and methods of analysis with respect to π^+ and π^- makes our results much more reliable. We believe that we have convincingly demonstrated the ability of differential studies of π^+ and π^- inelastic scattering to provide an insight into the neutron/proton or core/valence structures of excited nuclear states which has hitherto before not been possible. The pions may now be considered as beginning to deliver on their promise as a new and powerful tool in the study of nuclear structure!

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Long-Range Absorption in the Heavy-Ion Optical Potential

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A long-range imaginary optical potential approximating the effects of quadrupole Coulomb excitation is derived in closed form. An analytical closed form for sub-Coulomb elastic scattering is obtained by inserting this potential into a weak-absorption model.

A long-range absorption in the heavy-ion optical potential due to Coulomb excitation of a low-lying collective quadrupole state has been the subject of some interest recently. An experimental specimen is the elastic scattering data of 90-MeV ^{18}O on ^{184}W.[1] These data show a Fresnel pattern damped below the Rutherford cross section that is well reproduced by a coupled-channels calculation which includes Coulomb excitation of the 111-keV 2^+ rotational state in ^{184}W.

An alternative theoretical description is the construction of an optical-model component arising from two-step contributions to elastic scattering. This can be done using the Feshbach projection-operator formalism.[2] In this framework, Love, Terasawa, and Satchler have recently obtained a formula for a long-range imaginary potential (which we will refer to as the LTS potential) by making the approximation of using plane-wave intermediate states along with a classical correction for the Coulomb braking.[3] The potential obtained is dominantly negative imaginary, and, apart from finite-size corrections, has a radial dependence of $R^{-5}[1-(Z_1Z_2e^2/RE_{\text{c.m.}})]^{-1/2}$.

In this Letter we show that it is possible to derive a more exact expression for this long-range imaginary optical potential by making use of Coulomb-distorted scattering states and a Coulomb-distorted Green's function. The result shows some interesting differences from the LTS potential. Furthermore, we obtain an analytical closed form for the differential elastic cross section below the Coulomb barrier based on a weak-absorption model.

The potential component to be evaluated may be written[2]

$$V(r,r')=V_f(r)G_2(r,r')V_i(r'), \tag{1}$$

where G_2 is the Green's function for the intermediate 2^+ state, and V_f, V_i are the quadrupole operators connecting ground and excited states, i.e.,

$$V_i(r')=\frac{4\pi Z_p e[B(E2)\uparrow]^{1/2}}{5\sqrt{5}}\frac{1}{r'^3}\sum_{M'}Y_{2M'}(\hat{r}'), \tag{2}$$

and likewise *mutatis mutandis* for V_f. A partial-wave expansion of G_2 may be made in coordinate space

$$G_2=-\frac{2\mu}{k\hbar^2}\sum_{l'm'}F_{l'}(r_<)H_{l'}(r_>)Y_{l'm'}(\hat{r})Y_{l'm'}^*(\hat{r}'), \tag{3}$$

where $F_{l'}(r_<)$ and $H_{l'}(r_>)$ will be taken to be the regular and outgoing boundary Coulomb wave functions, respectively. One then obtains a nonlocal, l-dependent contribution to the optical potential

$$U_l(r,r')=-\frac{2\mu}{k\hbar^2}\frac{4\pi}{25}Z_p^2e^2B(E2)\uparrow\sum_{l'}\langle l0\,20|l'0\rangle^2\frac{1}{r^3}\frac{1}{r'^3}F_{l'}(r_<)H_{l'}(r_>). \tag{4}$$

A "trivially equivalent local potential"[3] may now be defined in perturbation theory:

$$U_l(r)F_l(r)=\int dr'\,U_l(r,r')F_l(r'). \tag{5}$$

Recalling that

$$H_{l'}(r_>)=G_{l'}(r_>)+iF_{l'}(r_>), \tag{6}$$

the local potential takes the form

$$U_l(r)=-\frac{2\mu}{k\hbar^2}\frac{4\pi}{25}Z_p^2e^2B(E2)\uparrow$$
$$\times\sum_{l'}\langle l0\,20|l'0\rangle^2\frac{1}{r^3}\left[i\frac{F_{l'}(r)}{F_l(r)}\int_0^\infty dr'\,F_{l'}(r')\frac{1}{r'^3}F_l(r')\right.$$
$$\left.+\frac{F_{l'}(r)}{F_l(r)}\int_r^\infty dr'\,G_{l'}(r')\frac{1}{r'^3}F_l(r')+\frac{G_{l'}(r)}{F_l(r)}\int_0^r dr'\,F_{l'}(r')\frac{1}{r'^3}F_l(r')\right]. \tag{7}$$

Notice here the clean separation of the real and imaginary parts of this potential. Obviously, both real components will oscillate in sign as a function of r; this behavior is confirmed in computer evaluation of Eq. (7). These real components merely serve to put insignificant "hair" on top of the real Coulomb potential and will not be considered further. On the other hand, we can evaluate the imaginary component in closed form.

For the sake of simplicity in derivation we assume no energy loss in the quadrupole transition. However, a semiclassical energy-loss factor $g_2(\xi)$ is applied to our results at the end.[3] $g_2(\xi)$ is merely the ratio $f_2(\eta,\xi)/f_2(\eta,0)$, where ξ is the adiabaticity parameter $\xi=\frac{1}{2}\eta\Delta E/E_{\text{c.m.}}$, and $f_2(\eta,\xi)$ is the standard factor of Alder *et al.*[4] which we assume for $\eta=\infty$. In the derivation of our closed form it will be assumed that either η or $\hat{l}=l+\frac{1}{2}$ are large, the usual semiclassical conditions.[4] Use is made of the closed forms for the $1/R^3$ Coulomb integrals[5,4] and the Coulomb wave-function recurrence relations. One obtains finally the long-range imaginary potential for a given partial wave l:

$$U_l(r)=-i\frac{2\mu}{k\hbar^2}\frac{\pi}{50}Z_p^2e^2B(E2)\uparrow g_2(\xi)\left[\left(\frac{\eta^2k^2(3\hat{l}^2+\eta^2)}{\hat{l}^2(\hat{l}^2+\eta^2)^2}-\frac{\eta k^2}{\hat{l}^3}\arctan\frac{\hat{l}}{\eta}\right)\frac{1}{r^3}+\frac{4\eta k\hat{l}^2}{(\hat{l}^2+\eta^2)^2}\frac{1}{r^4}+\frac{2\hat{l}^4}{(\hat{l}^2+\eta^2)^2}\frac{1}{r^5}\right], \tag{8}$$

where μ is the reduced mass of the system, Z_p is the projectile charge, k is the wave number, and η is the usual Sommerfeld parameter.

This potential at first sight seems quite different from the LTS potential. It is specifically l dependent with $1/R^5$, $1/R^4$, and $1/R^3$ radially dependent terms in contrast to the l-independent, dominantly $1/R^5$, LTS behavior. As $l \to \infty$, the $1/R^5$ term dominates in our potential and the ratio of the LTS potential to ours approaches $\frac{4}{3}$. The physical correspondence between the approximate LTS potential and our more exact form may be seen in Fig. 1. The LTS potential crosses our l-dependent potential several fermis outside of the classical turning point for the small and intermediate l values of interest. Paradoxically, due to the l dependence, our $1/R^5$ term has the longest range, while the $1/R^3$ term has the shortest range.

Our formula has been compared with the results of a computer evaluation of the imaginary part of Eq. (7) for the case in Fig. 1, and for all partial waves agreement is quite good [to within several percent, except for computationally unstable points where $1/F_l(r)$ becomes large]. For the lower partial waves in above-barrier scattering, one should properly consider nuclear effects both in the wave functions and in the quadrupole operator, but for the present we do not consider these questions, arguing that our potential is sufficient as it stands to describe the unambiguously long-range part of the imaginary optical potential.

The potential derived is suitable, as it stands, to be incorporated into already existing optical-model codes. However, we will not pursue this at present. Rather we will consider the effects of long-range absorption in the case of sub-Coulomb elastic scattering and derive a formula for the cross section in analytical closed form. This approach turns out to provide the most concise way to compare our potential with the LTS potential. Moreover, studying long-range absorption below the Coulomb barrier insures that the effect will not be obscured by the short-range nuclear surface absorption which dominates cross sections beyond the critical angle; this is a nuclear analogy to eclipsing the solar disk to observe better the corona.

Since our long-range imaginary potential is weak and smoothly varying in both r and $\hat{l}$, it will produce a nondiffractive quasiclassical absorptive effect on trajectories passing a few diffuseness lengths or more outside the nuclear surface. We have extended the strong-absorption formula of Frahn[6] to include this superimposed weak absorption, whose contribution to the complex phase shifts is calculated by inserting Eq. (8) into a perturbative Jeffreys-Wentzel-Kramers-Brillouin (JWKB) integral developed previously.[7] We will describe in detail the resulting formula for elastic scattering above the Coulomb barrier in a subsequent publication. Below the Coulomb barrier, our result becomes independent of nuclear surface parameters other than $B(E2)\uparrow$, and we obtain a simple form for the elastic scattering ratio to Rutherford cross section

$$\sigma(\theta)/\sigma_R(\theta) = \exp[-Kf(\theta)], \tag{9}$$

where all the specific parameters of the reaction are contained in the constant

$$K = \frac{16\pi}{225}\frac{k^4}{\eta^2}\left[\frac{B(E2)\uparrow}{Z_T^2}\,\frac{g_2(\xi)}{e^2}\right], \tag{10}$$

and $f(\theta)$ is a universal function of angle:

$$f(\theta) = \tfrac{9}{4}\Big((\cos\tfrac{1}{2}\theta)^4(\tfrac{4}{3}D^4 + \tfrac{104}{105}D^5) + (\sin\theta)^2[\tfrac{1}{4}\pi D^3 + (\tfrac{64}{30} - \tfrac{15}{30}\pi)D^4]$$
$$+ \{[3 + (\tan\tfrac{1}{2}\theta)^2](\sin\tfrac{1}{2}\theta)^4 - (\tan\tfrac{1}{2}\theta)^3(\tfrac{1}{2}\pi - \tfrac{1}{2}\theta)\}(D^2 + \tfrac{2}{3}D^3)\Big), \tag{11}$$

with

$$D = (1 + \csc\tfrac{1}{2}\theta)^{-1}.$$

This analytical form for $f(\theta)$ has the smooth behavior exhibited in Fig. 2(a).

We may also obtain an expression identical to the above for the cross section produced by the LTS potential except that a different universal function of angle $\bar{f}(\theta)$ is involved. We have plotted the universal below-barrier ratio $\bar{f}(\theta)/f(\theta)$ in Fig. 2(b). This ratio deviates from unity by up to $33\frac{1}{3}\%$ at forward angles, but this will not show up in most reactions because of the small magnitude of $f(\theta)$. At intermediate angles of about 40° to 110° the ratio deviates little from unity, implying

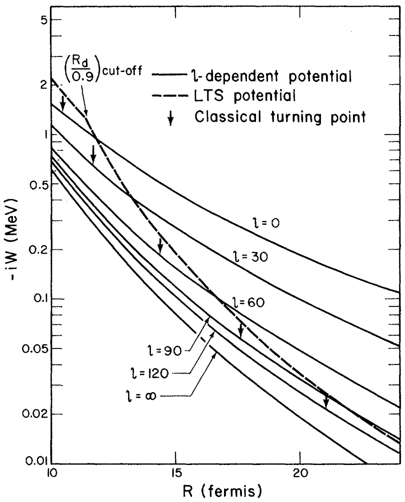

FIG. 1. l-dependent imaginary optical potential obtained from Eq. (8) compared with the LTS potential for $^{18}O + ^{184}W$ at 90 MeV.

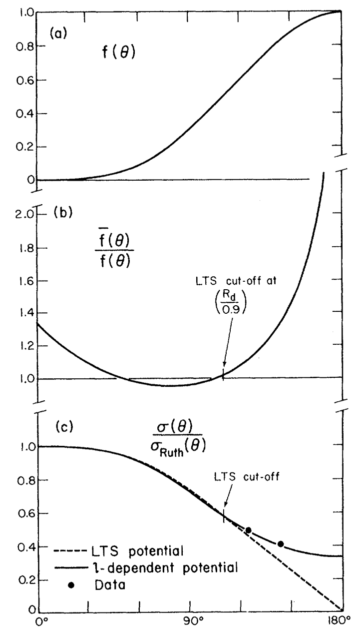

FIG. 2. (a) Universal function of angle, $f(\theta)$. (b) Ratio of $\bar{f}(\theta)$ for the LTS potential to $f(\theta)$ for our potential. (c) Elastic-scattering cross section for $^{16}O + ^{162}Dy$ at 48 MeV calculated from Eq. (9) incorporating $f(\theta)$ for our potential and $\bar{f}(\theta)$ for the LTS potential. Data are from Lee and Saladin (Ref. 8).

excellent agreement for the predictions of the two potentials. However, beyond 110° (corresponding to LTS cutoff of the Coulomb correction factor at $R_d/0.9$) there is no theory from the LTS potential but only a possible prescription. For the sake of analytical tractability we have merely ignored the cutoff in the ratio calculation. Clearly, without the arbitrary cutoff LTS predictions deviate substantially from those of our potential at very large angles as is illustrated in Fig. 2(c). Here are plotted cross sections in a realistic case for which data exist at two angles[8]: $^{16}O + ^{162}Dy$ at 48 MeV. There is also similar data for $^{16}O + ^{152}Sm$ for which σ/σ_R is 0.56(1) at 120°(lab) and 0.51(1) at 140° as compared with our calculated values of 0.57 and 0.49, respectively. Multiple Coulomb excitation plays some role in these cases, with a cross section to the 4^+ states in both cases about $\frac{1}{4}$ that to 2^+ state at 140°. Although the 4^+ cross section clearly depletes flux from the 2^+ cross section, its effect upon the elastic scattering is less direct. The excellent agreement between the calculated cross section and the existing data in these two cases seems to encourage further experiments to obtain complete angular distributions in sub-Coulomb elastic scattering.

The remarkable point of such experiments is that there is a nontrivial theory with no free parameters, which can be evaluated without a computer, that gives specific cross-section predictions. Indeed, in the situation where the long-range potential arises dominantly from a single state, sub-Coulomb elastic scattering analyzed in terms of our analytical expression might provide an alternative method of determining the experimental $B(E2)$ to that single state.

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