We have accepted a rather poor fit to the 2^+ inelastic cross section and did not increase the value of $|\beta_2 R|$ to obtain a larger 2^+ cross section as well as better agreement between our value of $|\beta_2 R|$ and those found at lower energies.[1] The reason is that the predictions for the 4^+ (and to a lesser extent, the 2^+) are quite sensitive in magnitude to assumed values for $|\beta_6 R|$. Increasing $|\beta_2 R|$ would cause the 4^+ cross section to become too large at angles beyond 28°. The fit to the 4^+ state could be recovered if a nonzero value for $|\beta_6 R|$ were assumed. However, in all the calculations reported here, $\beta_6 R = 0$. If future experiments reveal a 6^+ state in ^{12}C at about 30 MeV, and if the angular distribution can be measured, the value of $\beta_6 R$ can be determined and the present analysis redone. It is somewhat surprising to see such a relatively large sensitivity to the value assigned to β_6 in the CC calculations. It is interesting to compare the ground-state deformation obtained here to that found from analysis of electron inelastic scattering data on ^{12}C where qualitative agreement is found.[13]

In further work on medium-energy proton inelastic scattering from ^{12}C one should employ a microscopic, proton-nucleus optical potential,[14] generated from a nonspherical nuclear density $\rho(\vec{r})$ which is consistent with electron scattering results.[13] The present data could also be usefully extended one further cycle of diffraction. Finally, it would be profitable to study similar transitions in other deformed or highly collective nuclei, in order to discover whether the clean separation of single-step and multistep excitation is indeed a characteristic of medium energies.

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Parity Dependence of the Heavy-Ion Optical Potential

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Recently measured gross structure in the 180° excitation function of $^{28}Si + ^{16}O$ elastic scattering is fitted very well by including a small parity-dependent term in an energy-dependent and surface-transparent optical potential. This interpretation is in contrast to a previous proposal suggesting that the structure is due to potential shape resonances of varying principal quantum numbers n and angular momenta L.

The recently measured[1] gross structure in the 180° excitation function for elastic and inelastic scattering of $^{28}Si + ^{16}O$ has attracted much attention[2] because of its possible connection with quasimolecular shape resonances. In this Letter we present an alternative interpretation of the elastic data which may be of considerable theoretical interest.

The Pauli principle is not properly taken into account in present folding models which are applied widely in analyses of heavy-ion elastic scattering data.[3] It has, however, been known for some time[4] that, in light-ion scattering, exchange effects arising from this principle lead to a Majorana space-exchange term in the optical potential. Specifically for the $^3He + \alpha$ system it has

been shown[4] by resonating-group calculations that such effects may be included in the optical model by the addition of the term $U_{\text{exch}}(r) = U_a(r) + (-1)^L U_b(r)$ to an optical potential that is derived from a folding procedure (the L are the angular momenta of the scattering partial waves). The presence of the $U_a(r)$ term is probably the reason for the frequently observed need of renormalizing the folding potential strengths and for the differences between the geometries of folded potentials and phenomenological potentials. While the $U_a(r)$ term is taken into account in an approximate way in usual phenomenological potentials, this is not so for the parity-dependent term $(-1)^L U_b(r)$. This term may represent an elastic-transfer process[4] or it may stand for a parity-dependent repulsive core as predicted by a recent hydrodynamical model.[5] Such parity-dependent terms cause an odd-even staggering of the partial-wave scattering amplitudes which alters the near cancellation of these amplitudes at far backward angles.[6]

For $^{12}C + ^{13}C$ and similar systems[7] a parity-dependent term has been calculated in the DWBA (distorted-wave Born approximation) assuming a neutron exchange process. Because the exchange amplitude is very large in the $^{12}C + ^{13}C$ system, large differences are found between the envelopes of the even and odd partial-wave amplitudes and the differential cross sections for elastic scattering are affected by this process in the whole angular range. An odd-even staggering in the scattering amplitudes has also been found[8] in an analysis of $^{16}O + ^{18}O$ elastic scattering assuming either a two-neutron exchange mechanism or an L-dependent absorptive potential.[9] Furthermore, a parity-dependent potential has been applied in the analysis of the well known anomalous back-angle scattering for the $Ca + \alpha$ system.[10] For several heavy-ion systems such terms have recently been calculated.[11,12] In general, their strength and range are quite small. However, even a very small parity-dependent term can affect the far-backward-angle cross sections very strongly while leaving the forward-angle cross sections essentially unchanged.

In the present study, we employ a parity-dependent term in the optical potential to analyze the 180° excitation function for $^{28}Si + ^{16}O$ elastic scattering measured by Barrette *et al.*[1] These authors have observed a rather striking gross structure in the excitation function of elastic and inelastic scattering cross sections at 180°. Between $E_{\text{lab}} = 32$ and 58 MeV a series of peaks, approximately 1.5 MeV (lab) wide, with an average spacing of about 4 MeV (lab), and with a peak-to-valley ratio of about 5:1 were found (Fig. 1). Furthermore, at the energies of the 50- and 55-MeV peaks the backward-angle differential cross sections[13] have shapes which resemble closely the shapes of the squares of Legendre polynomials, $P_L^2(\theta)$, where L is close to the critical angular momentum L_{cr} for which the absorption coefficient $\eta_{L_{\text{cr}}} = 1 - |S_{L_{\text{cr}}}|^2 = 0.5$ (θ is the scattering angle and $S_{L_{\text{cr}}}$ is the elastic scattering matrix element at $L = L_{\text{cr}}$). The authors of Ref. 1 suggested this gross structure was due to the presence of single-L potential shape resonances of different principal quantum numbers and angular momenta L close to L_{cr}. They calculated the positions of the shape resonances in a potential which is the energy-independent real part of a strongly absorbing optical potential given by Satchler,[14] but no fit to the data was presented.

For an analysis of the 180° excitation function we modified the optical-model-parameter search code RAROMP[15] to include a simple parity-dependent term $(-1)^L[C_r V(r) + iC_i W(r)]$ in addition to the regular optical potential $V(r) + iW(r)$ for which we assume standard Woods-Saxon shapes. Of course other parametrizations, e.g., a parity-dependent geometry could also be used (see Ref. 5). The calculated cross sections were integrated over the rather large aperture (±5° c.m.) of the experiment.[1] Values of C_r and C_i of only a few percent were found to increase the backward-angle cross sections by several orders of magnitude. By gridding on C_r and C_i and searching on

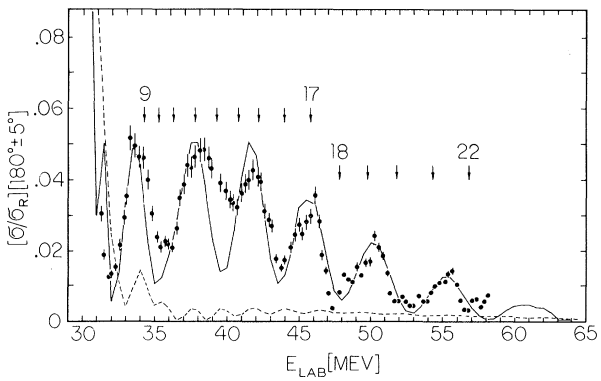

FIG. 1. Excitation function for $^{28}Si + ^{16}O$ elastic scattering at 180° ± 5°. Data of Ref. 1. Here E_{lab} is the average beam energy in the target. Solid line, optical model fit with parity-dependent potential; broken line, without parity-dependent part. Energies of $n = 0$ shape resonances are indicated by arrows several of which are labeled by the L value of the resonating partial wave ($L_{\text{res}} \approx L_{\text{cr}} - 3$).

V and W, starting with parameters from Shkolnik *et al.*,[16] good fits were obtained with energy-independent values of $C_r = C_i \approx -0.002$ and a quadratic energy dependence of the absorptive potential. Somewhat better fits (Fig. 1) were achieved with values of $C_r = C_i$ that decrease with energy as predicted by Tang, LeMere, and Thompson.[17] We used $C_r = 0.112 \exp(-0.073 E_{lab}/\mathrm{MeV})$ and real and imaginary well depths which increased systematically with energy. For convenience we used the following parametrizations (in units of MeV): $V = 22.1 + 0.453\,(E_{lab} - 45.0)$ and $W = 8.2 \times 10^{-3} \times \exp[0.905(E_{c.m.}/\mathrm{MeV} + 11.4)^{1/2}]$. The other parameters are $r_r = 1.323$ fm, $a_r = 0.485$ fm for the real-well geometry, $r_i = 1.350$ fm, $a_i = 0.300$ fm, for the imaginary geometry, and $r_c = 1.0$ fm for the Coulomb radius. This potential, especially its real part, is very similar to the surface-transparent potential which we derived[16] from forward-angle data between $E_{lab} = 45$ and 63 MeV and the backward-angle data at 50 and 55 MeV of Ref. 13.

The backward-angle elastic-scattering cross sections were affected considerably more by the parity-dependent part of the real potential than of the imaginary potential. No need was found to make C_r different from C_i. Because of ambiguities between the parameters of the parity-dependent strength and because of the lack of precise data in the whole angular range, the constants of the above parametrizations of C_r and W are not well determined individually. In particular the differences[18] between the backward-angle data of Refs. 1 and 13 make it difficult to extract parameters for W and C_r, since both have large effects on the backward-angle cross sections.

Fits to the angular distributions between 45 and 63 MeV are quite good even though we have not used the L dependence in the absorption[9] as in Ref. 16 because the L-dependence did not appear important for a fit to the 180° gross structure and because we wanted to reduce the number of free parameters. As an example of the quality of the fits to the angular distributions (which could be improved by using the L dependence) we show the results for 55 MeV (Fig. 2) with and without parity dependence. However, the angular distributions at 33 and 36 MeV and at the high energies (142.5 and 215.2 MeV) (Ref. 19) are not fitted well. Thus it is inappropriate to extrapolate the energy dependence of the optical potential as given above to these lower and higher energies. Both real and imaginary parts need to be larger at 33 and 36 MeV than predicted by the energy dependence derived from the 45–63-MeV data

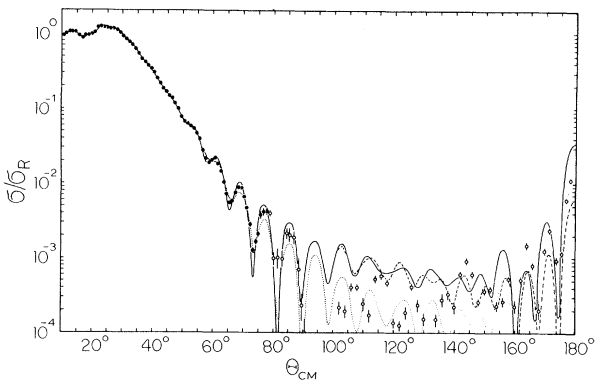

FIG. 2. Differential cross sections for elastic scattering of $^{28}Si + ^{16}O$ at $E_{lab} = 55.0$ MeV (incident ^{16}O) and $E_{lab} = 96.25$ MeV (incident ^{28}Si). Solid dots, data of Ref. 16; open dots, data of Ref. 13. Solid line, calculation with parity-dependent potential that fits the 180° excitation function; broken line, same parameters as for solid line, except $C_r = C_i = 0$; dotted line, fit to 55-MeV angular distribution with $C_r = C_i = -0.002$ and $W = 4.808$ MeV. [Here W is increased over the value used for fitting the excitation function since the angular distribution data (Ref. 13) at backward-angles are lower than the data of the excitation function (Ref. 1)].

and the potential needs to be strongly absorbing at 142.5 and 215.2 MeV. These as yet unresolved problems indicate that surface transparency extends over only a limited energy range as in the case of $^{16}O + ^{16}O$ (Refs. 9 and 20).

It is instructive to compare the calculated excitation function with and without the parity-dependent term in the potential. Without it the excitation function at 180° exhibits a structure which has about twice as many peaks as observed. Inclusion of the parity-dependent term suppresses or enhances approximately every other peak and produces an excellent fit to the data (Fig. 1).

There is, however, no simple relationship between the energies of the maxima in the calculated cross sections and the energies of the shape resonances of this potential. From the real phase shifts as a function of energy we find 14 resonances (indicated in Fig. 1 by arrows) with principle quantum number $n = 0$ (and L values which are about three units below L_{cr}) between 33 and 58 MeV but only ten peaks in the calculated curve with $C_r = C_i = 0$ and only six peaks in the experimental data. Thus it appears clear that the peaks in the 180° excitation function are not due to single resonating partial waves (see also McVoy[21] and Takemasa and Tamura[22]). Rather the interference[10] of contributions to the scattering amplitudes due to the parity-independent part of the potential and the small parity-dependent part reproduces

the structure in the 180° data.

The best-fit potential needs to be surface transparent to yield sufficiently large amplitudes at backward-angles from the parity-independent part. Furthermore, a systematically increasing real well depth is needed to reproduce the energy dependence of the intermediate-angle diffraction patterns[16] and the positions of the peaks of the backward-angle gross structure.

Since the parity-dependent term is possibly in part due to the elastic transfer of a ^{12}C between ^{16}O cores, the rather small values for C_r and C_i are consistent with the contention that such a transfer is a fairly rare event. An elastic exchange of a ^{12}C has been calculated[23] in the DWBA at one energy ($E_{lab} = 55$ MeV) but an unrealistically large spectroscopic factor was needed to obtain the experimental cross sections at backward angles. This is probably due to the use of the strongly absorbing potential $E18$ (Ref. 19). Work on coupled-channels calculations to fit the inelastic data is in progress.[24]

In summary, excellent fits have been obtained to the recently measured 180° excitation function for $^{28}Si + ^{16}O$ elastic scattering with an optical potential that is slightly more attractive for the odd partial waves than for the even partial waves, i.e., the optical potential contains a parity-dependent part $(-1)^L U_b(r)$. Both real and imaginary well depths of the parity-independent part of this potential increase systematically with energy between 45 and 63 MeV indicating a strong energy dependence of the $U_a(r)$ term in the energy range of this study. It should be noted that recent results on the $^{28}Si + ^{12}C$ system[1,25] and the $^{40}Ca + ^{16}O$ system[26] show that backward-angle gross structure in heavy-ion elastic scattering is not an isolated phenomenon. Of particular interest would be attempts to estimate the relative importance of cluster-exchange processes[4] as opposed to effects due to parity-dependent repulsive core.[5]

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