tion, an anomalous electron signal may also arise from decays of the heavy lepton τ. From preliminary measurements on $e\mu$ events at the $\psi(3772)$ [see M. L. Perl, in Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977 (to be published)], we estimate that 6% of the anomalous electron events can come from this source. Taking account of $\tau^+\tau^-$ production would also lead to a decrease in the value of $\sigma(D)$ by about 12%; the net effect is to raise our value for the branching ratio of D to electrons from 7.2% to 7.6%.

[9]With the assumption that the $\psi(3772)$ is a state of definite isospin (0 or 1), this average corresponds to $(0.56 \pm 0.03)B_{D^0\to e} + (0.44 \pm 0.03)B_{D^+\to e}$, as noted in Ref. 2.

Meron Pairs and Quark Confinement

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We discuss the consequences of our classical meron-pair model on the quark-confinement phase transition in an SU(2) Yang-Mills quantum field theory.

A phase transition of merons has been proposed by Callan, Dashen, and Gross[1] as a mechanism for quark confinement. We study the Wilson loop integral

$$\oint \exp(i\oint_C A_\mu dx^\mu)d\mu = e^{-TE(L)}, \quad (1)$$

where $d\mu = Z^{-1}\exp(-\frac{1}{4}\int F\cdot F\,dx)[\mathfrak{D}A]$ and C is an $L\times T$ rectangular path. We propose a picture which yields a confining potential

$$E(L)\sim L^\epsilon, \quad 0\leq\epsilon=\epsilon(L)\leq 1. \quad (2)$$

Our value of ϵ depends on the density $\nu(a)$ of meron pairs with separation a, and is limited by $\epsilon\leq 1$ in our analysis.

In a previous paper[2] we obtained multiple meron configurations for classical SU(2) gauge fields. In studying (1) and (2) it is fundamental to minimize action plus entropy. Entropy of the classical meron configuration is associated with the location of each meron in R^4 and also with the assignment of bags to join the merons pairwise (cf. Ref. 2). Because we minimize action plus entropy, we do not insist that the configurations allowed are solutions of the classical Yang-Mills equations (derived by minimization of the action alone) (cf. Ref. 1 and Mueller[3]). In particular we analyze configurations formed from meron pairs whose bags may overlap.

Since the action for a meron pair is logarithmic in the separation a, and since the effective coupling constant $\bar{g}(a)$ for distance a is a slowly increasing function of a by a renormalization-group argument, we postulate a density

$$\nu(a)\,da = a^{-4+\epsilon}\,da \quad (3)$$

for meron pairs with separation a. Here $\epsilon=\epsilon(a)$ is a slowly increasing function of a, and (3) results from neglecting the interaction of distinct meron pairs. Note that $\epsilon<0$ corresponds to nonoverlapping pairs while $\epsilon=3$ corresponds to complete meron unbinding.

We have two main results. The first is similar to Ref. 1 and shows that (3) leads to a quark-confinement potential (2) if $\epsilon>0$. The second is that at $\epsilon=1$ (if not sooner for some $\epsilon<1$) a phase transition to the strong-coupling regime occurs, in which the meron picture no longer applies. In this regime a linear confinement potential is known.[4]

In order to evaluate (1), we regard $A=\sum A_j$ as a sum of potentials A_j, each due to a single meron pair at random points and orientations, and subject to the constraint (3). Then

$$\langle\exp(i\oint A)\rangle\sim\exp[-\langle\tfrac{1}{2}(\oint A)^2\rangle]\sim\exp[-\tfrac{1}{2}\sum\langle(\oint A_j)^2\rangle],$$

and so

$$E(L)\sim\sum_{t=0}\langle(\oint A_j)^2\rangle. \quad (4)$$

Here $\sum_{(t=0)}$ restricts the sum over meron pairs to those pairs centered on the $t=0$ hyperplane, and we have taken T to infinity with fixed L, and center the loop at the origin.

Let r be the distance from the midpoint of the meron pairs to the time axis. A calculation using $\vec{A}$ as given in Ref. 2, below Eq. (25), shows that for meron-pair solution

$$|\oint A_j| = O(1)\left(\frac{a^2}{a^2+r^2}\right)\left(\frac{L}{L+r}\right), \quad (5)$$

up to a factor of $\ln L$, which we ignore. Then substituting (5) and (3) in (4) gives

$$E(L)=O(1)\int_0^\infty\int_0^\infty \frac{a^4}{(a^2+r^2)^2}\frac{L^2}{(L+r)^2}\,r^2\,dr\,\nu(a)\,da=O(L^\epsilon),\quad \epsilon>0. \tag{6}$$

Here $4\pi r^2\,dr$ gives the integral over meron location, and $\nu(a)\,da$ gives the integral over meron density.

To show that $\epsilon<1$ in the low-temperature phase, we recall that low temperature means an ordered phase for A and a disordered phase for the dual variables—such as instantons or merons. Thus we take the mean squared fluctuation $\langle(\bar{A})^2\rangle$ as the criterion for a phase transition. For a fixed length a_0, the intermeron distance determined by (3) is $a_0^{1-\epsilon/4}$, and the mean distance to a member of a meron pair of separation a is $a^{1-\epsilon/4}$. To study merons of separation $a\geqslant a_0$, we can average over cubes of length $a_0^{1-\epsilon/4}$, but no larger. Thus $\bar{A}$ is defined by averaging A over a cube of this length, and A is the sum of all random meron pair potentials A_j with $a\geqslant a_0$ which are *not* located in the cube defining $\bar{A}$. We find that

$$\langle\bar{A}^2\rangle=O(a_0^{-1+\epsilon}). \tag{7}$$

Thus $\epsilon>1$ leads to large fluctuations in A and instability of the low-temperature phase (i.e., to a transition from weak to strong effective coupling).

To establish (7), we first determine the contribution of each scale size $a\geqslant a_0$, and then integrate over a using (3). Let $r_0=a_0^{1-\epsilon/4}$. The number of r_0 blocks separating a meron pair with separation a is $a^{1-\epsilon/4}/r_0$. Thus in units of r_0, the a merons have density $(r_0a^{-1+\epsilon/4})$.[4] The maximum distance for influence of an a meron is a, which in units of r_0 is a/r_0. Thus, by summing over merons in distinct cubes of size r_0, we obtain

$$\langle\bar{A}^2\rangle=O(1)\int_{a_0}^\infty da\sum_{j=1}^{a/r_0} j^3(r_0/a^{1-\epsilon/4})^4(jr_0)^{-2}.$$

Here the factor $\sum_j j^3$ gives the volume summation, the second factor gives the meron density, and the final factor equals $(A_{\rm meron})^2$ for distances $jr_0\leqslant a$, i.e., within the meron pair. Evaluation of this expression yields (7).

The string, or bag, connecting the meron pairs is a phase boundary in terms of the equation $\Delta\psi=r^{-2}\psi(\psi^2-1)$. For a bag of radius a, the phase boundaries fill all space at $\epsilon=0$. This fact will be elaborated into a droplet picutre of the confinement phase transition, and it suggests that $\epsilon=0$ may be the correct location for the transition, with our value $\epsilon\leqslant 1$ as only an upper bound. The importance of meron or instanton overlap also appears in the work of Callen, Dashen, and Gross.[1]

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