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Negative-Parity *NN* Resonances and Extraneous States

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Angular momentum excitations of six quarks in a bag give a rich resonance structure in the NN channel above $P_{\rm lab} \simeq 1.3$ GeV/c. Some states occur with quantum numbers foreign to NN, which we refer to as extraneous states. The lowest negative-parity states are an $(NN)^1P_1$ resonance and two extraneous $I=0$ states with $J^P = 0^-$ and 2^-, all around 2.2 GeV. The latter two states mainly decay in $NN\pi$. The lowest $(NN)^3F_3$ resonance lies at $M=2.34$ GeV.

Increasing experimental evidence[1] for some resonances in the NN system is being reported. The results on pp scattering with polarized targets and beams above $P_{\rm lab} \simeq 1$ GeV/c show a surprisingly rich structure. Analyses[2,3] of these pp data indicate the existence of a resonance in one of the uncoupled spin-triplet partial waves ($^3P_1$, $^3F_3, \ldots$) at $M \simeq 2.3$ GeV and possibly also a resonance in one of the spin-singlet states ($^1S_0, {}^1D_2, \ldots$) around $M \simeq 2.4$ GeV. Angular distributions and polarizations indicate that the $^3F_3$ assignment for this uncoupled spin-triplet state is favored. Deuteron photodisintegration experiments[4] give evidence for a resonance around 2.38 GeV.

Conventionally[5] one tries to explain these resonances as strong interactions with a channel coupled to the NN channel. A (not too deeply) bound state in the coupled channel will show up as a resonance in NN, in the neighborhood of the threshold for that channel. We note that the NN system cannot have the quantum numbers $I=0$, $J^P = 0^+, 0^-, 2^-, 4^-, \ldots$ and $I=1$, $J^P = 1^+, 3^+, 5^+, \ldots$. We call states with those quantum numbers extraneous to the NN channel and will denote them by X_{IJP}. Recently Jaffe[6] proposed a quite novel type of resonance. He indicated the possible existence of bound states of six quarks in a bag. One assumes here the quarks to be in s states of a spherical bag. These resonances necessarily have positive parity. One finds $I=1$ resonances in the $^1S_0$ ($M=2.17$ GeV) and $^1D_2$ ($M=2.36$ GeV) waves and $I=0$ resonances in the $^3S_1$ ($M=2.24$ GeV) and $^3D_3$ ($M=2.36$ GeV) waves.

The masses of these states which contain $N=6$ nonstrange quarks can be calculated in the spherical-bag approximation in which the mass operator is given by

$$M = \frac{4\pi}{3}BR^3 - \frac{Z_0}{R} + N\frac{\alpha_n}{R} + \alpha_c \frac{M_{00}}{R}\Delta, \tag{1}$$

where $B = 59.2$ MeV fm^{-3}, $Z_0 = 363$ MeV fm, and $\alpha_c = 2.20$ are the parameters determined from the q^3 and $q\bar{q}$ spectrum.[7] The nonstrange-quark energy is measured by $\alpha_n = 403$ MeV fm and the color magnetic-interaction strength by $M_{00} = 34.9$ MeV fm. The color magnetic mass splitting is determined by

$$\Delta = -\tfrac{1}{3}N(6-N) + \tfrac{1}{3}\vec{J}^2 + \vec{I}^2 + \tfrac{1}{2}F^2. \tag{2}$$

F^2 is the color SU(3) quadratic Casimir operator. In the bag model the mass of a colorless state is found by minimizing the eigenvalue of the mass operator with respect to R, which gives

$$M = \tfrac{4}{3}\pi R^3(4B)$$
$$= \tfrac{4}{3}(4\pi B)^{1/4}[N\alpha_n - Z_0 + \alpha_c M_{00}\Delta]^{3/4}. \quad (3)$$

This shows that in a spherical bag the equilibrium energy density is $4B$. In order to present the basic features of the calculations we will assume one constant bag radius for all N quark states. Equation (1) then simplifies to

$$M = A_0 + B_0[I(I+1) + \tfrac{1}{3}J(J+1)] \quad (4)$$

for a colorless state with $A_0 = 2.126$ GeV and $B_0 = 58.7$ MeV for $N = 6$. This approximation works fairly well[8] and the radius for an N-quark system can be parametrized as

$$R = r_0 N^{1/3},$$

with $r_0 = 0.72$ fm.

To find the isospin and spin content of the states it is convenient to consider the SU(4) isospin-spin irreps to which the colorless $N = 6$ states belong. The Pauli principle requires the color $\underline{1}$ irrep to couple uniquely with the SU(4) isospin-spin irrep $[\underline{50}]$.[7-9] The decomposition of this irrep into SU(2) isospin I and SU(2) spin J gives the following (I,J) content:

$$[\underline{50}] = (0,1) + (0,3) + (1,0) + (1,2) + (2,1) + (3,0).$$

In order to get negative-parity states in the bag model one has to consider either $q^7\bar{q}$ states or q^6 states with angular momentum (L) excitations. The lowest nonstrange $q^7\bar{q}$ states have a mass of 2.8 GeV[10] and can be discarded as possible candidates for the aforementioned resonances. To introduce L excitations in a six-quark bag we will follow a semiquantitative argument given by Johnson and Thorn.[11] For high L the bag (Fig. 1) gets stretched to a rodlike shape of length l with at each end a certain number of quarks coupled to opposite nonzero color charges. Taking the energy density $E^2/2$ of the color electric field in between to be equal to $4B$ [see Eq. (3)] it follows that the cross section $A = \pi R_0^2$ depends only on the color charges at the ends. The mass M is then given by

$$M = 4BAl = l(8\pi\alpha_c B f^2)^{1/2}, \quad (5)$$

where f^2 is the eigenvalue of the color SU(3) operator F^2 of the color irreps $\underline{n}$ and $\underline{n}^*$ at the ends. Qualitatively the connection between M^2 and L can be found by simply using the moment of inertia $I = Ml^2/12$ of a rod and if we assume that the ends of the rod have velocity c then $l\omega/2 = 1$ and L

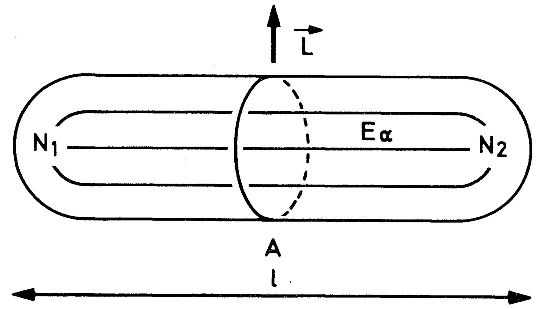

FIG. 1. The stretched bag for $L \neq 0$ with N_1 quarks at one and N_2 quarks at the other end.

$= Ml^2\omega/12 = Ml/6 = M^2/(24AB)$. This leads to linear trajectories $L \sim \alpha' M^2$ with the slope $\alpha' = (24AB)^{-1}$. This slope depends on the color charge at the ends of the bag. For ordinary mesons and baryons with the color configuration $\underline{3}$-$\underline{3}^*$ this becomes $\alpha' = 0.9$ GeV^{-2}, which is exactly the universal slope.

To estimate the masses of the stretched bag states we make the following assumptions (Jaffe[12]). The Regge trajectories are asymptotically straight with $1/\alpha' = 1.1$ GeV^2. As the color magnetic interaction is short ranged, a quark will only feel the color magnetic interaction of those quarks which are in the same end of the bag. In a stretched bag we can write Eq. (2) as $\Delta = \Delta_1 + \Delta_2$ where Δ_1 contains the summation over the N_1 quarks in one end and Δ_2 that over the N_2 quarks in the other end of the bag. The N_i quarks couple their spins to the total spin S_i and their isospin to the total isospin I_i, $i = 1,2$. The intercept for $L = 0$ is then assumed to be the square of the mass (M_0^2) of a spherical bag in which the color magnetic interaction is restricted in the aforementioned sense. The lowest intercepts are found for those states with color charges $\underline{3}$ and $\underline{3}^*$ at the ends. Then

$$\Delta = \Delta_0(N_1, N_2; \underline{n}\text{-}\underline{n}^*) + (\vec{I}_1^{\,2} + \vec{I}_2^{\,2}) + \tfrac{1}{3}(\vec{S}_1^{\,2} + \vec{S}_2^{\,2}),$$

where $\Delta_0(1,5;\underline{3}\text{-}\underline{3}^*) = -2$ and $\Delta_0(2,4;\underline{3}^*\text{-}\underline{3}) = -4$. The intercepts found in this way are not the physical states for $L = 0$, but they are useful to estimate the mass of the $L \neq 0$ resonances. For $L = 0$ the physical mass will depend on the isospin I and spin $J = S$ of the total N-quark system, whereas for $L \neq 0$ it is determined by I_1, S_1, I_2, S_2. The trajectories are then labeled by $N_1, N_2, f^2, I_1, I_2, S_1, S_2$, leading to $(2I_1+1)(2I_2+1)(2S_1+1)(2S_2+1)$ degenerate levels with different IJ^P values, where $\vec{J} = \vec{L} + \vec{S}$ with $\vec{S} = \vec{S}_1 + \vec{S}_2$ and $P = (-1)^L$. Spin-spin and spin-orbit forces and coupling with other levels will probably remove most of the degeneracy.

To find the possible values for I_i and S_i we again turn to SU(4). For $N_1 = 2$ states coupled to the antisymmetric color SU(3) irrep $\underline{3}^*$, the Pauli principle requires that they belong to the symmetric SU(4) irrep $[\underline{10}]$ with the (I_1, S_2) content

$$[\underline{10}] = (0,0) + (1,1).$$

The $N_2 = 4$ states coupled to the color irrep $\underline{3}$ must be in the isospin-spin irrep $[\underline{45}]$ with the (I_2, S_2) content

$$[\underline{45}] = (0,1) + (1,0) + (1,1) + (1,2) + (2,1).$$

For the q-q^5 trajectories the $N_2 = 5$ states belong to the color irrep $\underline{3}^*$ and therefore must couple to the SU(4) isospin-spin irrep $[\underline{60}]$ with (I_2, S_2) content

$$[\underline{60}] = (\tfrac{1}{2},\tfrac{1}{2}) + (\tfrac{1}{2},\tfrac{3}{2}) + (\tfrac{1}{2},\tfrac{5}{2}) + \tfrac{3}{2},\tfrac{1}{2}) + \tfrac{3}{2},\tfrac{3}{2}) + \tfrac{5}{2},\tfrac{1}{2}).$$

The values $\Delta = \Delta_1 + \Delta_2$ of the q^2-q^4 ($\underline{3}^*$-$\underline{3}$) and q-q^5 ($\underline{3}$-$\underline{3}^*$) trajectories are given in Table I, together with the I, S values of these trajectories.

Starting from $M_0 = A_0 + B_0\Delta$ for the intercepts [see Eq. (4)] the masses of the L-excited dibaryon levels can be written approximately in the form

$$M_L = A_L + B_L \Delta, \tag{6}$$

where we take $A_L^2 = A_0^2 + 1.1L$. In order to satisfy the condition of linear trajectories $M_L^2 = M_0^2 + 1.1L$, we find $B_L \simeq B_0 A_0 / A_L$. The relevant values for A_L and B_L are given in Table II. The masses M_L for $L = 1$ and 2 are given in the last two columns of Table I. The pattern of levels that is obtained is displayed in Fig. 2. In this figure the q^2-q^4 and q-q^5 states are given. The lowest states are q^2-q^4 states. In the upper part of the level scheme q^2-q^4 ($\underline{6}$-$\underline{6}^*$) and q^3-q^3 ($\underline{8}$-$\underline{8}$) levels also will appear.

In the $I = 0$ NN channel the lowest negative-parity resonance appears in the $^1P_1$ wave at 2.2 GeV. Degenerate with this resonance two extraneous states X_{00}- and X_{02}- appear. These states could show up in $pp\pi^-$ invariant-mass plots for the reaction $pp \to pp\pi^+\pi^-$.

In the $I = 1$ channel we note that the lowest negative-parity levels exhibit equal spacing, starting from a level at 2.266 GeV, showing up as a resonance in the $^3P_1$ wave. More resonances occur in the ^3P waves at 2.301, 2.336, and 2.371 GeV. The

TABLE II. The coefficients A_L and B_L occurring in Eqs. (4) and (6), for $L = 0, 1, 2$.

L	A_L(GeV)	B_L(MeV)
0	2.126	58.7
1	2.371	52.6
2	2.592	48.1

TABLE I. The quantum numbers of the q^2-q^4 and q-q^5 trajectories with color charges $\underline{3}$ and $\underline{3}^*$ at the ends, and the masses for $L = 1$ and 2.

q^2-q^4									q-q^5								
I_1	S_1	I_2	S_2	Δ	I	S	M_1 [GeV]	M_2 [GeV]	I_1	S_1	I_2	S_2	Δ	I	S	M_1 [GeV]	M_2 [GeV]
0	0	0	1	-10/3	0	1	2.196	2.432	1/2	1/2	1/2	1/2	0	0,1	0,1	2.371	2.592
0	0	1	0	-2	1	0	2.266	2.496			1/2	3/2	1	0,1	1,2	2.424	2.640
0	0	1	1	-4/3	1	1	2.301	2.528			1/2	5/2	8/3	0,1	2,3	2.511	2.720
1	1	0	1	-2/3	1	0,1,2	2.336	2.560			3/2	1/2	3	1,2	0,1	2.529	2.736
0	0	1	2	0	1	2	2.371	2.592			3/2	3/2	4	1,2	1,2	2.581	2.784
1	1	1	0	2/3	0,1,2	1	2.406	2.624			5/2	1/2	8	2,3	0,1	2.792	2.977
1	1	1	1	4/3	0,1,2	0,1,2	2.441	2.656									
0	0	2	1	8/3	2	1	2.511	2.720									
1	1	1	2	8/3	0,1,2	1,2,3	2.511	2.720									
1	1	2	1	16/3	1,2,3	0,1,2	2.652	2.849									

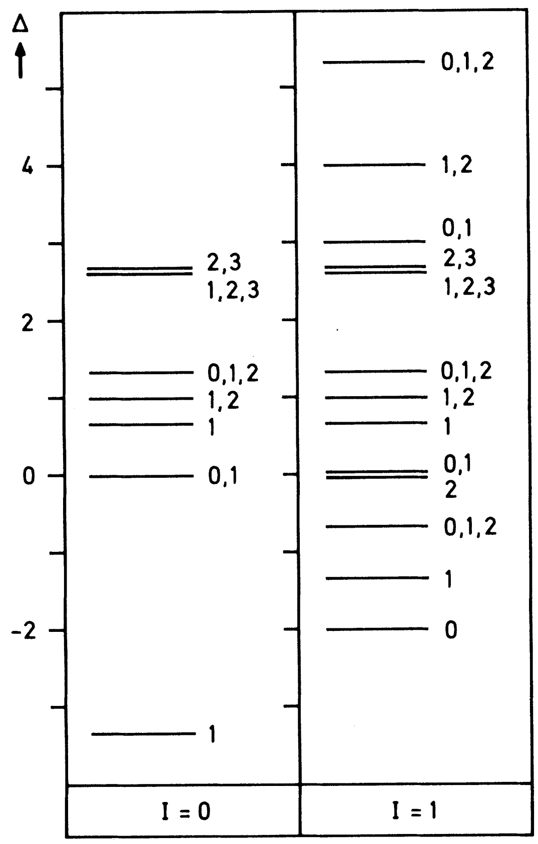

FIG. 2. The $L = 1$ ($P = -$) q^2-q^4 and q-q^5 levels. The splitting is proportional to Δ according to Eq. (6). The total spin S of the states has been given in the figure. To find J, the spin S has to be combined with $L = 1$.

resonances at 2.34 and 2.37 GeV also appear in the $^3F_3$ wave. Because of the appearance of several resonances in the energy region below 2.4 GeV, the analysis of the pp data is much more complicated than expected. It is noteworthy that most of the low-lying resonances appear in the triplet uncoupled states[3] $^3P_1$ and $^3F_3$ and that the highest J in this region[2] is 3.

Still below the ΔΔ threshold some extraneous states occur with $I=2$, for instance X_{20^-}, X_{21^-}, and X_{22^-} at $M = 2.41$ GeV, which might appear in $pp\pi^+$ invariant-mass plots.

To conclude we want to stress that this is an *estimate* of the NN resonance spectrum. In making this estimate many effects have been neglected such as the coupling between the color $\underline{3}$-$\underline{3}$* and $\underline{6}$-$\underline{6}$* trajectories, spin-spin, and spin-orbit splittings. At present we do not know how to handle all the details properly.

Concerning the general validity of our approach, we think that our arguments are valid if the bag is really stretched, i.e., if $l > 2R_0$, where R_0 is the tube radius. For the mass this requires $M > 8\pi B R_0^3$ or for color $\underline{3}$-$\underline{3}$* configurations $M > 2.1$ GeV. We think that this condition is almost independent of the number of quarks at each end. Below 2.1 GeV exchange forces between the quarks have to be included.[13] For the dibaryons (lowest mass ~2.2 GeV) we therefore believe that the main features are correct and we hope that these results can be useful for the future analyses of the NN scattering experiments.

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[1] I. P. Auer *et al.*, Phys. Lett. 67B, 113 (1977), and 70B, 475 (1977); D. Miller *et al.*, Phys. Rev. Lett. 36, 763 (1976), and Phys. Rev. D 16, 2016 (1977); M. Albrow *et al.*, Nucl. Phys. B23, 445 (1970).
[2] K. Hidaka *et al.*, Phys. Lett. 70B, 479 (1977); N. Hoshizaki, Prog. Theor. Phys. 58, 716 (1977).
[3] W. Grein and P. Kroll, University of Wuppertal Report No. WUB 77-6 (to be published).
[4] T. Kamae *et al.*, Phys. Rev. Lett. 38, 468 (1977).
[5] For instance the analysis of Ref. 4 in T. Kamae *et al.*, Phys. Rev. Lett. 38, 471 (1977); J. J. de Swart *et al.*, *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, New York, 1971), Vol. 60, p. 138.
[6] R. L. Jaffe, Phys. Rev. Lett. 38, 195, 617(E) (1977).
[7] T. DeGrand *et al.*, Phys. Rev. D 12, 2060 (1975).
[8] A. Aerts, P. Mulders, and J. J. de Swart, Phys. Rev. D 17, 260 (1978).
[9] V. Matveev and P. Sorba, Lett. Nuovo Cimento 20, 435 (1977), and Fermilab Report No. PUB 77/56 (unpublished).
[10] P. J. G. Mulders, A. Th. M. Aerts, and J. J. de Swart, to be published.
[11] K. Johnson and C. B. Thorn, Phys. Rev. D 13, 1934 (1976).
[12] R. L. Jaffe, MIT Report No. CTP 657, 1977 (to be published).
[13] R. E. Cutkosky and R. E. Hendrick, Phys. Rev. D 16, 2902 (1977).