Problem of Strong P and T Invariance in the Presence of Instantons

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(Received 29 November 1977)

The requirement that P and T be approximately conserved in the color gauge theory of strong interactions without arbitrary adjustment of parameters is analyzed. Several possibilities are identified, including one which would give a remarkable new kind of very light, long-lived pseudoscalar boson.

One of the main advantages of the color gauge theory of strong interactions is that so many of the observed symmetries of strong interactions seem to follow automatically as a consequence of the gauge principle and renormalizability—P, T, C, flavor conservation, the $\underline{3}\oplus\underline{3}^*$ structure of chiral symmetry breaking, and asymptotic scale invariance. As a result of this, gauge theories of the weak and strong interactions mesh nicely, and effects such as parity-nonconserving and flavor-changing processes can be calculated to be small, potentially dangerous renormalization effects being under control.[1] This attractive picture, however, is based upon neglect of possible interactions of the form

$$\mathcal{L}_{\mathrm{int}} = (\theta/32\pi^2)\,\mathrm{tr}\,G_{\mu\nu}\tilde{G}_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the gauge field and $\tilde{G}_{\mu\nu}$ its dual, written in matrix form. Although this interaction is formally a total divergence,

$$\mathrm{tr}\,G_{\mu\nu}\tilde{G}_{\mu\nu} = \partial_\mu \mathrm{tr}\,\epsilon_{\mu\nu\lambda\sigma}A_\nu(\partial A_\sigma + \tfrac{2}{3}A_\lambda A_\sigma), \tag{2}$$

recent work[2-6] has made it clear that it cannot be neglected. The interaction in Eq. (1) breaks P and T but conserves C, so it contributes directly to the neutron electric dipole moment d_n. The extremely good experimental limits on d_n require a very small θ

$$\theta/32\pi^2 \lesssim 10^{-7}. \tag{3}$$

In this Letter I will analyze whether such a small quantity can emerge in a credible way from gauge theories, or if θ can be made zero in some sense. Three possibilities can be distinguished: (i) If the interactions which break P and T lead to infinite renormalization of θ, we shall say strong P and T invariance is *unnatural.* (ii) If the interactions which break P and T lead to a small finite renormalization of θ, we shall say strong P and T invariance is *natural.* In this case, if a bare value $\theta = 0$ is imposed as a symmetry requirement, a physically acceptable theory may result with no further adjustment. (iii) In a certain class of theories[4,5,7] the parameter θ is physically meaningless,[4,5] or dynamically determined.[7] In this case, if the strong interaction conserves P and T, we shall say the conservation is *automatic.*

I regard a theory of type (i) as very unattractive. Below I shall argue that a theory of type (ii) requires that either P or T be softly broken — that is, that the breaking occurs through a dimensional coupling in the bare Lagrangian or spontaneously. A theory of type (iii) requires that the mass of some quark be zero or that a remarkable new kind of particle (a 0^{-+} meson of mass ~ 100 keV, which we call an *axion*) exist. So if our arguments are correct, at least one of the following four conditions must hold: (i) P is softly broken—this condition leads to some awkwardness in understanding the two-component neutrino. (ii) T is softly broken. (iii) The mechanical mass[8] of some quark is zero—this does not agree with current-algebra estimates,[9] but it is not completely clear that it is excluded, given the uncertainties of these estimates. If this case is realized, it gives an interesting parallel between the quark and lepton sectors (massless quark and massless neutrino). (iv) An axion, with properties to be detailed below, exists.[10] This is in some ways the most attractive and certainly the most exciting possibility. Among these four alternatives, P and T conservation for strong interactions is natural in the first two and automatic in the second two.

Renormalization of θ.—Naively, one might expect that since $\mathrm{tr}\,G_{\mu\nu}\tilde{G}_{\mu\nu}$ is a dimension-four interaction it will get infinitely renormalized (logarithmic divergences) unless it does not conserve quantum numbers— here the only candidates are P and T invariance— which are only softly broken. I believe that this is correct, but it requires some special discussion since the vertex of $\mathrm{tr}\,G_{\mu\nu}G_{\mu\nu}$ vanishes (as do all instanton effects) in the usual Feynman perturbation theory. A convenient method for recognizing the divergences is to evaluate the renormalization of the vertex $\mathrm{tr}\, G_{\mu\nu}\tilde{G}_{\mu\nu}$ at an infinitesimal but nonzero momentum transfer.

As examples of theories with natural and unnatural strong P and T conservation I take the usual Weinberg-Salam[11] model with one Higgs doublet and all fermion–Higgs-particle Yukawa couplings relatively real, and the extended CP-nonconserving model of Kobayashi and Mashawa,[12] respectively. In the first case, once I put $\theta = 0$ there is no T nonconservation whatsoever—notice especially that the phase of the Higgs vacuum expectation value has no physical significance even in the presence of instantons (since a change in this phase is implemented by a weak T_3 rotation, under which the instanton interactions are invariant). In the second case there are relatively complex Yukawa couplings and hard P and T nonconservation. The infinite renormalizaction first potentially shows up in very high order, so that it has not been computed explicitly. It seems a safe conjecture, however, that in the absence of a symmetry preventing them, the expected divergences do indeed show up.

We regard these arguments for intrinsic renormalization of θ as indicative rather than conclusive. In any case, there is another, equally dangerous effect—instanton effects prevent us from rotating away an overall phase from the quark-mass matrix.

It is very interesting to note that infinite renormalization of the interaction

$$\mathcal{L}_{\mathrm{int}} = K \int \sqrt{g}\, \epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma} R^{\alpha\beta\mu\nu} R^{\gamma\delta\rho\sigma} \tag{4}$$

which is also formally a total divergence, has recently been found in the theory of gravity.[13]

Automatic theories (axions).— The deep conection between instantons and chiral invariance discovered by 't Hooft[3] has the consequence that in theories with a chiral invariance the angle θ may be eliminated in favor of other parameters in the theory. The key relation here is the triangle anomaly in the divergence of the axial current[14]

$$\partial_\mu J_{\mu 5} = \frac{g^2 n}{32\pi^2}\, \mathrm{tr}\, G_{\mu\nu}\tilde{G}_{\mu\nu} + \text{naive divergence}, \tag{5}$$

where n is the number of quark bilinears in the axial current, properly normalized. If some quark (most plausible, the u quark) has bare mass zero, then by making a suitable chiral rotation we may, according to Eq. (5), add or subtract an interaction of the form $\mathrm{tr}\, G_{\mu\nu}\tilde{G}_{\mu\nu}$ from our Lagrangian. For instance, in the Kobayashi-Maskawa (KM) model with a single Higgs doublet and a massless u quark, the rotation of u_R by a phase allows us to eliminate the strong P and T nonconservation found earlier, leaving everything else in the Lagrangian unchanged.

There is one neglected question that I wish to emphasize. Is it meaningful (i.e., independent of subtraction point) to uphold the condition $m_u = 0$ even though the associated chiral invariance is spoiled by instanton effects? The answer is yes —the instanton-generated chiral breaking is much softer than that given by an explicit mass term in the Lagrangian and can in principle be distinguished from it, for instance, in deeply inelastic scattering.

Peccei and Quinn[7] have made the ingenious observation that instead of a quark of bare mass zero, we might also consider a chiral symmetry. This leads to a special kind of Higgs boson (which we are calling the *axion*) with zero bare mass.[10] This proposal has some remarkable physical implications.

For notational simplicity, I shall analyze in some detail a simplified model and then mention the straightforward generalization to a realistic case. Let us consider a truncated Weinberg-Salam–type model with just two quarks, and a Higgs-particle–fermion interaction Lagrangian of the form

$$\mathcal{L}_{\mathrm{int}} = g_1 (\overline{ud})_L \varphi_1 u_R + g_2 (\overline{ud})_L \tilde{\varphi}_2 d_R + \mathrm{H.c.} + V(\varphi_1, \varphi_2) \tag{6}$$

with two Higgs doublets φ_1, φ_2, where $V(\varphi_1, \varphi_2)$ is insensitive to separate phase rotations of φ_1 and φ_2. This is the most general gauge-invariant form consistent with the symmetry

$$u_R \to e^{i\alpha} u_R,\quad d_R \to e^{i\beta} d_R,\quad \varphi_1 \to e^{-i\alpha}\varphi_1,\quad \varphi_2 \to e^{i\beta}\varphi_2. \tag{7}$$

By redefining φ_1 and φ_2, if necessary, we may assume g_1, g_2 real. Furthermore, according to Eqs. (5) and (7) we may make a chiral rotation so that $\theta = 0$. Although $V(\varphi_1, \varphi_2)$, even corrected by all radiative corrections due to the local interactions in the Lagrangian, is insensitive to the relative phase of φ_1 and φ_2, this degeneracy is lifted when instanton effects are taken into account (Fig. 1). The relative phase of φ_1 and φ_2 is therefore dynamically determined, by minimizing the full Higgs potential. *If* the vacuum expectation values v_1 and v_2 are relatively real (I then may take them both real by convention), then the theory conserves P and T. The relative reality of v_1 and v_2 was shown by Peccei and Quinn[7] in an unrealistic approximation, and to show it in general seems very difficult. I will simply assume it.

The two separate phase rotations in Eq. (7) give us two potential Nambu-Goldstone bosons, since the vacuum is not invariant under them. One of these, coupling to hypercharge, is absorbed by a vector meson according to the usual Higgs mechanism. The orthogonal combination,

$$\alpha = -\sin\lambda \operatorname{Im}\varphi_1^{\,0} + \cos\lambda \operatorname{Im}\varphi_2^{\,0}, \tag{8}$$

$$\tan\lambda = v_2/v_1, \tag{9}$$

where, e.g., $\varphi_1^{\,0}$ is the neutral component of φ_1, is physical and has zero bare mass. Its mass is generated entirely by processes such as shown in Fig. 1. Counting the visible coupling constants and supposing that instanton effects are characterized by a typical strong-interaction scale $\mu = 200$ MeV, we get an order-of-magnitude estimate

$$m_a \approx G_F^{1/2}\mu^2 \approx 100 \text{ keV} \times 10^{\pm 1}. \tag{10}$$

The most straightforward generalization of the coupling scheme Eq. (6) is to a sequential Weinberg-Salam–type model letting φ_1 couple to all the charge-$\frac{2}{3}$ right-handed quarks and φ_2 to the charge-$(-\frac{1}{3})$ right-handed quarks and to the right-handed charged leptons. The axion coupling to fermions is then

$$\mathcal{L}_{\text{int}} = 2^{1/4} G_F^{1/2}\alpha\Big(\tan\lambda \sum_{\substack{Q=2/3\\ \text{quarks}}} m_i \bar{q}_i\gamma_5 q_i + \cot\lambda \sum_{\substack{Q=-1/3\\ \text{quarks}}} m_j \bar{q}_j\gamma_5 q_j + \cot\lambda \sum_{\substack{Q=-1\\ \text{leptons}}} m_k \bar{l}_k\gamma_5 l_k\Big). \tag{11}$$

Unless the axion is significantly heavier than estimated in Eq. (10), it will decay overwhelmingly into two photons. The width for this is crudely (ignoring strong-interaction renormalization effects!)

$$\Gamma(\alpha\to\gamma\gamma) \simeq \frac{\alpha^2 G_F\sqrt{2}}{9\pi^3}\,\frac{N^2}{\sin^2 2\lambda}\,m_\alpha^{\,3}, \tag{12}$$

where N is the number of quarks (and lepton) doublets. For $N=3$, $\lambda=\pi/4$, and $m_a = 100$ keV, this gives $\Gamma \sim 3\times 10^{-20}$ MeV or α lifetime of order 2×10^{-2} sec.

Regarding the phenomenology of these particles, we have the following remarks: (i) The pseudoscalar character of the couplings of α makes several classes of experiment designed to search for light Higgs particles with scalar couplings much less powerful.[15] In particular coherence effects are lost and the interactions mediated by virtual axions fall faster than $1/r$.[16]

(ii) Because of the explicit mass factors in the couplings, effects of the axion will be most readily seen in heavy-quark systems. One amusing effect is that since the axion is CP odd and very light, it can act as a sort of spurion, introducing large mock CP-nonconserving effects if it is undetected in heavy-particle decays.

(iii) Similarly to the ordinary Higgs particle, the best method to look for axions is probably in radiative decays of heavy vector mesons, $V\to\alpha+\gamma$.[17] It can be calculated similarly to Ref. 17. Surprisingly, despite the change from scalar to pseudoscalar coupling the same result emerges:

$$\frac{\Gamma(V\to\alpha+\gamma)}{\Gamma(V\to\mu^+ + \mu^-)} = \frac{G_F}{\sqrt{2}\,\pi\alpha}\,m_q^{\,2}, \tag{13}$$

where V is a vector meson composed of quarks of mass m_q, and $\lambda = \pi/4$ for convenience.

(iv) Along with a, even in this minimal model, we are left with one charged and two other neutral massive Higgs mesons. The sequential version of Eq. (6) has the advantage that the neutral Higgs particles do not mediate flavor-changing interactions.[18]

(v) Axions can be produced by bremsstrahlung-like mechanisms in high-energy collisions, roughly at a rate $\sim 10^{-7}$–10^{-6} of pion production. Once produced, they interact $\sim 10^{4-5}/E$(GeV) as strongly as neutrinos at the same energy E for typical accelerator energies. Thus there is a good possibility to observe them in "beam-dump" experiments.[19] Moreover, axions could contaminate focused neutrino beams through the decay $K^+ \to a\pi^+$, which could also be searched for directly.

(vi) If the axion is heavier than two electron

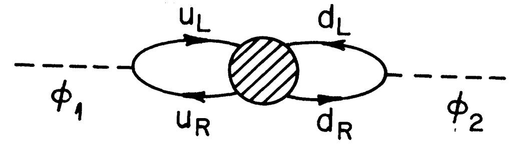

FIG. 1. Instanton interaction generating the axion mass.

masses it will decay into e^+e^- pairs at the level

$$\Gamma(a\to e^+e^-)=\frac{G_F\sqrt{2}}{8\pi}m_a m_e^2\cot^2\lambda\left(1-\frac{4m_e^2}{m_a^2}\right)^{1/2}. \tag{14}$$

We thank C. Quigg for his hospitality at Fermilab, where this work was begun, and C. Bernard, D. Gross, S. Treiman, and especially C. Callan for indispensable clarifying discussions. We thank S. Weinberg for informing us of his work prior to publication and for emphasizing to us the possibility that an axion as described here might have escaped detection. The author acknowledges receipt of an Alfred P. Sloan Foundation fellowship.

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