to be made; e.g., see the following papers by Vary,[11] and Kauffmann and Gyulassy.[12] It is our hope that the experimental data presented here will stimulate further theoretical activity and will provide useful new information for the development of meaningful theoretical models.

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Multiple-Collision Model for Pion Production in Relativistic Nucleus-Nucleus Collisions

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A simple model for pion production in relativistic heavy-ion collisions is developed based on nucleon-nucleon data, nuclear density distribution, and the assumption of straight-line trajectories. Multiplicity distributions for total pion production and for negative-pion production are predicted for ^{40}Ar incident on a Pb_3O_4 target at 1.8 GeV/nucleon. Production through intermediate baryon resonances reduces the high-multiplicity region but insufficiently to yield agreement with data. This implies the need for a coherent production mechanism.

Considerable speculation has appeared suggesting that novel phenomena may be found in relativistic nucleus-nucleus collisions.[1] The possible detection of nuclear shock waves or new condensate states of hadronic matter are two notable examples. Such speculation is unfettered by any detailed knowledge of the nuclear equation of state at densities beyond saturation density. Critical experimental tests of these speculations have not been proposed and universally accepted but it should be clear that the results must deviate from a simple multiple-collision analysis in order to be conclusive.

Since a full treatment of internuclear cascades will involve substantial numerical work by computer, it is important to have approximate estimates which strive for an analytic framework. It should then be possible to focus the experimental and theoretical effort rapidly on specific types of measurements. Therefore, we present a simple multiple-collision model (MCM) and extract the pion production spectra which are then compared with experiment in order to look for "unusual," i.e., nonsimple features in the data. We also introduce a saturation mechanism, production through baryon resonances, which inhibits pion production, but find it, by itself, to be an inadequate explanation of the data. We then argue that a coherent production mechanism is required in order to describe the data in a multiple-scattering framework.

In this classical impulse-approximation picture of independent collisions of A projectile nucleons with B target nucleons, we collide static density distributions $\rho_A(\vec{r})$ and $\rho_B(\vec{r}')$ (normalized to A and B, respectively) at impact parameter $\vec{b}$ [$\vec{r}=(\vec{b},z)$]. Then the probability that a given nucleon of A collides with a given nucleon of B is $Y_{AB}(b) \equiv \sigma T_{AB}(b)/AB$ where σ is the total nucleon-nucleon (N-N) cross section and

$$T_{AB}(\vec{b})=\int \rho_A(|\vec{r}-\vec{r}'|)\rho_B(|\vec{r}'|)\,d^3r'\,dz, \qquad (1)$$

which has cylindrical symmetry. Then the probability distribution for i N-N Collisions in an A-B inelastic collision is

$$P(i,AB)=\frac{\int d^2b\,\binom{AB}{i}[Y_{AB}(b)]^i[1-Y_{AB}(b)]^{AB-i}}{\int d^2b\,\{1-[1-Y_{AB}(b)]^{AB}\}}, \qquad (2)$$

where the denominator is the Glauber total reaction cross section $\sigma_{AB}{}^R$.[2] The mean number of N-N collisions is given by the analytic result

$$\bar{\nu}_{AB}=\sum_i iP(i,AB)=AB\sigma/\sigma_{AB}{}^R, \qquad (3)$$

which is especially useful for simple estimates. For example, assuming, as we do here, that σ and $\bar{n}$ (the mean multiplicity of pions produced per N-N collision) are energy independent for a region of interest, then an estimate of $\bar{N}(AB)$, the mean multiplicity of pions produced in inelastic AB collisions, is

$$\bar{N}(AB)\simeq \bar{n}\bar{\nu}_{AB}. \qquad (4)$$

Since this single moment of the multiplicity distribution is insufficient, by itself, to compare theory and experiment, we wish to incorporate the distribution of pions produced in each N-N collision and eventually obtain the pion distributions in an A-B collision. Analysis of the experimental p-p data[3] reveals that $\bar{n}$ is about 0.7 between 1 and 3 GeV/c incident lab momentum p_L and is approximately a constant. Therefore, we concentrate on incident nucleon momentum p_L/A at 3 GeV/c and neglect energy dependence in the N-N pion production distribution as the internuclear cascade proceeds. This simplified MCM may be expressed in terms of $p(\vec{n})$, the probability that $\vec{n}$ pions ($0\leq\vec{n}\leq\vec{n}_{max}$) are produced in an N-N collision in this energy range. The available data[3] yield $p(0)$, $p(1)$, $p(2)$, and $p(\vec{n}_{max}=3)$ equal to 0.386, 0.455, 0.136, and 0.023 respectively.

The probability that N pions are produced in an A-B collision where there were i N-N collisions is

$$R(i,N)=\sum_{j=j_0}^{\min(N,i)}\sum_{\{A\}}\frac{j!}{k!\,l!\cdots r!}[p(n_{max})]^k[p(n_{max}-1)]^l\cdots[p(1)]^r[p(0)]^{i-j}\binom{i}{j}, \qquad (5)$$

where $j_0=\max(0,(N-1)/n_{max}+1)$ rounded down and j represents the subset of the i collisions pro ducing pions; $\{A\}$ represents the set of all unique arrangements (i.e., fixed set of values for k, l, $\ldots,r$) satisfying the two conditions

$$kn_{max}+l(n_{max}-1)+\cdots+r=N,$$
$$k+l+\cdots+r=j. \qquad (6)$$

Then the total probability that there are N pions produced in an A-B collision at $p_L/A=3$ GeV/c is

$$P_N(AB)=\sum_i R(i,N)P(i,AB). \qquad (7)$$

We present P_N as the solid curve in Fig. 1(a) for ^{40}Ar incident on Pb_3O_4 at 3 GeV/c per nucleon, which is the weighted sum of results for a ^{208}Pb and a ^{16}O target. All three density distributions were taken from fits to electron-scattering data and scaled to the total mass. This total distribution P_N is characterized by several rather distinct regions: a steep, exponentially falling low-multiplicity region ($1\leq N\leq 10$), a rather flat medium-multiplicity region ($10\leq N\leq 40$), another somewhat steep region ($40\leq N\leq 50$), another flat region ($50\leq N\leq 200$), and a final falloff to zero ($N>200$) which is not shown. Further analysis reveals that the first flat region arises primarily from central $^{40}Ar+{}^{16}O$ collisions while the second arises from central $^{40}Ar+{}^{208}Pb$ collisions. As N increases beyond about 120 we expect the simplifications of the MCM to be increasingly severe; so we do not further discuss that region here.

Because of the difficulties in distinguishing the π^+ from the proton in high-multiplicity events and in detecting the π^0, it is desirable to have a pre-

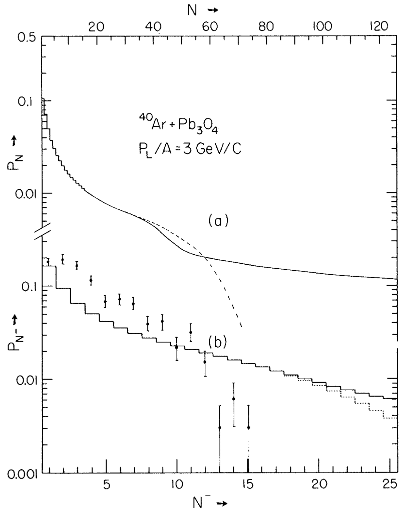

FIG. 1. Probability of a given event versus total number of pions (N) in that event or versus the number of negative pions (N^-) in that event. The probabilities P_N and P_{N^-} are given by Eqs. (7) and (8), respectively, in the text. The solid curves give the results of the full MCM while the dashed curves represent the results of an approximate resonance production model as described in the text. The data for the negative-pion multiplicity spectra are from Ref. 4.

diction for the π^- spectrum alone.[4] A reasonable way to extract this spectrum would be to define $f(\pi^-,pp)$, $f(\pi^-,pn)$, and $f(\pi^-,nn)$ as the probability that a produced pion is a π^- in a pp, pn, or nn collision, respectively, at this energy. Based on the available data,[3] interpolations, and symmetries, we roughly estimate these values to be $\frac{1}{9}$, $\frac{1}{3}$, and $\frac{2}{3}$, respectively. Then, with the charge and baryon number of projectile and target nuclei, one obtains the distribution of collisions of each type and extracts $\bar{p}_{AB}(\pi^-)$, the probability in an A-B collision that a pion produced in an *arbitrary* N-N collision is a π^-, which is 0.375 for ^{40}Ar + ^{16}O and 0.405 for ^{40}Ar + ^{208}Pb. Then, using the results for $P_N(AB)$, we construct $P_{N^-}(AB)$, the spectrum of produced π^-, from

$$P_{N^-}(AB) = \sum_{N=N^-}^{N_{\max}} \binom{N}{N^-}(\bar{p})^{N^-}(1-\bar{p})^{N-N^-}P_N(AB). \quad (8)$$

The weighted sum of these results, denoted simply as P_{N^-} and renormalized by $1-P_{N^-=0}$ for ^{40}Ar incident on Pb$_3$O$_4$ at 3 GeV/c per nucleon, is given as the solid curve in Fig. 1(b) and is compared with the available data.[4] It was found that the shape of P_{N^-} is very insensitive to changes in $\bar{p}$ by as much as 25%. Consistent with our reservations about the MCM for the total spectrum, I choose to present and discuss these results only up to $N^-=25$. The distinctive regions in the total spectrum have now been smeared to a certain extent. Nevertheless, the significant probability for rather high-multiplicity events appears as a striking result of the MCM in contrast with the data. For example, the area under the curve for $N^->15$ indicates about 20% of the events will have >15 negative pions whereas there are no events experimentally beyond 15. Approximately 10% are predicted in the range 16 to 25.

To test sensitivity of the spectra in the high-multiplicity regime one may construct models with different particle production mechanisms involving admixtures of direct production and production through resonances as well as admixtures of collective and independent production processes.[5] Then, if the MCM can be viewed as the ultra-independent model, it is worth considering, for comparison, what dynamics may significantly alter its predictions. For example, suppose that pion production proceeds primarily through the formation of baryon resonances which decay after the A-B collision is completed. This has been proposed[6] for high-energy p-nucleus collisions and is in reasonable agreement with experiment.[7] Furthermore, isobar models have been employed in hadron-nucleus cascade calculations at lower bombarding energies with some success.[8]

In addition, let us assume that the resonance-nucleon total cross section is considerably smaller than the N-N cross section. Again, recent p-nucleus data[9] support this assumption. Then, after an incident nucleon experiences about two collisions on the average, it will have been excited to a resonance and it then proceeds through the target matter with greatly reduced effect. For ^{40}Ar incident on Pb$_3$O$_4$ it seems reasonable to obtain an estimate of the net effect of this mechanism within the MCM by truncating to ~80 the maximum number of N-N collisions i that can contribute to pion production. The effects on P_N and P_{N^-} are indicated as a dashed line in Fig. 1. As may be expected, the net result is to increase the rate of falloff of the spectra as a function of N or N^- beyond some appropriate value. However, substantial reduction of the N^- spectra only occurs for N^- greater than 20 to 25.

This method should be viewed as providing a generous estimate of the quenching of high-multiplicity events due to production through baryon resonances. Hence, we are led to suggest an alternative explanation—a coherent production mechanism. For example, if one studies particle production in nucleus-nucleus collisions by colliding coherent tubes[5,10] of nucleons, one obtains two dramatic effects. First, there is a substantial cross section for "subthreshold" production—that is, production when the energy per nucleon alone would be insufficient to produce particles by independent N-N collisions. Second, and of direct relevance to the situation here, at energies per nucleon well above threshold (where the mean multiplicity in an N-N collision is relatively flat) there is a dramatic reduction in the cross sections for particle production from the MCM result.[5] The exact reduction depends on the amount of coherent production one introduces. In the limit that the whole projectile acts coherently on the whole target the mean multiplicity at 3 GeV/c for ^{40}Ar + ^{208}Pb will be reduced by *more than one order of magnitude.* This indicates that there is ample flexibility to describe the π multiplicity data with an admixture of a coherent production mechanism into the MCM.

I conclude that there are distinctive features of the total pion spectrum in the simple multiple-collision model of relativistic A-B collisions. On the other hand, these features are somewhat smeared in the negative-pion spectrum. For ^{40}Ar incident on Pb_3O_4 at 3 GeV/c per nucleon, the introduction of a resonance production model in an approximate fashion to inhibit high-multiplicity events does not dramatically affect the N^- spectrum below $N^-=25$. I propose, therefore, that the significant deviation in the data from these simple spectra for high-multiplicity events is indicative of nonsimple behavior in central A-B collisions. In particular I argue that this deviation is evidence of the presence of a coherent production mechanism.

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Pion Multiplicity Distributions in Heavy-Ion Collisions

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We calculate the π^- multiplicity distribution for Ar + Pb_3O_4 at 1.8 GeV/nucleon. Good agreement with data is found. Effects of multipion correlations are investigated.

Recently, the negative-pion multiplicity distribution $P(n)$ has been measured[1] for a variety of heavy-ion targets and projectiles at beam energies of ~2 GeV/nucleon. [$P(n)$ is the probability of producing n negative pions in a given heavy-ion collision.] In order to assess the implications of the data and to help in planning future experimetns, we present here the results of model calculations of $P(n)$. Our study is motivated by the following questions: (1) Can a simple model account for the measured $P(n)$? (2) How sensitive is $P(n)$ to the details of heavy-ion dynamics? (3) What information does $P(n)$ contain that is not available from single-pion inclusive cross sections[2]? And (4) how are multipion correlations reflected in $P(n)$?

We show below that a thermodynamic "fireball" model[3,4] gives $P(n)$ in good agreement with data.[1] Then through a dynamical calculation of $P(n,t)$ based on a master equation, we show that there exists a large class of models that lead to the same result. In particular, we prove that for a fixed impact parameter b, $P(n,t;b)$ is a Poisson distribution *independent* of the dynamical