reducing $b_{\rm max}$ shifts the whole distribution $P(n)$ to higher

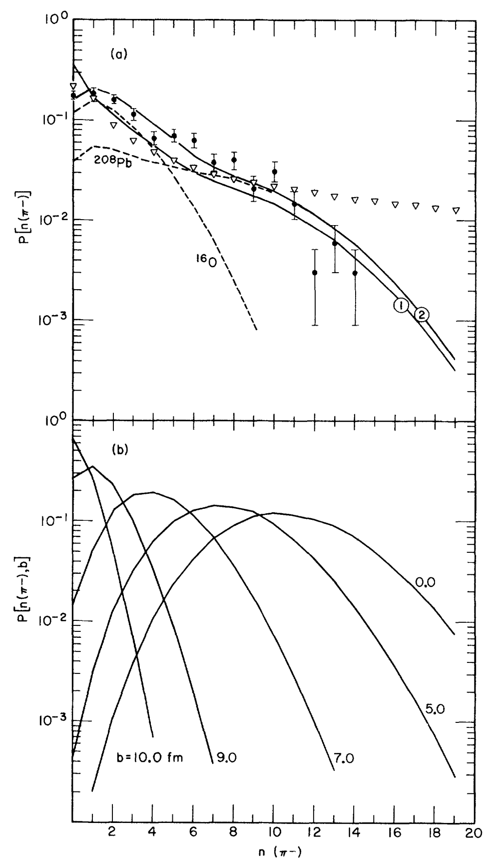

FIG. 1. (a) π^- multiplicity distribution for Ar + Pb_3O_4 at 1.8 GeV/nucleon: closed circles, data from Ref. 1; triangles, Ref. 7; curve 1, Eq. (6), no b cutoff, $\rho_c = 0.16 m_\pi^3$; curve 2, Eq. (6) with $b_{\rm max}({\rm Pb}) = 9.6$ fm, $b_{\rm max}(0) = 5.0$ fm; dashed curves give contributions to 2 from Pb and O targets. (b) π^- multiplicity distribution as a function of impact parameter b for Ar + Pb at 1.8 GeV/nucleon.

n as seen in Fig. 1(a).

Having found good agreement with data using the fireball model, we turn now to the question of whether the assumption of equilibrium is necessary. To this end we formulate a dynamical theory of multiplicities. We only outline the main results here. Details will be given elsewhere. Our approach is motivated by the results of Malfleit and Karant.[6] In Ref. 6, the number of binary collisions in a heavy-ion scattering was shown to be Poisson distributed by solving a master rate equation. We here extend the method to time-dependent pion production and absorption rates to study the general conditions under which $P(n,t)$ is a simple Poisson distribution.

The dynamical model is specified by a set of average rates, $\gamma_k^{\pm}(t)$, for producing and absorbing correlated groups of k pions. For example,

$\gamma_1^{\pm}(t)$ could specify the average rates for $NN \leftrightarrow NN\pi$, while $\gamma_k^{\pm}(t)$ for $k \geq 2$ could describe multipion resonance production and absorption [such a resonance would have to include k π^-'s in its possible decay products to be included in $\gamma_k^{\pm}(t)$ for the data studied here]. We expect that all rates must be strongly time dependent since the nucleon space and momentum distribution $f(\vec{x},\vec{p},t)$ varies rapidly during heavy-ion collisions.

The probability of producing a correlated group of k pions between times t and $t+dt$ is then $\gamma_k^+(t) \times dt$. Also, the probability of coherently absorbing a group of k pions between t and $t+dt$, where there were n pions at t, is $C(n,k)\gamma_k^-(t)dt$, where $C(n,k) = n!/[(n-k)!k!]$. The master equation for $P(n,t)$ may then be obtained by expressing $P(n,t+dt)$ in terms of $P(n\pm k,t)$ and the above probabilities:

$$\frac{dP(n,t)}{dt} = \sum_{k=1}^{\infty} \gamma_k^+(t)[P(n-k,t) - P(n,t)] + \sum_{k=1}^{\infty} \gamma_k^-(t)[C(n+k,k)P(n+k,t) - C(n,k)P(n,t)], \tag{8}$$

where $P(n,t) \equiv 0$ for $n<0$ is understood. Note that in this formulation, we ignore *statistical* correlations due to Bose statistics and consider only *dynamic* correlations due to coherent production and absorption mechanisms.

The time-dependent generating function, Eq. (2a), then satisfies

$$\frac{\partial F(\lambda,t)}{\partial t} = \sum_{k=1}^{\infty} (\lambda^k - 1)\left[\gamma_k^+(t)F(\lambda,t) - \gamma_k^-(t)\frac{1}{k!}\frac{\partial^k F}{\partial \lambda^k}\right]. \tag{9}$$

The boundary conditions $F(1,t)=1$ and $F(\lambda,0)=1$ stem from the normalization of $P(n,t)$ and the initial condition $P(n,0)=\delta_{n0}$.

The general solution of Eqs. (8) and (9) will be discussed elsewhere. Here, we note the result that if multipion absorption can be neglected—i.e., $\gamma_k^- = 0$ for $k \geq 2$—then the solution of Eq. (9) is given by Eq. (2c) with $N(k)$ replaced by $N(k,t)$. These $N(k,t)$ satisfy the system of rate equations

$$dn(k,t)/dt = \gamma_k^+(t) + \gamma_1^-(t)\{(k+1)N(k+1,t) - kN(k,t)\}. \tag{10}$$

Thus, $P(n,t)$ is given by Eq. (4) with $N(k) \to N(k,t)$. Note that $N(k,t)$ converges to a unique equilibrium solution as $t \to \infty$. However, $N(k,\infty)$ for $k>1$ arises here from dynamic correlations rather than from the statistical correlations leading to Eq. (3).

Our main result can be stated as follows: When correlations between pions can be neglected, i.e., $N(1,t) \gg N(k \geq 2,t)$ at the observation time $t = t_{\mathrm{obs}}$ [as, for example, when $\gamma_k^{\pm}(t) = 0$ for $k \geq 1$], then $P(n,t_{\mathrm{obs}})$ is a *simple* Poisson distribution with mean $N(1,t_{\mathrm{obs}})$ (at a fixed impact parameter). This means that a very large class of dynamical models exists that give identical results to the fireball model. In particular, equilibrium need not be reached! The only requirements on those models is that $N(1,t_{\mathrm{obs}}) \gg N(k \geq 2, t_{\mathrm{obs}})$ and that the average pion multiplicity $\langle n\rangle \approx N(1,t_{\mathrm{obs}})$ is correctly given.

We can now comment on the multiplicity calculation of Vary[7] represented by the triangles in Fig. 1(a). That calculation is based on an independent nucleon-nucleon collision model with pion absorption neglected. In terms of Eq. (9), γ_1^+ is taken as a constant, while $\gamma_1^- = 0$. Consequently, this model predicts too many high-multiplicity events. While the condition $N(1,t_{\mathrm{obs}}) \gg N(k \geq 2, t_{\mathrm{obs}})$ is satisfied in that model, $\langle n\rangle$ is overestimated for central collidions because of the neglect of pion absorption.

To conclude, we suggest that future experiments on pion multiplicities should be conducted in a "highly inelastic" trigger mode in order to measure $P(n,b\approx 0)$ directly. Any deviations from a simple Poisson form would then be a truly significant signal of unusual coherent pion processes in heavy-ion collisions.

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[1]S. Y. Fung, W. Gorn, G. P. Kierman, F. F. Liu, J. J. Lu, Y. T. Oh, J. Ozawa, R. T. Poe, C. Van Dalan,

L. Schroeder, and H. Steiner, second proceding Letter [Phys. Rev. Lett. 40, 292 (1978)].

[2]H. Bowman, J. G. Ioannou, S. Kaplan, J. O. Rasmussen, J. Chiba, S. Nagamiya, K. Nakai, K. Sugimoto, and I. Tanihata, to be published.

[3]G. D. Westfall *et al.*, Phys. Rev. Lett. 37, 1202 (1976); J. Gosset *et al.*, Phys. Rev. C 16, 629 (1977); W. D. Myers, Lawrence Berkeley Laboratory Report No. LBL-6569, 1977 (to be published). Note that the notion of transparency introduced by Westfall *et al.* that led to a two-fireballs model to explain 2.1-GeV/nucleon data has been abandoned in the more recent analysis by Gosset *et al.* The most current thermodynamic calculations that reproduce well single-particle inclusive cross sections at 0.25–2.1 GeV/nucleon are those of J. Gosset, J. Kapusta, and G. D. Westfall (to be published) employing a diffuse firestreak model (see Myers) without transparency. In our calculations we found that $P(n)$ differs by $\lesssim 10\%$ between the fireball and firestreak models. We therefore discuss here only the single-fireball model for simplicity.

[4]J. I. Kapusta, Lawrence Berkeley Laboratory Report No. LBL-6504, 1977 (to be published).

[5]R. Hagedorn, *Cargèse Lectures in Physics*, edited by E. Schatzman (Gordon and Breach, New York, 1973), Vol. 6, p. 713.

[6]R. Malfliet and Y. Karant, Lawrence Berkeley Laboratory Nuclear Science Annual Report No. LBL-5075, 1975 (unpublished), p. 270; we are grateful to Y. Karant for bringing this work to or attention.

[7]J. P. Vary, preceding Letter [Phys. Rev. Lett. 40, 295 (1978)].

Measurement of the Correlation between Nuclear Alignment and Electron Direction in ^{12}B Decay as a Direct Search for the Second-Class Axial-Vector Current

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The *alignment* correlation term $A\alpha_- EP_2(\cos\theta)$ has been measured in the β^- decay $^{12}\mathrm{B} \to {}^{12}\mathrm{C}(\mathrm{g.s.})$ by a novel technique. The result, $\alpha_-(E_0) = -(0.1 \pm 0.2)/\mathrm{GeV}$, indicates that the conserved vector current weak-magnetism form factor, F_M, is essentially compensated here by the weak electricity (induced tensor) form factor F_E. Using the decomposition of $F_E = F_E^{(1)} + F_E^{(2)}$ into first- and second-class parts, as well as a reliable estimate of $F_E^{(1)}$, we conclude that $F_E^{(2)}$ is small compared to F_M, and compatible with zero.

There is currently much interest in second-class currents in β decay,[1] in particular in the possible presence of an induced tensor (or "weak electricity",[2] WE) coupling. Although arguments can be made on the basis of $(ft)_\pm$ differences, the most compelling evidence is to be obtained from correlation experiments with oriented mirror nuclei. Specifically, for a $1^+ \to 0^+$ $\beta^\mp$ decay $[{}^{12}\mathrm{B} \to {}^{12}\mathrm{C}(\mathrm{g.s.}) \leftarrow {}^{12}\mathrm{N}]$ one has the relation[3]

$$W(\theta, E)_\mp = F_\mp(E)[1 \mp P(1 + \alpha_\mp E)P_1(\theta) + A\alpha_\mp EP_2(\theta)], \qquad (1)$$

where P (A) is the polarization (alignment) of the 1^+ nucleus, $P_l(\theta)$ are Legendre polynomials, and E is the electron kinetic energy. The terms $\alpha_\mp E$ are contributed by *gradient* couplings (WE, and weak magnetism, WM) and recoil terms; in the "elementary particle" treatment one has[3]

$$\alpha_\mp = \pm 2(F_M - F_E^{(2)})/3F_A - 2F_E^{(1)}/3F_A, \qquad (2)$$

where the F_i's are form factors (A denotes axial vector). Note the presence of a *first-class* WE term; in the impulse approximation,[4] $F_E^{(1)}/F_A = (1/2M)[1 + 2i\int \vec{r}\,\vec{\sigma}\cdot\vec{p}/\int\vec{\sigma}] \equiv y/2M$. With CVC (conservation of vector current), $F_M/F_A = 3.8/2M = 2.0/\mathrm{GeV}$, so that $\alpha_\mp$ is typically expected to be of the order of a few times $0.1\%/\mathrm{MeV}$. This smallness mades $\alpha_\mp$ determinations from the E dependence of the up-down asymmetry (with $P \neq 0$, $A \simeq 0$) particularly difficult and scale dependent.[5,6] Since the *alignment* correlation coefficient in (1) is *entirely contributed* by the quantity of interest, it is of obvious advantage to measure it *directly*; such an approach is reminiscent of direct measurements of the lepton g-factor anomaly. We

