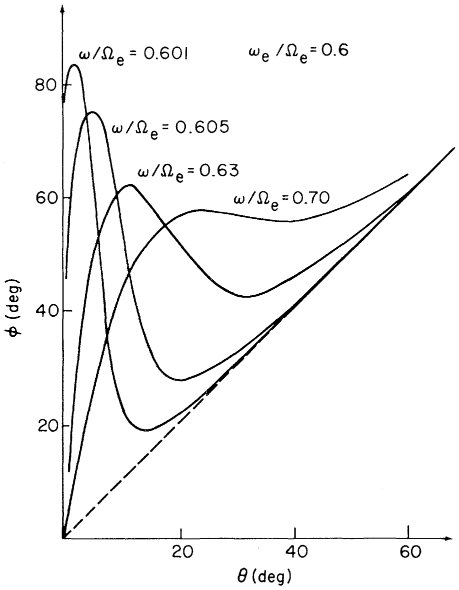

FIG. 3. Graph of φ (defined as the direction of energy transport with respect to $\vec{B}_0$) vs θ for $\omega_e/\Omega_e = 0.6$ and several values of the frequency.

direction of the group velocity. If we let φ denote the angle between the group velocity and the ambient magnetic field, then it can be shown that[12]

$$\tan(\theta-\varphi)=\frac{2\alpha^2(1-\alpha^2)\sin\theta\cos\theta}{\rho[2(1-\alpha^2)-\beta^2(\sin^2\theta+\rho)]}. \tag{11}$$

A plot of φ versus θ is presented in Fig. 3 for parameters used in the calculation of the emissivity. Evidently, φ ranges between about 70° and 83° for frequencies which correspond to the high levels of emission, and we conclude that the direction of energy transport is predominantly perpendicular to the ambient magnetic field. As a result, the radiation should be easily detectable to an observer looking into the plasma along the direction perpendicular to the magnetic field.

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Effect of Magnetic Shear on Dissipative Drift-Wave Instabilities

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We report the results of a linear radial-eigenmode analysis of dissipative drift waves in a plasma with magnetic shear and spatially varying density gradient. The results of the analysis are shown to be consistent with a recent experiment concerned with dissipative drift-wave instabilities in a toroidal stellarator.

Recently Vojtsenya *et al.*[1] have reported the results of an interesting experiment concerned with the effect of magnetic shear on dissipative drift-wave instabilities in a collisionally dominated plasma in a toroidal stellarator. Their results indicate that drift waves are localized near the maximum of the drift-wave frequency ω_* (the variation of ω_* arising due to a spatially varying density gradient) and are strongly reduced in amplitude when the shear in the magnetic field exceeds a critical value. In this Letter we present the results of a linear radial-eigenmode analysis[2] of dissipative drift waves in a plasma with magnetic shear and a spatially varying density gradient, and propose a possible interpretation of the above experiment.

The local theory of dissipative drift waves is well known (see, e.g., Kadomtsev[3]); this theory is applicable as such only to shearless situations and with no spatial variation of the drift-wave frequency ω_*. Moiseev and Sagdeev[4] have carried out a radial-eigenmode analysis including the spatial variation of ω_* but with no magnetic shear. The inclusion of shear strongly modifies their results. Further, the previous analyses[5,6] with magnetic shear have been rather qualitative and incomplete.

We consider a plane plasma slab with the density variation and magnetic shear along x direction; i.e., $n_0(x)=n_0(0)\exp[x/L_n(1-x^2/3L_*^2)]$ and $\vec{B}(x)=B_0(\vec{1}_z+\vec{1}_y x/L_s)$. Here L_s, L_n, and L_* are the shear length, the density scale length, and the scale length for variation of density gradient, respectively. $x=0$ is taken as the mode-rational surface; i.e., surface on which $\vec{k}\cdot\vec{B}=0$. For simplicity, we have assumed the density gradient to peak at $x=0$. As usual, we shall assume electron thermal conductivity parallel to the lines to be very large to justify using isothermal electrons and approximate the ions as a cold species ($T_i=0$). If we use the linearized equations of continuity and motion for electrons and ions, take perturbations of the form $\varphi(\vec{x},t)=\varphi(x)\exp[i(k_y y-\omega t)]$, and finally assume the quasineutrality condition $n_e\approx n_i$, we obtain the eigenmode equation

$$\rho_s^2\left(\frac{d^2}{dx^2}-k_y^2\right)\varphi-\frac{x^2}{x^2-ix_R^2}\left[1-\frac{\omega_{*0}}{\omega}\left(1-\frac{x^2}{L_*^2}\right)-\frac{x^2}{x_s^2}\right]\varphi=0, \tag{1}$$

where

$$k_\parallel=k_\parallel' x\equiv k_y x/L_s,\quad x_s^2=\omega^2/k_\parallel'^2C_s^2,\quad \omega_{*0}=k_y\rho_s C_s/L_n,$$

$$x_R^2=\omega\nu_{ei}/k_\parallel'^2 v_{eT}^2,\quad v_{eT}^2=T_e/m_e,\quad C_s^2=T_e/m_i,\quad \rho_s^2=C_s^2/\omega_{ci}^2,$$

and the other symbols have obvious meanings.

The eigenmode equation is unchanged if instead of using fluid equations we use a drift kinetic equation for electrons and take account of electron-ion collisions by a number-conserving Krook operator and work in the limit $\nu_{ei}>\omega, k_\parallel v_{eT}$. In the weakly collisional limit,[7] other collision operators may give somewhat different results but this does not seem to be true in the strongly collisional limit $\nu_{ei}>\omega, k_\parallel v_{eT}$. The first term comes from the ion-polarization drift, the coefficient $x^2/(x^2-ix_R^2)$ is a consequence of the resistive parallel dynamics of electrons and the x^2/x_s^2 term has its origin in the parallel ion motion. Moiseev and Sagdeev[4] solved Eq. (1) in the limit $x\ll x_R$, $k_\parallel' x=k_\parallel=$const (no shear), and $x_s=\infty$ (ignoring the parallel ion dynamics). For $x_s\to\infty$, $x\ll|x_R|$, and ω_{*e} constant, Coppi[8] has indicated that no growing eigenmodes exist for this equation.

We first examine the eigenvalue problem for $L_*=\infty$; i.e., ω_* independent of x. After changing variables and rearranging the terms, Eq. (1) may be rewritten as

$$\left(\frac{d^2}{d\xi^2}+\delta-\frac{\xi^2}{4}-\frac{\Lambda}{\xi^2+\xi_R^2}\right)\varphi=0, \tag{2}$$

where

$$\xi=x\exp(i\tfrac{1}{4}\pi)/\lambda,\quad \lambda^2=\tfrac{1}{2}\rho_s x_s,\quad \xi_R^2=x_R^2/\lambda^2,$$

$$\delta=i(\lambda^2/\rho_s^2)(1-\omega_*/\omega+k_y^2\rho_s^2-ix_R^2/x_s^2),$$

and

$$\Lambda=i(x_R^2/\rho_s^2)(1-\omega_*/\omega-ix_R^2/x_s^2).$$

When $\xi_R^2=2(m_e/m_i)(\nu_{ei}/\omega_*)(L_s/L_n)\ll1$, the eigenvalue problem associated with Eq. (2) may be solved by the following matching procedure. In the outer region $\xi\gg\xi_R$ [so ξ_R may be neglected in Eq. (2)] and the solution which properly decays away for $x\to\infty$ and $\mathrm{Im}\,\omega>0$ is given by

$$\varphi_0=A_0(\tfrac{1}{2}\xi^2)^{(2\alpha+1)/4}U(a,b,\tfrac{1}{2}\xi^2)\exp(-\tfrac{1}{4}\xi^2), \tag{3}$$

where U is Kummer's confluent hypergeometric function[9] and $\alpha=-\frac{1}{2}(1+4\Lambda)^{1/2}$, $b=1+\alpha$, and a

$=\frac{1}{2}(b-\delta)$. In the inner region $\xi \sim \xi_R \ll 1$ and the solution may be approximated as

$$\varphi_I = (1+\xi^2/\xi_R^2)^{1/2}[A_I P_\nu^1(\xi/i\xi_R) + B_I Q_\nu^1(\xi/i\xi_R)], \tag{4}$$

where the P_ν^1 and Q_ν^1 are the associated Legendre functions and the order $\nu = -(\frac{1}{2}+\alpha)$. The constants B_I and A_I are related by the parity condition at the origin, viz, $\varphi_I(0)=0$ for odd-parity modes and $d\varphi_I/d\xi\,|_{\xi=0}=0$ for even-parity modes. Matching the outer-region solution φ_0 with $\xi \ll 1$ to the asymptotic form of the inner-region solution (for $1 \gg \xi \gg \xi_R$), one gets the eigenvalue condition

$$\frac{\Gamma(a)}{\Gamma(a+\nu+\frac{1}{2})} = \left(-i\,\frac{\xi_R}{2^{3/2}}\right)^{1+2\nu} \frac{\Gamma(\nu)\Gamma(-\frac{1}{2}-\nu)\Gamma(\frac{1}{2}-\nu)}{\Gamma(-1-\nu)\Gamma(\frac{1}{2}+\nu)\Gamma(\frac{3}{2}+\nu)}\left(1+\pi\,\frac{B_I}{A_I}\cot\pi\nu\right), \tag{5}$$

where Γ denotes the usual gamma function. Note the definitions of ν, a, and other related quantities following Eqs. (4), (3), and (2), respectively.

The eigenvalue condition (5) is, in general, difficult to solve analytically for the complex frequency ω. However, approximate results may be obtained in certain interesting limits. When $|\Lambda| \simeq \frac{1}{2}k^2\rho_s^2(L_s/L_n)|\xi_R^2| \ll 1$, one finds $\nu \simeq \Lambda$. Evaluating B_I/A_I for even-parity modes, one obtains the approximate eigenvalue condition

$$a \simeq -n + (-1)^{n+1}2^{-3/2}\,\frac{\pi\Lambda}{\xi_R\Gamma(\frac{1}{2}-n)}, \qquad n=0,1,\ldots.$$

For the lowest mode we then obtain the dispersion relation

$$1-\frac{\omega_*}{\omega} \cong -\left(i\,\frac{L_n}{L_s}+k_y^2\rho_s^2\right)\left[1+\left(\pi\,\frac{\nu_{ei}}{\omega_*}\frac{m_e}{m_i}\frac{L_s}{L_n}\right)^{1/2}\right] = \frac{\delta\omega}{\omega_*}. \tag{6}$$

Equation (6) leads to the surprising result that in a sheared geometry, resistivity only *enhances* the shear damping of drift waves by a factor proportional to $\nu_{ei}^{1/2}$ and that no unstable mode exists. A detailed analysis[2] shows that the expected growth terms are proportional to ν_{ei} and are always subdominant to the resistivity-induced enhancement of shear damping. For $k^2\rho_s^2 \gg L_n/L_s$, the other simple and instructive limit $|\Lambda| \gg 1$ can be used. Here again one finds for the lowest mode $\nu \simeq \Lambda^{1/2}-\frac{1}{2}$, $a \simeq -\Lambda^{1/2}$ and the approximate dispersion relation

$$1-\frac{\omega_*}{\omega} \simeq -k_y\rho_s\left[k_y\rho_s+(1+i)\left(2\,\frac{m_e}{m_i}\frac{\nu_{ei}}{\omega_*}\right)^{1/2}\right] = \frac{\delta\omega}{\omega_*}. \tag{7}$$

Even in this limit, no unstable eigenmodes exist. Furthermore, it has been shown[2] that for $\Lambda \gg 1$ and $\xi_R^2 > 1$, a WKB analysis predicts the same eigenvalue condition as Eq. (7).

Our analysis thus leads us to the surprising result that for $2m_e\nu_{ei}L_s/m_i\omega_*L_n \ll 1$, electrostatic drift waves in a plane plasma slab with magnetic shear only exist as damped eigenmodes. We have verified this result by a direct numerical integration of the differential equation (2) using a shooting method and also by a complex WKB treatment.[2] Figure 1 gives a plot of the numerically obtained collisional enhancement of the damping rate $\bar{\gamma}/\omega_* = \gamma/\omega_* - L_n/L_s$ versus the electron-ion collision frequency ν_{ei}/ω_*. The analytical results in various limits are shown by dotted lines; the agreement is very good. The numerical calculations also show that the modes are stable even in the range $2m_e\nu_{ei}L_s/m_i\omega_*L_n > 1$.

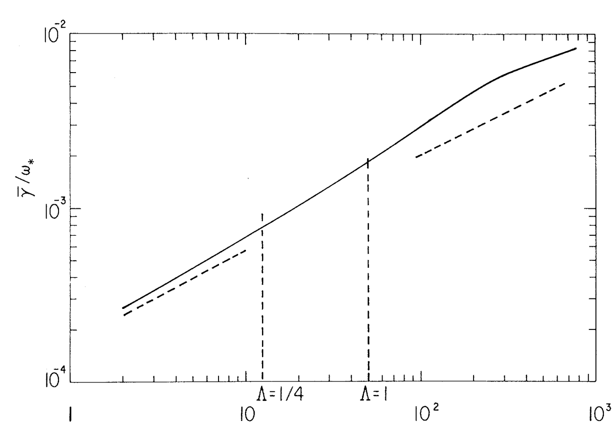

FIG. 1. Collision-enhanced damping rate $\bar{\gamma}/\omega_* = \gamma/\omega_* - L_n/L_s$ vs electron-ion collisional frequency ν_{ei}/ω_* for $k_y^2\rho_s^2 = 0.02$, $m_e/m_i = 10^{-4}$, and $L_s/L_n = 100$.

None of the above investigations, however, forbids a convective amplification of wave packets of drift waves.[10] Physically the eigenmodes are damped because of a resistivity-induced enhancement of shear damping. An obvious way to recover growing eigenmodes is thus to make shear damping ineffective. This may be done by introducing a proper profile for ω_*, i.e., by discussing the case with $L_* \neq \infty$.

Equation (1) may be written as

$$\left(\frac{d^2}{d\eta^2}+\delta_0-\frac{\eta^2}{4}-\frac{\Lambda_0}{\eta^2-i\eta_R^2}\right)\varphi(\eta)=0, \tag{8}$$

where

$$\eta = x/\lambda_0, \quad \lambda_0^2=\tfrac{1}{2}x_0\rho_s, \quad x_0^{-2}=(\omega_{*0}/\omega)L_*^{-2}-x_s^{-2}, \quad \eta_R=x_R/\lambda_0,$$

$$\delta_0=-(\lambda_0/\rho_s)^2(1-\omega_{*0}/\omega+k_y^2\rho_s^2+ix_R^2/x_0^2), \quad \Lambda_0=i(x_R/\rho_s)^2(1-\omega_{*0}/\omega+ix_R^2/x_0^2).$$

To nullify shear damping, we must require that $\mathrm{Re}\,x_0^{-2}>0$; i.e.,

$$L_*^2<x_s^2\omega_{*0}/\omega. \tag{9}$$

For $|\eta_R|^2 \ll 1$, we can use the same matching procedure as above. The analog of Eq. (6) is

$$1-\frac{\omega_{*0}}{\omega}\simeq-\left(k_y^2\rho_s^2+\frac{\rho_s}{x_0}\right)\left[1+\frac{\pi^{1/2}}{2}(1-i)\left(\frac{\nu_{ei}}{\omega_{*0}}\frac{m_e}{m_i}\frac{L_s^2}{L_n^2}\frac{\rho_s}{x_0}\right)^{1/2}\right]. \tag{10}$$

Equation (10) shows that shear damping is absent and that resistivity contributes to growth of the eigenmode. For shorter wavelengths, the analog of Eq. (7) takes the form

$$1-\frac{\omega_{*0}}{\omega}\simeq-k_y^2\rho_s^2+(i-1)k_y\rho_s\left(\frac{\rho_sL_s}{x_0L_n}\right)\left(\frac{\nu_{ei}}{\omega_{*0}}\frac{m_e}{m_i}\right)^{1/2} \tag{11}$$

which again shows an eigenmode growth rate $\propto \nu_{ei}^{1/2}$. It is clear that inequality (9) is the critical condition for the existence of growing eigenmodes. When it is not obeyed, we again revert to the previous case of shear damping only.

The experiment of Vojtsenya *et al.*[1] was done on an argon plasma in a stellarator with typical parameters: $T_e \approx 6$ eV, $n_e \approx 2\times10^{11}$ cm^{-3}, $T_e \gg T_i$, $B_0=3.1$ kG. This justifies[3] the use of an electrostatic drift-wave model to describe their experiment, since $\beta=8\pi nT/B_0^2<m_e/m_i$. Our assumption of cold ions is also obviously good. The experiment demonstrated the excitation of dissipative drift waves ($\omega \sim \omega_* < \nu_{ei}$) localized in the neighborhood of a maximum of ω_*. It was also discovered that when the shear parameter $\theta=L_n/L_s$ exceeded 0.05, a substantial reduction in the level of drift-wave oscillations takes place. We now propose that the transition from a growing eigenmode to a stable one may lead to a similar reduction in the oscillation level. In the former case, the instability keeps growing until nonlinear effects saturate the waves. In the latter case, there is only convective amplification of waves and they may saturate even at low levels, by simply convecting out of the unstable region. In the experiment, the transition can occur because inequality (9) switches as $\theta=L_n/L_s$ is increased. The condition of shear stabilization of density-gradient-localized drift waves may thus be written $L_*/\rho_s>\Delta\theta^{-1}$, where Δ is a numerical factor taking account of the fact that (i) the experimental density-gradient variation is not parabolic, (ii) the transition to a *low*-saturation convective mode does not occur exactly at $L_*^2=x_s^2\omega_{*0}/\omega$, (iii) the experimental situation may not be well represented by a one-dimensional slab model, and (iv) there may be inaccuracies in our estimates of various parameters. For $L_* \cong 2$ cm and the other experimental parameters given above, we find agreement with the experimental value of $\theta_c = 0.05$, if we choose $\Delta \sim \frac{1}{5}$. Considering the above uncertainties, this seems pretty reasonable.

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Laser Fusion Experiments at 4 TW

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DT-filled glass microspheres have been imploded at power levels exceeding 4 TW using the Lawrence Livermore Laboratory 1.06-μm Argus laser. Thermonuclear neutron yields in excess of 1.5×10^9 have been observed implying a DT burn efficiency of 1.6×10^{-5}. Neutron and α time-of-flight measurements indicate DT burn temperatures of 4–8 keV, implying that a DT gain of approximately 10^{-2} and a $n\tau$ of 10^{12} were obtained.

As part of the effort to understand the physics of laser imploded targets, a series of experiments examining both basic laser-plasma interaction phenomena and the parameter space of exploding-pusher experiments have been performed over the last three years, both at Lawrence Livermore Laboratory and other laboratories.[1-8] A laser irradiated target is said to operate in the exploding-pusher[9] mode when the pusher significantly decompresses in the process of compressing the fuel. This is characteristic of a high rate of energy addition to the pusher. Laser absorption mainly by collective processes producing superthermal electrons, early energy deposition in the hsell by these superthermal electrons, a near supersonic electron thermal wave driven by electron thermal conduction from the laser-absorption region, and significant shock compression of the fuel causing a large entropy change are other characteristics of the exploding-pusher mode. This results in a limited density increase, but significant heating of the fuel. Early exploding-pusher experiments with DT-filled glass microshells indicated that for fixed DT fill, the target performance, measured in terms of neutron yield, should increase[10] as $r_0^{10/3}w^{2/3}\langle\sigma v\rangle T^{-1/2}$. Here r_0 is the target radius, w the shell wall thickness, T the time- and space-averaged final DT fuel temperature, and $\langle\sigma v\rangle$ the Maxwell averaged DT cross section. It is assumed that T is proportional to the useful specific absorbed energy, $\mathcal{E}_C$. The useful fraction is essentially the absorbed energy corrected for any temporal mismatch between the input laser pulse length and the characteristic target-implosion time scale, and is thus the time in the laser pulse beyond which further absorption can no longer influence the final implosion phase. This time is found empirically to be roughly determined by the amount of energy absorbed until the pusher has traversed ~30% of the initial target radius, with the instantaneous pusher velocity assumed proportional to the energy absorbed up to that time. Since pusher velocities are $\simeq(2.5-3.5)\times10^7$ cm/sec,[8,11] a 90-μm-diam target would find nearly all the energy "useful" for laser pulses with a full width at half-maximum (FWHM) $\lesssim$ 40 ps. Experiments performed at KMS Fusion, Inc. also gave results consistent with this description.[12] Targets irradiated at a fixed peak power, with pulse lengths increasing in steps of 40 ps up to 240 ps, showed no increase in neutron yield for pulse lengths beyond 80 ps. Both to confirm the "useful energy" hypothesis and to extend our data