an interband term. The onset of the interband transitions is in fair agreement with the idea that alloying monovalent Au with tetravalent Si increases the concentration of conduction electrons. However, the Drude term itself was found to be enhanced significantly over that expected from such a simple interpolation scheme. Also the strength of the interband transitions was found to be reduced. Both these features apparently stem from the fact that the conduction electrons have extremely short lifetimes.

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# Intersubband-Cyclotron Combined Resonance in a Surface Space-Charge Layer

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In a magnetic field tilted with respect to the sample surface we observe combined resonance transitions because of a coupling of Landau levels and subband states. In the high-density, interacting electron gas on the semiconductor surface these experiments give information on the depolarization and excitonlike shifts involved in the resonant excitation of the system.

Cyclotron resonance[1] and intersubband resonance[2] for charge carriers on semiconductor surfaces are thoroughly documented phenomena. For electrons on Si(100) the cyclotron motion in a perpendicular magnetic field $H$ is independent of the $z$-directed, electric-field–induced subband quantization. Energies are given as $E_{n,l} = E_n + (l + \frac{1}{2})\hbar\omega_c$.

The presence of an additional, parallel component of $H$ leads to a coupling of the cyclotron and subband motions. With $H$ tilted at an angle $\theta$ with respect to the sample normal, a combined intersubband-cyclotron resonance mode, involving a simultaneous change of subband index and Landau-level number, can be excited. The effect has been observed for electrons on liquid helium.[3] A pair of small satellite peaks, displaced symmetrically by about $\pm\hbar\omega_c$ from the subband transition, has been observed. For Si(100) the existence of such transitions has been speculated on in previous work.[4]

The origin of the combined resonance can be understood in simple terms. The field $\vec{H}=(0,H_{\parallel},H_{\perp})$ is described by the potential $\vec{A}=(-H_{\perp}y+H_{\parallel}z,0,0)$. Following Ando,[5] one treats $H_{\parallel}$ as a perturbation in the sense that one neglects its influence on the $z$ part of the wave function. In the magnetic Hamiltonian, $z$ is replaced by $\langle z\rangle$. There results a small diamagnetic increase of the subband energy together with a lateral shift on the $k_x$ scale for each subband. The relative displacement of the $n=0$ and $n=1$ states is $\Delta k_x=(eH_{\parallel}/\hbar c)(\langle z\rangle_1-\langle z\rangle_0)$. The $y$ coordinate of the orbit center is shifted accordingly (Fig. 1). This $y$ displacement gives nonvanishing matrix elements for $\Delta l\neq 0$ combined resonances as excited by $z$-polarized radiation. The amplitude of the pure intersubband mode $\Delta l=0$, $\Delta n=1$ is reduced as the spatial overlap of the Landau functions decreases with increasing $\Delta y$. The simple picture predicts symmetrical $\pm|\Delta l|\hbar\omega_c$ satellite resonances. It provides an adequate account of the helium experiments.

Electrons on helium represent a low-density gas with large $r_s$ parameter. Exchange and correlation do not measurably affect the subband separations. The low density also implies that the depolarization effect involved in the resonant excitation of the electrons does not shift the resonance from the intersubband energy appreciably.[6] The space-charge layer on Si is substantially different in this respect. For the high densities

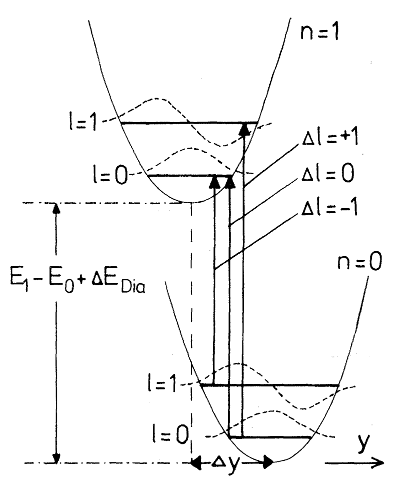


FIG. 1. Landau energies and wave functions (dotted lines) of the $n=0$ and $n=1$ electric subbands in a tilted magnetic field. $H_{\parallel}$ causes a small increase $\Delta E_{\rm Dia}=(2m_c^*)^{-1}(eH_{\parallel}/c)^2[(\Delta z)_1^2-(\Delta z)_0^2]$ in the separation of the subbands and a lateral displacement $\Delta y=(H_{\parallel}/H_{\perp})(\langle z\rangle_1-\langle z\rangle_0)$. Transitions $\Delta l=0$, $\pm 1$ are indicated.

usually encountered, $r_s$ is of order unity and it has been established that many-body effects decisively influence the subband energies. The resonant excitation of the subband states involves an excitonlike shift[7] as well as a depolarization correction.[6] The observed resonance is thought to differ noticeably from the energy separation of the subbands. According to Ando[7] the resonant absorption of energy due to $n=0$ to $n=1$ transitions occurs at $\hbar\omega\equiv\tilde{E}_{01}=(1+\gamma)^{1/2}E_{01}$, where $E_{01}\equiv E_1-E_0$. The factor $\gamma\equiv\alpha+\beta$ represents the sum of the depolarization shift $\alpha$ and the inherently negative excitonlike shift $\beta$. Other than the reasonable agreement of calculated and observed resonance energies, there exists no evidence for this correction factor $\gamma$. Ando's calculation[7] gives $\gamma$ as positive and of order 0.1 for electron densities $N_s\sim 1\times 10^{12}$ cm$^{-2}$ on $n$-type (100) Si.

A recent treatment[8] of combined resonance, for the interacting surface electron gas and including the depolarization effect, yielded a remarkable result. For the example of an inversion layer on Si(100), Ando[8] finds a sensitively $\gamma$-dependent spectrum of resonances. The calculation places the combined resonances $\Delta l=\pm 1$ at the unshifted energies $E_{01}\pm\hbar\omega_c$, with the main $\Delta l=0$ resonance situated asymmetrically between them at $(1+\gamma)^{1/2}\times E_{01}$. Not only is the position shifted according to the sign and value of $\gamma$, but there is found a large asymmetry of the amplitudes of the $\pm$ satellites. For a realistic, positive $\gamma=0.2$ the $+1$ peak is an order of magnitude larger than its $-1$ counterpart. Additional work,[9] where Ando considers an accumulation layer for parameters typical of the present experiments, confirms the essential observations—combined resonances at the unshifted $E_{01}\pm\hbar\omega_c$ placed unsymmetrically with respect to the $\Delta l=0$ line and with a large amplitude asymmetry.

These results for Si show the combined resonance phenomenon in the interacting electron system to be fundamentally different from its helium counterpart. We expect the combined resonances to be an experimental handle on the $\gamma$ problem. In so far as the fit to calculation involves sensitively an accurate description of energies and binding lengths in addition to polarization and many-body effects, it should prove a definitive test of theory.

Our experiments examine an accumulation layer on $n$-type (100) Si. We measure the absorption derivative $dP/dV_g$ in a sweep of the gate voltage $V_g$ for fixed $\hbar\omega=10.45$ and 15.81 meV. The magnetic field is provided by a split-coil superconducting magnet. The sample can be tilted to arbitrary angle $\theta$ with respect to the field.

Figure 2 gives a sequence of curves for $H_\perp = 5$ T at $\hbar\omega = 10.45$ meV. $H_\parallel$ is raised until $\theta = 40°$. The major peak represents the subband resonance $\tilde{E}_{01}$. With rising $\theta$ this line remains fixed at $N_s = 5.3\times10^{11}$ cm$^{-2}$. The new resonance at lower $N_s$ we identify as the $\Delta l = +1$ combined peak. No $\Delta l = -1$ peak can be resolved. The combined resonance occurs displaced by $\Delta N_s = 1.7\times10^{11}$ cm$^{-2}$ below the intersubband mode. Using the experimentally determined $\tilde{E}_{01}(N_s)$ [2] we convert this $\Delta N_s$ to an energy displacement of 2.2 meV. $\hbar\omega_c$ at 5 T is 2.8 meV. The small separation and marked amplitude asymmetry in favor of $\Delta l = +1$ are features of Ando's calculation for positive $\gamma$.

The variation of the combined resonance amplitude with $H_\parallel$ provides additional confirmation of its origin. The oscillator strength of a transition from the $l$th level in the lower subband, to $l+1$ in the higher subband grows according to $(l+1)H_\parallel^2/H$ as the displacement $\Delta y$ increases from zero (compare Fig. 1). We have confirmed the field dependence implied by this relation.

The fact that $\gamma$ is positive and finite follows from qualitative features of Fig. 2, i.e., the amplitude asymmetry in favor of the $\Delta l = +1$ peak and the closer than $\hbar\omega_c$ spacing of the two peaks. Both of these observations are beyond reasonable doubt, and in this way the experiments provide evidence for the existence of the combined polarization and excitonlike effects. They confirm Ando's prediction of a positive $\gamma$. The numerical evaluation of $\gamma$, in as much as it depends on the empirically derived observation (based on a limited number of calculated curves) that the $\Delta l = +1$ resonance occurs at precisely $E_{01} + \hbar\omega_c$, is subject to considerable uncertainty. Based on the data in Fig. 2 we determine $E_{01} = 7.65$ meV at 3.7 $\times10^{11}$ cm$^{-2}$. The energy $\tilde{E}_{01}$ at that same density can be estimated from frequency-dependence data such as those in Ref. 2 as 8.25 meV. It follows $\gamma = 0.16$, a number that we would trust to only $\pm 50\%$.

Figure 3 extends the experiments to $\hbar\omega = 15.81$ meV and a different $H_\perp = 3.5$ T. Smaller $H_\perp$ al-

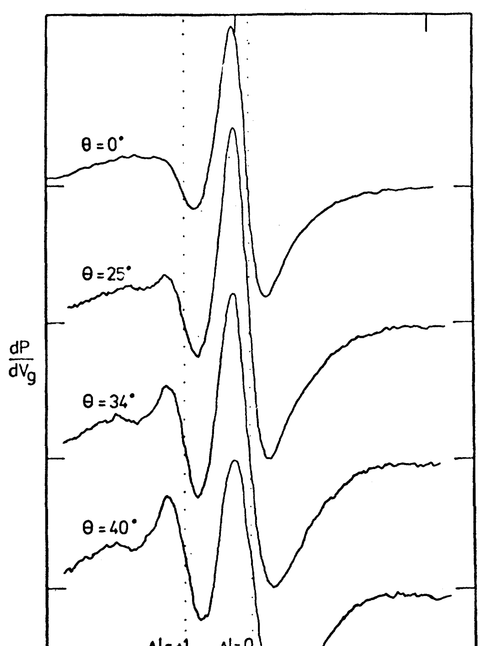


FIG. 2. Absorption derivative curves for fixed $H_\perp = 5$ T at various tilt angles $\theta$, $T = 4.2$ K and $\hbar\omega = 10.45$ meV. The $\theta = 0°$ curve is identical with the $H = 0$ case. Both the $\Delta l = 0$ intersubband mode and the $\Delta l = +1$ combined resonance remain at fixed $N_s$.

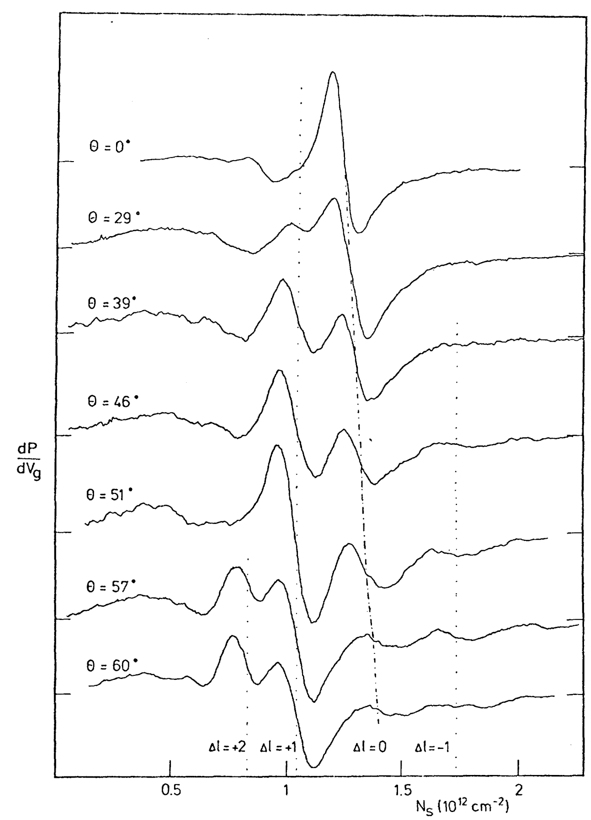


FIG. 3. Absorption derivative curves for $H_\perp = 3.5$ T at various tilt angles. $T = 4.2$ K and $\hbar\omega = 15.81$ meV.

lows a larger range of $\theta$ and thus a more rigorous test of the theoretical description. The $\Delta l = 0$ intersubband peak remains at fixed $N_s$ only for small $\theta$. With increasing $H_\parallel$ it shifts to higher $N_s$, while its amplitude decreases. When it disappears at large $\theta$, it is located symmetrically between the $\Delta l = \pm 1$ peaks. The shift is much larger and opposite to what can be ascribed to the diamagnetic energy terms. The amplitude decrease is a "tuning out" of the resonance as the spatial overlap between a pair of equivalent Landau wave functions diminishes with rising $\Delta y$. In the absence of any published theoretical description of such a shift we can only suggest a possible reason. Depolarization and the excitonlike shift are function of the oscillator strength and vanish with decreasing resonance amplitude. In the limit where the excitation becomes very weak the resonance represents the true energy separation of the levels. This point remains to be verified.

The combined resonance $\Delta l = +1$, and for larger $H_\parallel$ also a $\Delta l = -1$ peak, are identified in the data of Fig. 3. Note the asymmetric placement of the strong $\Delta l = 0$ peak at $\theta = 0$ relative to the fixed $\Delta l = \pm 1$ positions. Estimating the energy separations as before, $\Delta l = +1$ occurs $0.7\hbar\omega_c$ displaced to lower $N_s$ while $\Delta l = -1$ is found at $1.6\hbar\omega_c$ above. The sum of the separations is expected[8,9] to be $2.0\hbar\omega_c$ instead of our estimate $2.3\hbar\omega_c$. The difference is more than possible errors in the location of the $\pm 1$ peaks. A likely source of the discrepancy is the fact that the $\tilde{E}_{01}(N_s)$ curve[2] and not $E_{01}(N_s)$ has been used for the energy estimate. A more accurate check can only be achieved in a frequency-sweep experiment. Proceeding as before we estimate $\gamma = 0.08$ at $N_s = 10.4\times10^{11}$ cm$^{-2}$ and $\gamma = 0.16$ at $N_s = 17.2\times10^{11}$ cm$^{-2}$.

Figure 3 shows at large $\theta$ an additional strong peak, which we have labeled $\Delta l = +2$. Its dependence on $H_\perp$ in the limited range 2.5–3.5 T suggested to us a higher-order combined resonance. The estimated energy separation of the $+2$ and $+1$ peaks is only $0.8\hbar\omega_c$, i.e., less than the expected $\hbar\omega_c$. Ando has privately pointed out that, in the presence of a field, the subband resonance $\tilde{E}_{02}$ occurs at about the same energy and may make a distinct contribution to the spectrum.

A great deal more experimental and theoretical work remains to be done in order to provide a precise, quantiative description of all the effects. It will follow in due course. What the present work has demonstrated is that combined resonance in an interacting, high-density electron gas is fundamentally and characteristically different from the dilute, clean limit of the electrons on helium. It has been shown experimentally that, for electron accumulation layers on Si(100) at densities in the $10^{12}$-cm$^{-2}$ range, the polarization-exciton shfit parameter $\gamma$ is finite, positive, and of order of magnitude 0.1. The observation on the shift of the intersubband mode suggest an experimental way to obtain the subband spacing directly by, so to speak, "tuning out" the resonance.

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