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Spontaneous Emission near the Electron Plasma Frequency in a Plasma with a Runaway Electron Tail

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Spontaneous emission of radiation with frequencies near the electron plasma frequency is studied for a plasma which consists of both thermal and runaway electrons. It is found that a substantial enhancement of the spontaneous radiation intensity can occur in this frequency regime via a Cherenkov resonance with the runaway electrons. Numerical analysis indicates that, for reasonable estimates of densities and energies, the plasma-frequency radiation can attain levels greater than the peak thermal emission at the second gyroharmonic.

A great deal of interest has recently been focused on the question of high-frequency radiation from tokamak plasmas. In particular, recent observations have detected intense radiation at frequencies in the vicinity of the central electron plasma frequency of the device.[1-6] Since these emissions are correlated with the presence of high-energy runaway electrons, it has been proposed that the observed radiation is due to induced processes arising from the highly anisotropic nature of the runaway-electron distribution function.[7-9] In this work, however, we suggest that the emissions near the plasma frequency can be explained by a spontaneous-emission process. Specifically, the high energies characteristic of the runaway electrons permit the spontaneous emission of synchrotron radiation via a relativistic Cherenkov resonance. Numerical analyses, based upon reasonable estimates of tokamak operating parameters, indicate that the radiation levels in the vicinity of the plasma frequency can be comparable to the intensity of the emission at the peak in the thermal spectrum near the second gyroharmonic, which is in agreement with experiment.

The physical configuration we consider is that of a magnetized plasma which, in addition to a thermal background, contains a small population of suprathermal runaway electrons. We assume that the scale lengths for variation of the ambient magnetic field $\vec{B}_0$ ($\equiv B_0\hat{e}_z$) and electron density are much greater than the wavelengths of interest. Furthermore, it is assumed that the thermal energy of the background electrons is of the order of 1 keV, while the runaway energies may be, typically, several hundred keV. As a result, relativistic effects may be generally neglected in the computation of the thermal emissivity, but they must be retained in all phases of the computation of the runaway emissivity.

It is known that under the combined influence of an external electric field and Coulomb scattering, the runaway-electron distribution function is generally expected to possess a long flat tail in the

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direction of the ambient magnetic field. Since we find that the expression for the emissivity does not depend sensitively on the detailed structure of the model distribution function, we shall adopt a distribution which facilitates the computation without loss of generality. The chosen model distribution function, in the laboratory frame, is as follows[8]:

$$F_r(u_\perp,u_\parallel)=2(\pi u_p^2)^{-1}(\pi u_1^2)^{-1/2}\left(1-\frac{u_2}{u_1}\right)^{-1}\exp(-u_\perp^2/u_p^2)[\exp(-u_\parallel^2/u_1^2)-\exp(-u_\parallel^2/u_2^2)] \tag{1}$$

when $u_\parallel>0$, and zero otherwise. In (1), $\vec{u}$ is the relativistic velocity ($\equiv\vec{p}/m_e$), u_p characterizes the perpendicular momentum spread, u_1 and u_2 characterize the parallel momentum and momentum spread; it is assumed that $u_p\ll u_1$ and $u_2\ll u_1$. Hereafter, we adopt the subscript (or superscript) "r" to denote quantities associated with the runaway electrons. Distribution (1) displays the essential features of runaway electrons; in particular, it has a long flat tail when $u_2\ll u_1$ and vanishes for small and negative $u_\parallel$.

Since the density of the runaways is much less than that of the thermal background, we assume that the effect of the runaways on the dielectric properties of the plasma can be ignored. For simplicity, we employ the cold-plasma approximation in determining the dielectric properties of the system, which leads to the Appleton-Hartree dispersion relation[10]

$$n_\pm^2=1-\frac{2\alpha^2(1-\alpha^2)}{2(1-\alpha^2)-\beta^2(\sin^2\theta\pm\rho)}, \tag{2}$$

where n ($\equiv ck/\omega$) is the index of refraction, $\alpha^2\equiv\omega_p^2/\omega^2$, $\beta^2\equiv\Omega_e^2/\omega^2$, $\omega_e^2\equiv4\pi e^2n_b/m_e$, $\Omega_e\equiv|eB_0/m_ec|$, $\rho^2\equiv\sin^4\theta+4(\omega^2/\Omega_e^2)(1-\alpha^2)^2\cos^2\theta$, θ denotes the angle between the wave vector $\vec{k}$ and the ambient magnetic field, and n_b represents the number density of the background electrons. In (2), the "+" and "−" refer to the ordinary and extraordinary modes, respectively.

Before we proceed with the discussion, some consideration of the appropriate relativistic gyroresonance and the range of validity of the cold-plasma approximation is appropriate. The cold-plasma approximation is justified in the limit in which $k_\perp^2v_b^2\ll\Omega_e^2$ and $k_\parallel v_b\ll|\omega-m\Omega_e|$, where v_b is the thermal speed of the background electrons and m is an integer. In the presen work we are interested in the case in which $\omega_e\lesssim0.6\Omega_e$ and for thermal energies of the order of 1 keV (i.e., $v_b^2/c^2=3.9\times10^{-3}$), so that the cold-plasma approximation is justified for $\omega\sim\omega_e$. Consequently, we may write the relativistic gyroresonance as follows:

$$\left(1+\frac{u_\parallel^2}{c^2}\right)^{1/2}=n_\pm\cos\theta\,\frac{u_\parallel}{c}+\frac{m\Omega_e}{\omega}, \tag{3}$$

where terms of order u_p^2/c^2 have been neglected. Since we require that the resonant $u_\parallel$ for the runaways be positive, the Cherenkov ($m=0$) resonance yields the minimum resonant $u_\parallel$ when $n_\pm\times\cos\theta>1$ and, therefore, dominates in the computation of the emissivity. Because the index of refraction exceeds unity only for the extraordinary mode when $\omega\sim\omega_e$, we shall limit the discussion to this case. It should also be pointed out that the contribution to the emissivity from the thermal electrons is negligible in this frequency regime, and only the runaways need be considered.

With these considerations in mind, we solve (3) for the resonant $u_\parallel$

$$\frac{u_m}{c}=\frac{|m|\,\Omega_en_-\cos\theta+[m^2\Omega_e^2+\omega^2(n_-^2\cos^2\theta-1)]^{1/2}}{\omega|n_-^2\cos^2\theta-1|}, \tag{4}$$

and write the synchrotron emissivity for $\theta<\pi/2$ [11]

$$\eta(\omega,\theta)=\frac{e^2n_r}{2\pi^{3/2}}\frac{\omega u_p^2n_-}{c\rho(u_1-u_2)}\sum_{m=0}^{\infty}|u_m-c\gamma_mn_-\cos\theta|^{-1}H[m^2\Omega_e^2-\omega^2(1-n_-^2\cos^2\theta)]$$
$$\times[\exp(-u_m^2/u_1^2)-\exp(-u_m^2/u_2^2)]\exp(-\mu)\{A_{1m}(m^2/\mu)I_m(\mu)+A_{2m}[I_m(\mu)-I_m'(\mu)]\}, \tag{5}$$

where n_r is the runaway density, H is the Heaviside function, I_m and I_m' are the modified Bessel function and its derivative, and

$$\mu \equiv \frac{1}{2}\frac{\omega^2}{\Omega_e^2}\frac{u_p^2}{c^2}n_-^2\sin^2\theta, \tag{6}$$

$$A_{1m} \equiv \frac{1}{2}\left[\left(1+\frac{X_m^2}{(1-\alpha^2)^2\cos^2\theta}\right)\rho-\left(1-\frac{X_m^2}{(1-\alpha^2)^2\cos^2\theta}\right)\sin^2\theta\right], \tag{7}$$

$$A_{2m} \equiv \mu(\rho+\sin^2\theta)-2\frac{\omega}{\Omega_e}mX_m, \tag{8}$$

$$\gamma_m^2 \equiv 1+u_m^2/c^2, \tag{9}$$

$$X_m \equiv (1-\alpha^2)-\gamma_m(\omega/m\Omega_e)n_-^2\sin^2\theta. \tag{10}$$

For reasonable choices of the relevant parameters, the synchrotron emissivity (5) has been evaluated in the vicinity of the electron plasma frequency as a function of frequency and angle, and the results of the computation are shown in Figs. 1 and 2. Here we note that the peak in the thermal spectrum occurs at the second gyroharmonic in the limit of perpendicular propagation; we normalize our result to this value of the peak emissivity for purposes of comparison.

In Fig. 1 we plot the normalized emissivity as a function of frequency for several values of θ ranging from 1° to 15°, and for $n_r/n_b = 10^{-5}$, $\omega_e/\Omega_e = 0.6$, $v_b^2/c^2 = 3.9\times10^{-3}$, $u_p^2/c^2 = 3.95\times10^{-2}$, $u_1^2/c^2 = 1.92$, and $u_2^2/c^2 = 2.05\times10^{-1}$. This corresponds to a mean perpendicular energy of 10 keV and a mean parallel energy of 200 keV for the runaways. It is highly significant that the emissivity peaks for $\omega \gtrsim \omega_e$, which may explain recent observations.[6] The dependence of the emissivity on θ and the mean parallel energy is shown in Fig. 2, in which we plot the peak emissivity versus θ for $u_1^2/c^2 = 0.88$, 1.92, and 7.68 (which correspond to mean parallel energies of 100, 200, and 600 keV, respectively). We note that for $\theta \lesssim 5°$ and for frequencies corresponding to the peak emission, the emissivity is relatively independent of the mean perpendicular energy.

It is important to recognize that the computation is based on a conservative estimate of n_r/n_b; nevertheless, the runaway emissivity is comparable to the peak thermal emissivity. Further, we remark that this enhancement of the radiation near the electron plasma frequency cannot be explained by the classical Schott-Trubnikov formula.

As shown in the figures, the plasma-frequency emission is characterized by a sharp peak in θ, and the principal enhancement is expected to occur for $\theta \lesssim 15°$. In order to consider the direction of energy transport, however, we calculate the

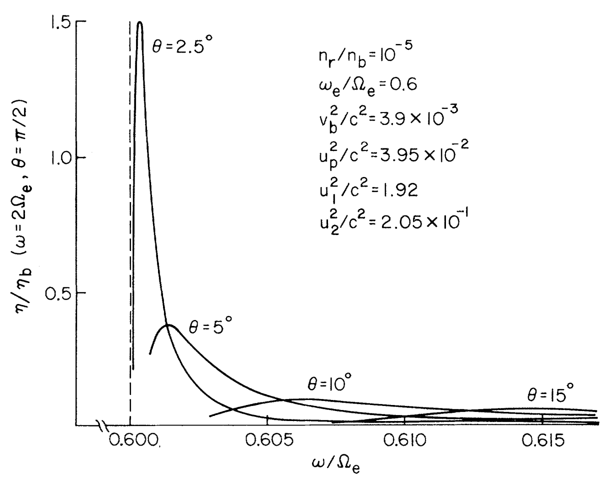

FIG. 1. Plot of the normalized emissivity due to the runaway electrons as a function of frequency for various angles of propagation.

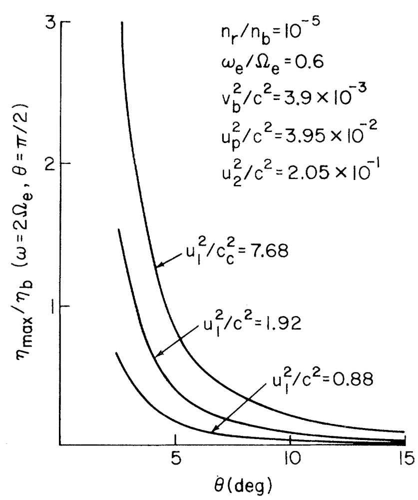

FIG. 2. Plot of the maximum value of the normalized emissivity as a function of θ for several values of the mean parallel energy of the runaway electrons.

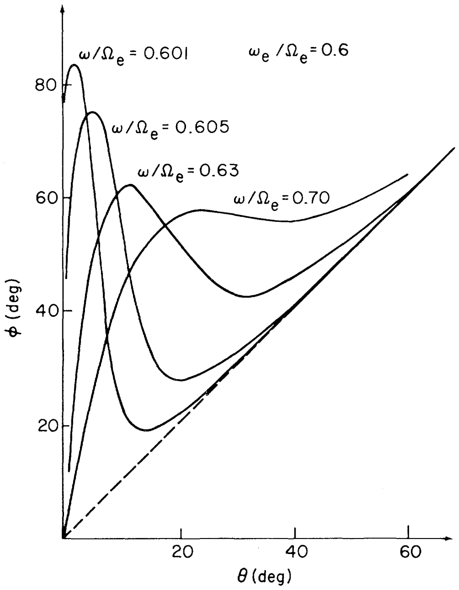

FIG. 3. Graph of φ (defined as the direction of energy transport with respect to $\vec{B}_0$) vs θ for $\omega_e/\Omega_e = 0.6$ and several values of the frequency.

direction of the group velocity. If we let φ denote the angle between the group velocity and the ambient magnetic field, then it can be shown that[12]

$$\tan(\theta - \varphi) = \frac{2\alpha^2(1-\alpha^2)\sin\theta\cos\theta}{\rho[2(1-\alpha^2) - \beta^2(\sin^2\theta + \rho)]}. \qquad (11)$$

A plot of φ versus θ is presented in Fig. 3 for parameters used in the calculation of the emissivity. Evidently, φ ranges between about 70° and 83° for frequencies which correspond to the high levels of emission, and we conclude that the direction of energy transport is predominantly perpendicular to the ambient magnetic field. As a result, the radiation should be easily detectable to an observer looking into the plasma along the direction perpendicular to the magnetic field.

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Effect of Magnetic Shear on Dissipative Drift-Wave Instabilities

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We report the results of a linear radial-eigenmode analysis of dissipative drift waves in a plasma with magnetic shear and spatially varying density gradient. The results of the analysis are shown to be consistent with a recent experiment concerned with dissipative drift-wave instabilities in a toroidal stellarator.

Recently Vojtsenya *et al.*[1] have reported the results of an interesting experiment concerned with the effect of magnetic shear on dissipative drift-wave instabilities in a collisionally dominated plasma in a toroidal stellarator. Their results indicate that drift waves are localized near the