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Empirical Renormalization of the One-Body Gamow-Teller β-Decay Matrix Elements in the $1s$-$0d$ Shell

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β-decay data from nuclei with $17 \leq A \leq 39$ are analyzed with full *sd*-space shell-model wave functions to determine the effective values of the single-particle matrix elements of the Gamow-Teller operator between all combinations of *sd*-shell orbitals.

A fundamental problem of nuclear physics is to understand the effects which the nuclear medium has, via mesonic exchange currents, upon the properties of the individual constituent nucleons of the nucleus. It is also of basic importance to understand the effects introduced by the necessity of treating the many-nucleon system with model-space truncations. Discussions of these effects are based on comparisons of experimental results to predictions based on the assumptions of the classical nuclear shell model and the properties of the neutron and proton in free space. Because of the difficulties inherent in treating many active particles in the framework of the shell model, study of these problems has to date been confined primarily to "single-particle" nuclei such as (to take examples from the *sd* shell) ^{17}O, ^{17}F, ^{39}Ca, and ^{39}K. Our attention here is focused upon possible mesonic and truncation effects in Gamow-Teller (GT) β decay in the region $17 \leq A \leq 39$. We have been able, by taking advantage of recent advances in shell-model calculations,[1,2] to analyze data[3] from *all* the *sd*-shell nuclei with the same model approximations which are conventionally used for the single-particle systems, and can thereby base our conclusions upon a much larger data base than is provided by the $A=17$ and 39 decays alone.

Following the pioneering work in this field of Wilkinson,[4,5] we express the ft value for β decay as

$$ft_{1/2}(1+\delta_R)=\frac{C}{B(\mathrm{F})(1-\epsilon)+B(\mathrm{GT})}, \tag{1}$$

where $B(\mathrm{F})^{1/2}$ is the Fermi decay matrix element, $B(\mathrm{GT})^{1/2}$ is the Gamow-Teller decay matrix element, $t_{1/2}$ is the partial half-life, f is the integrated Fermi function, and δ_R is the model-independent outer radiative correction up to third order in the fine-structure constant.[3] The constant $C=6177\pm14$ is chosen to reproduce the $0^+\to0^+$ Fermi transitions in the sd shell. The factor ϵ is a correction for the lack of isobaric symmetry,[5] taken to have the value $1-\epsilon=0.995\pm0.003$.

We assume that the GT operator is an arbitrary *one-body* operator of rank $\lambda=(\Delta J, \Delta T)=(1,1)$. The theoretical $B(\mathrm{GT})^{1/2}$ matrix elements in the many-nucleon systems can then be reduced to linear combinations of triple-barrel (i.e., reduced in angular momentum and isospin) single-particle matrix elements (SPME) $\langle j|||O_{\mathrm{GT}}|||j'\rangle$ via the one-body transition densities (OBTD)

$$\frac{\langle\psi_f|||(a_j{}^\dagger\otimes a_{j'})_\lambda|||\psi_i\rangle}{[(2\Delta J+1)(2\Delta T+1)]^{1/2}}$$

for given initial- and final-state wave functions. This yields

$$B(\mathrm{GT})_{\mathrm{th}}{}^{1/2}=\frac{1}{[2(2j_i+1)]^{1/2}}\begin{pmatrix}T_f & 1 & T_i\\ -T_{zf} & \Delta T_z & T_{zi}\end{pmatrix}\sum_{jj'}\frac{\langle\psi_f|||(a_j{}^\dagger\otimes a_{j'})_\lambda|||\psi_i\rangle}{[(2\Delta J+1)(2\Delta T+1)]^{1/2}}\langle j|||O_{\mathrm{GT}}|||j'\rangle. \tag{2}$$

In the present analysis we generate theoretical GT matrix elements with OBTD[3] calculated from the multiplarticle wave functions of Chung and Wildenthal.[2] These wave functions were obtained by diagonalizing empirically established Hamiltonians in the complete space of $d_{5/2}$-$s_{1/2}$-$d_{3/2}$ configurations. The use of the full sd-shell space is critical for this investigation because the GT matrix elements are sensitive to the ratio of $d_{5/2}$ to $d_{3/2}$ occupancy. We assume that these OBTD values correctly represent the model-space wave functions of the states involved, analogously to the conventional assumption of unit values of the OBTD for the single-particle nuclei of the model space, $A=17$ and 39. Mesonic and/or extra-sd-shell configuration-admixture corrections which can be expressed in terms of effective one-body operators then enter the scheme of Eq. (2) as renormalizations of the conventional or "free-nucleon" values of the SPME of the GT operator. These "free-nucleon" values are given by the expression

$$\langle J|||O_{\mathrm{GT}}|||j'\rangle_{\mathrm{free}}=|g_A/g_V|\langle j|||\sigma\tau|||j'\rangle, \tag{3}$$

where $|g_A/g_V|=1.251\pm0.009$ (Ref. 5), as determined from the beta decay of the free neutron.

We have obtained empirical estimates for these renormalizations, i.e., the "effective" values of the five SPME which can enter Eq. (2), by treating the SPME as the variables in a least-squares fit of the $B(\mathrm{GT})_{\mathrm{th}}{}^{1/2}$ to corresponding experimental values taken from selected sets of data. The data used are all or portions of a set of 54 relatively large experimental matrix elements which were selected on a strength criterion similar to that of Wilkinson[5] from all (about 200) of the available values of logft for GT transitions in the region $17\leq A\leq39$. These values of $B(\mathrm{GT})_{\mathrm{expt}}{}^{1/2}$ were calculated[3] via Eq. (1) from the latest experimental information on lifetimes, branching ratios, and Q values.

Our results are summarized in Table I and Fig. 1. Illustrated in Fig. 1 is the degree to which a representative subset of the 54 selected data (the mirror transitions in the sd shell) is reproduced by the shell-model DBTD in conjunction with either the free-nucleon or the effective GT SPME. The variations in the theoretical values originate in the different OBTD, of course; the SPME values are A independent. Obviously, the shell-model wave functions together with the *free-nucleon* (dashed-line predictions) SMPE give a good qualitative accounting of observed absolute magnitudes and a close reproduction of observed relative magnitudes.

The present shell-model wave functions yield this same type of agreement not only with these and the remaining comparable GT transition data but also[2] with the gamut of data from the other usual observables of nuclear spectroscopy. This leads us to conclude that, within the assumed model-space context, the wave functions (and hence the OBTD) yield good descriptions of the many-body structure of these nuclear states. This in turn motivates us to test the hypothesis

TABLE I. Values of the single-particle matrix elements (SPME) of the GT operator as empirically determined from fits to multinucleon decay data and as calculated from the free-nucleon approximation.

Mass region	number of data	R[a]	Δ[a]	$\langle j\|\|\|0_{GT}\|\|\|j'\rangle$ 2j-2j'=5-5	1-1	5-3	3-3	1-3
				("free nucleon" assumption)				
17-39	54	1.0	0.204	8.88	7.51	-9.50	-4.75	0
				fitted empirical values				
17	1	0.878		7.79(6)[b]				
39	1	0.675					-3.20(4)[b]	
17-39	54	0.89	0.051	7.52(12)[c]	6.08(17)	-6.57(15)	-3.54(14)	-0.15(15)
17-39	54	0.89	0.053	7.49(11)	6.10(16)	-6.59(15)	-3.53(14)	[0][d]
17-23	23	0.88	0.060	7.58(18)	5.49(52)	-6.79(28)	[-3.54]	[0]
23-33	24	0.94	0.043	7.39(15)	5.97(23)	-6.34(23)	-3.88(28)	[0]
33-39	16	0.82	0.044	[7.52]	5.90(36)	-6.67(27)	-3.50(13)	[0]

[a]The definitions of R and Δ are

$$R \equiv \frac{1}{n}\sum_{i=1}^{n}[B(\mathrm{GT})_{\mathrm{expt}}/B(\mathrm{GT})_{\mathrm{th,free}}]^{1/2},$$

$$\Delta \equiv \text{rms of } |B(\mathrm{GT})_{\mathrm{th}}{}^{1/2} - B(\mathrm{GT})_{\mathrm{expt}}{}^{1/2}|.$$

[b]Experimental error only.

[c]Fit error only.

[d]A parameter enclosed by brackets [ ] was held fixed to that value in that search.

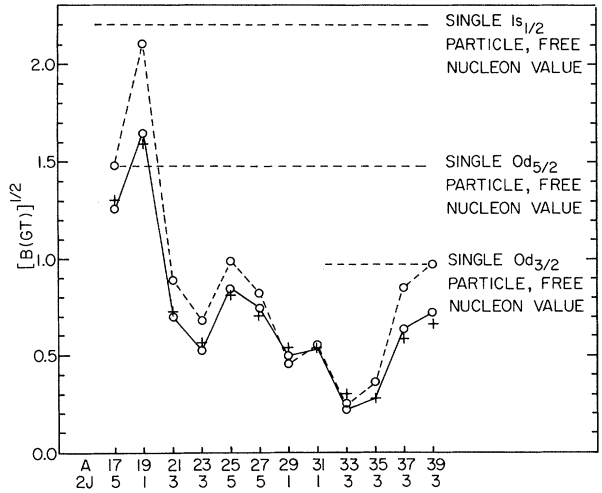

FIG. 1. Values of $[B(\mathrm{GT})]^{1/2}$ for mirror nuclei. The crosses denote experimental points do not attempt to indicate the small experimental errors. The dashed line connects the shell-model results which use free-nucleon single-particle matrix elements. The solid line connects the shell-model results which use fitted single-particle matrix elements obtained from the five-parameter fit to 54 data between $A = 17$ and $A = 39$.

that some of the remaining deviations between experiment and shell-model/free-nucleon predictions can be eliminated by renormalizations of the SPME. This hypothesis seems to be validated by our results: *Effective* SPME values, which result from changes which are small and stable with respect to different data sets, yield a significant improvement in agreement between theory and experiment, as shown by the summaries of Table I and the solid-line predictions in Fig. 1.

A detailed inspection of the renormalizations imposed upon the various SPME by the multiparticle data shows that the effective "$A = 17-39$" $d_{5/2}$-$d_{3/2}$ matrix element is 3.5% smaller than the "$A = 17$" value and 15% smaller than the "free" value. The absolute value of the effective "$A = 17-39$" $d_{3/2}$-$d_{3/2}$ matrix element is 10% larger than that of "$A = 39$" but 26% smaller than the "free" value. These results suggest that effects from "deformed-state admixtures," which should be largest at the very beginning or very end of the *sd* shell, are appreciable for $A = 39$ but not for $A = 17$ and that in both cases the larger part of the deviations from the "free" values comes from a relatively state-independent "average" alteration of nucleon properties.

The value obtained for the "l-forbidden" $s_{1/2}$-$d_{3/2}$ matrix element is small and consistent with zero when this degree of freedom is included in the fit. The effective values obtained for the $s_{1/2}$-$s_{1/2}$ and $d_{5/2}$-$d_{3/2}$ matrix elements are similar for the various data sets considered. The values from the $A = 17-39$ fits are 19% and 31% smaller, respectively, than the "free" values.

Wilkinson has previously made an extensive analysis[5] of GT decay for $A \leq 22$ by considering an average state-independent renormalization (ρ) which is the same as our R in Table I. Our present results for the region $17 \leq A \leq 23$ ($R = 0.89$) are identical to those he obtained. His use of wave functions obtained from a different effective Hamiltonian[6,7] provides a reassuring inference that conclusions from this sort of study are not overly sensitive to the detailed aspects of the shell-model analysis. The assumption of an orbit-independent renormalization, such as R, introduces attractive simplicities into the analysis of the data and into the interpretation of the results, along the lines of a renormalization of g_A. However, the totality of the data unambiguously requires orbit-*dependent* renormalizations.

We conclude with some comments on how our results can be related to theoretical calculations of mesonic[8-12] and major-shell configuration-mixing[13,14] effects. It must be realized that our results represent the *composite* renormalizations in the context of effective *one-body* operators. In general we have no method for distinguishing between mesonic and core-polarization effects in the present type of data; current theoretical work typically concentrates upon only one or the other. When theoretical results are not couched in terms of one-body effects, and to some extent the processes can be intrinsically two-body in nature, we can only note the degree to which the assumption of a constant one-body operator accounts for experimental observation.

Complete calculations for the effects of either configuration mixing or mesonic currents have been reported only for the $d_{5/2}$-$d_{5/2}$ and $d_{3/2}$-$d_{3/2}$ GT SPME. Shimizu *et al.*[13] and Arima *et al.*[14] have found large configuration-mixing effects originating in the tensor force. The $2\hbar\omega$ contribution was calculated with the Kuo-Brown G matrices,[13] yielding reductions of 4% and 6%, respectively, to the $d_{5/2}$ and $d_{3/2}$ "free" values. The Hamada-Johnston (HJ) potential was then used to estimate contributions up to $\infty\hbar\omega$ (Refs. 13 and 14), yielding final 12% and 24% reductions, respectively, to the "free" values. The renormalizations predicted from this approach are thus quite large and are, by themselves, as big as our empirical values, which are $(-12\pm2)\%$ and $(-22\pm3)\%$ respectively, when corrected[10] for relativistic effects.

The nuclear Lorentz-Lorenz effect in π-nucleus scattering is the basis for estimates by Rho[9] of 15% and 20% reductions of mesonic origin in $A=24$ and $A=40$, respectively. Again, reductions of this magnitude are by themselves consistent with the empirical results. On the other hand, alternative procedures for estimating mesonic effects lead Blin-Stoyle and Barroso[10,11] to predict small [(1–5)%] alterations, usually *enhancements*, to the Gamow-Teller strengths for the $d_{5/2}$ and $d_{3/2}$ terms.

A further evolution in the thoery of renormalization effects thus appears necessary before unambiguous tests via our empirical results become possible. Mutually consistent treatment of the mesonic and configuration-mixing effects, which at the deepest level may not be completely distinct,[4,5] are desirable, as is a more complete working out of the different sources of mesonic effects and of the extent to which they can be translated into the formulation of one-body operators. Obviously, a complete test of these theories will require comparisons, and hence calculations, for all of the SPME of the space, including the "l-forbidden" term.

Completely different types of experimental information, such as magnetic inelastic electron scattering, ultimately may provide more powerful tools for studying these problems. In the context of the present approach, the closely related phenomena of static magnetic moments provides further insight into the sources of renormalization when put into conjunction with the present Gamow-Teller results. An analysis of the sort reported here has been made of all the magnetic moments in the sd shell. The same shell-model wave functions were employed and the *empirically effective* single-particle matrix elements of the $M1$ operator obtained, as was here done for the GT operator.

The $\sigma\tau$ component of the isovector $M1$ SPME has thus been analyzed with the same OBTD values as were used to obtain the effective GT SPME. The configuration-mixing renormalizations to the wave functions are the same, of course, but the mesonic corrections for the vector $M1$ and axial-vector GT operators need not be identical. Comparison between the GT and $M1$ renormalization values is straightforward for the $s_{1/2}$ SPME, in which the $l\tau$ component of the $M1$ operator does not contribute. From Table I we note a reduction for the free-nucleon GT SPME of $\delta_{\mathrm{GT}}{}^{\sigma\tau}(s_{1/2})=(19\pm2)\%$. This is to be compared to an *enhancement* of $\delta_{M1}{}^{\sigma\tau}(s_{1/2})=(3\pm3)\%$ (Ref. 2) determined from the magnetic moment data. The difference, $\delta_{\mathrm{GT}}{}^{\sigma\tau}(s_{1/2})-\delta_{M1}{}^{\sigma\tau}(s_{1/2})=(-22\pm4)\%$, should originate from a difference in the mesonic corrections for the two operators. A similar analysis was recently reported for the $0p$ shell[15] in which $a\simeq-20\%$ difference also was found between GT and $M1$ matrix elements.

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Observation of Oscillations in the Charge Dependence of Total Electron-Capture Cross Sections

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An oscillatory dependence on ionic charge of total electron-capture cross sections has been observed for multiply charged heavy projectiles incident on an atomic hydrogen target. The oscillatory behavior is attributed to interference caused by the long-range Coulomb and a short-range (screened Coulomb) force acting on the captured electron. The oscillations are superimposed on a monotonically increasing variation with ionic charge. This monotonic increase is also found for lighter ions.

Total cross sections for single-electron capture by fast multiply charged projectiles incident on various gas targets have been measured by numerous investigators.[1-5] For collision velocities roughly in excess of the Bohr velocity, the previously observed cross sections $\sigma_{q,q-1}$ increase smoothly with projectile charge q, except for sharp dips that occur at projectile charge states corresponding to closed-shell configurations of the incident ion. Such a monotonic increase with q is predicted by a number of theories,[6-9] including that of Oppenheimer, Brinkman, and Kramers.[10] During our measurements of electron capture cross sections for fast ($v > 2 \times 10^8$ cm/s), multiply charged ions of C, N, O, Si, Fe, Mo, Ta, W, and Au incident on atomic and molecular hydrogen, an unexpected oscillatory deviation from this monotonic dependence on q was observed for the heavier projectiles (Ta, W, and Au). Such oscillatory behavior has not been previously reported in the literature.

The experimental techniques have been discussed in detail in previous reports.[11,12] A magnetically analyzed heavy-ion beam of the desired energy was obtained from the Oak Ridge National Laboratory's model EN tandem Van de Graaff accelerator, collimated, and further stripped by passage through a foil or gas stripper. The desired charge state was selected by 20° magnetic analysis and directed through the atomic or molecular hydrogen target. The various charge states in the beam emerging from the target were dispersed by a pair of electrostatic plates onto a solid-state position-sensitive detector. Reference 11 gives details of the implementation and calibration of the atomic hydrogen target.

The projectile-charge dependences of the present cross sections for the lighter projectiles Si^{+q}, Fe^{+q}, and Mo^{+q} incident on H and H_2 are similar to previous results for various projectile-target combinations at comparable collision velocities.[1-5,11] Data for Mo^{+q}+H, shown in Fig. 1, exhibit the features typical of the lighter projectiles. At a given velocity, the cross sections increase smoothly with q and can be represented by a power law $\sigma_0 q^p$ (where σ_0 and p are fitted parameters). Deviations from this behavior in the form of sharp dips at Mo^{+6} ($4p^6$) and Mo^{+14} ($3d^{10}$) closed-shell configurations are similar to those in previously reported data.[1-4,13]

The q dependences of the cross sections for the heavier projectiles are more complicated: Superimposed on a monotonic increase with q (as observed in the lighter projectile data), all three of the heavier projectiles investigated (Ta, W, and Au) exhibit a common oscillatory structure that is apparently independent of the ion species. Representative data for Ta^{+q} incident on H are shown in Fig. 1. To emphasize these deviations, the measured cross sections divided by the least-