to be explained by trapped electrons or an increase in the $Z_{\rm eff}$. Enhanced slowing-down of ions has been observed to result from beam-plasma instabilities[8] but the threshold fast-ion density ($n_f/n_e \gtrsim 10^{-3}$) appears to be somewhat higher than in the present experiment ($n_f/n_e \approx 5 \times 10^{-4}$). A more likely explanation might be the distortion of the electron distribution from that of a shifted Maxwellian. There is evidence from the work of Fomenko[9] that such a distortion increases the backward electron current for $T_e \gg 5$ eV and so a full Fokker-Planck treatment for the present regime might bring theory into better agreement with experiment. For small distortions, such a theory would still reproduce the energy loss rates obtained with a Maxwellian distribution and would not conflict with the classical energy loss rates observed by Klavan *et al.*[10]

The authors gratefully acknowledge the contribution to this work by N. R. Ainsworth, H. B. Rabbett, and the operating staff of the Levitron led by R. E. Bradford. We also wish to thank J. G. Cordey for many helpful discussions.

[1]T. Ohkawa, Nucl. Fusion 10, 185 (1970).

[2]D. R. Sweetman, in *Proceedings of the Symposium on Plasma Heating and Injection, Verenna, Italy, 1972* (Laboratorio di Fisica del Plasma ed Elettronica Quantistica, Milan, Italy, 1973).

[3]L. Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1956).

[4]D. P. Hammond *et al.*, in *Proceedings of the Ninth Symposium on Fusion Technology, Garmisch-Partenkirchen, West Germany, 1976*, EURATOM Publication No. EUR 5602 (EURATOM, Ispra/Varese, Italy, 1976), pp. 827–833.

[5]D. E. T. F. Ashby, W. H. W. Fletcher, and T. N. Todd, to be published.

[6]Y. Itikawa and O. Aono, Phys. Fluids 9, 1259 (1966).

[7]E. W. Laing and W. P. S. Tan, J. Phys. D 10, 1619 (1977).

[8]M. Yamada and S. W. Seiler, Phys. Rev. Lett. 39, 808 (1977).

[9]V. V. Fomenko, Nucl. Fusion 15, 1091 (1975).

[10]I. L. Klavan *et al.*, Phys. Rev. Lett. 28, 1254 (1972).

Stabilization of the Linear Drift Tearing Mode by Coupling with the Ion Sound Wave

M. N. Bussac
Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau Cedex, France

and

D. Edery
Association EURATOM, Commissariat à l'Energie Atomique, 92260 Fontenay aux Roses, France

and

R. Pellat
Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau Cedex, France

and

J. L. Soule
Association EURATOM, Commissariat à l'Energie Atomique, 92260 Fontenay aux Roses, France
(Received 14 December 1977)

The resistive drift tearing mode is shown to be stabilized by the ion motion along magnetic field lines. The effects of an electron temperature gradient are included in the discussion of the results.

The tearing mode[1] is important in the theory of plasma confinement in tokamaks because of the increased radial transport in the resulting "island structure" and the probable involvement of this mode in the internal[2,3] and external[4,5] disruptions. In this Letter we consider again the stability of the "drift collisional" tearing mode $l \geq 2$. Experimentally,[6] during a "typical good plasma discharge," the mode $l = 2$ is observed to be oscillating and stable or saturated to a very small amplitude (which corresponds to a magnetic island of some millimeters). At the end of the discharge the mode may become strongly unstable just before the external disruption. Different kinds of stabilizing mechanisms have already been proposed. In the usual theoretical approach, one matches a solution computed in a resistive layer with a macroscopic solution of the equations of ideal magnetohydrodynamics (MHD). The condition for stability is given by $\Delta'<0$, with $\Delta' = (d\Psi_1^+/dt - d\Psi_1^-/dr)\Psi_1^{-1}$, where the radial derivatives of the perturbed helical flux function Ψ_1 are taken on both sides (+, −) of the resistive layer in the linear theory, or of the magnetic island in a quasilinear theory.[7] Stable theoretical current profiles are computed[8] but, in fact, Δ' is found to be positive in the experiments with $|\Delta'|r_0$ of order unity, r_0 being the radius of the magnetic surface where the safety factor q takes the value 2. A more recent work[5] has shown that a nonlinear saturation of the tearing mode could follow the island sampling of the exterior solution, giving a quasilinear decrease of Δ'. The toroidal coupling[3] between the internal $l=1$ and the $l=2$ modes may also be important but an $l=1$ mode is not always observed during the external disruptions. In any case, the observed destabilization does not seem to involve a drastic modification of the current profile and it appears necessary to look inside the resistive layer for stabilizing effects. The average toroidal magnetic-field curvature provides a stabilizing mechanism[9] of the standard tearing mode. Nevertheless in the quoted experiments the resistive growth rate is smaller than the electron diamagnetic frequency and the mode becomes a drift tearing mode.[10] In such a case the stabilizing effect of the curvature is negligible if $r_0\delta/\rho_i Rq<1$,[3] where δ is the thickness of the resistive layer and ρ_i is the ion Larmor radius. In this Letter we introduce a *new* stabilizing effect due to the linear coupling of the tearing mode with the ion sound wave. Let us recall that the ion sound wave is already at the origin of the shear stabilization of the electrostatic drift wave. We start from the two-fluid model including the parallel electron heat flow and the parallel exchange of momentum between ions and electrons. More precisely, we keep the term $\nu_{ei}u_{i\parallel}^1$ in the equation for the electron parallel momentum and take also into account the ion contribution to the parallel current: $j_\parallel^1 = n_e^0 l(u_{i\parallel}^1 - u_{e\parallel}^1)$; here $u_{i\parallel}^1$ and $u_{e\parallel}^1$ are, respectively, the ion and electron velocities of the tearing mode, and n_e^0 is the electron density at equilibrium.

The geometrical model is the standard current-carrying cylindrical plasma column of length $2\pi R$ confined by a magnetic field with both axial B_z and azimuthal B_θ components. We make a Fourier transform of the equations upon time and space:

$$T(r,\theta,z,t)\to T(r,l,n,\omega)\exp[-i\omega t + i(l\theta - nz/R)].$$

We obtain the following equations for the scalar potential Φ and the parallel vector component $A_\parallel$ of the tearing mode ($A_\parallel \equiv \Psi$):

$$\Delta_\perp A_\parallel = \frac{V_e^2}{\rho_i^2 V_A^2}\,\frac{\omega-\omega_e^* - Z(\omega-\omega_i^*)k_\parallel^2 c_s^2/\omega^2}{\omega + i\nu_{ei} - k_\parallel^2 V_e^2/\omega}\left(A_\parallel + \frac{k_\parallel \Phi}{\omega}\right), \tag{1}$$

$$\Delta_\perp A_\parallel = -\frac{\omega-\omega_i^*}{k_\parallel V_A^2}\,\Delta_\perp\Phi, \tag{2}$$

where ν_{ei} is the electron-ion collision frequency and V_e the electron thermal velocity. For simplicity, the plasma is taken to be isothermal; c_s is the sound speed $c_s=(T_e/m_i)^{1/2}$; $\Delta_\perp$ is the Laplacian operator working perpendicular to $\vec{B}$; V_A is the Alfvén speed, $V_A = B/(\mu_0 n_i^0 m_i)^{1/2}$; ω_j are the diamagnetic frequencies,

$$\omega_j = \frac{l}{\Gamma}\,\frac{T_j}{q_j B}\,\frac{1}{n_j^0}\,\frac{dn_j^0}{dr},$$

with $q_e=-e$ and $q_i = Ze$; and finally, $k_\parallel = (n/R)(1-q_0/q)$, with $q_0\equiv q(r_0) = l/n$.

To illustrate the effect of the ion sound wave it is sufficient to restrict our analysis to the drift tearing mode. We introduce dimensionless quantities: $x=(r-r_0)/(-i)^{1/2}\delta$ with $\delta = |\omega^*\nu_{ei}|^{1/2}/k_\parallel' V_e$, δ being the width of the "electron current channel"[11]; $k_\parallel' = dk_\parallel/dr$; and $\xi = (-i)^{1/2}\delta k_\parallel'\Phi/\omega A_\parallel(0)$. Then we obtain, in the usual constant-Ψ approximation which holds for $l\geq 2$,

$$\overline{\Delta}' = \frac{\rho_i^2\Delta'\omega}{\beta_e Z(\omega-\omega_i^*)} = g(\alpha,\mu) = \sqrt{-i}\,\mu\int \frac{d^2\xi}{dx^2}\frac{dx}{x}, \tag{3}$$

$$\frac{d^2\xi}{dx^2} = \frac{x}{1+x^2}\left(i\alpha + \frac{x^2}{\mu}\right)(x\xi+1), \tag{4}$$

with the following definitions:

$$\alpha = -(\delta/\rho_i)^2(\omega-\omega_e^*)/(\omega-\omega_i^*),$$

$$\mu^{-1} = Z(m_e/m_i)(\delta/\rho_i)^2(\nu_{ei}/\omega).$$

Here β_e is the ratio between the electron and the magnetic pressures; we have made $\omega + i\nu_{ei} \sim i\nu_{ei}$. Equation (4) is solved with the boundary condition $\xi \to -1/x$ for $|x| \to \infty$.

Let us first summarize the results already known in the literature [$\mu \to \infty$ in Eq. (4)]. We set $\omega - \omega_e{}^* = |\delta\omega| \exp(i\Phi)$, with $|\delta\omega| \ll \omega_e{}^*$ and $0 < \Phi < \pi$ unstable. In the usual drift-tearing limit (α large, x small) we solve Eq. (4) with the Hermite functions and obtain $g = 2\pi[\Gamma(\frac{3}{4})/\Gamma(\frac{1}{4})]|\alpha|^{3/4}\mu \times\exp[i(3\Phi/4 - 5\pi/8)]$ and consequently $\Phi = 5\pi/6$. In the weakly collisional regime investigated recently[11] (α small, x of order unity), we neglect ξ in Eq. (4) and obtain $g = \pi\mu|\alpha|\exp[i(\Phi - 3\pi/4)]$ and $\Phi = 3\pi/4$. One then concludes that there always exists an unstable mode when $\overline{\Delta}' > 0$.

As we now find, the effect of the ion sound wave increases the stability of the drift tearing mode by giving a positive threshold $\overline{\Delta}' > \overline{\Delta}_0' > 0$. For small values of α and if $\overline{\Delta}'$ is not too large, we have to keep the term $\mu^{-1}x^2$ in Eq. (4) because we know from the previous discussion that $\alpha \sim \overline{\Delta}'/\pi\mu$. To compute numerically the threshold we first choose a value of μ, solve Eq. (4) for different real values of α, and then look for the value which gives a real g: The results are given in Fig. 1, where the new stable domain of parameters is shaded. In the weakly collisional regime (for small values of α and μ^{-1}) it is possible to give a simple explanation of the result.[12] Inside the electron current channel ($x \lesssim 1$), ξ (being odd) is small and, by keeping μ^{-1} smaller than α, one can still compute the contribution by taking $d^2\xi/dx^2 \simeq \alpha x/(1+x^2)$ and obtain $g_1 = \alpha\pi\mu\exp(i\pi/4)$. For large x, Eq. (4) can also be simplified: $d^2\xi/dx^2 \simeq \mu^{-1}i(x\xi + 1)$. For intermediate $x \sim (\alpha\mu)^{1/2}$ its solution has to be matched with the solution of the previous equation. But if we rescale x so that $x = \mu^{1/4}\bar{x}$, its solution holds now for all finite $\bar{x}$. To the lowest order in expansion (in both α and μ^{-1}), we have to take $\xi = 0$ at $\bar{x} = 0$. The equation is now easily solved (taking also into account the condition at $x \to \infty$) and we obtain, for the contribution of large values of x to g, $g_2 = 2\pi[\Gamma(\frac{3}{4})/\Gamma(\frac{1}{4})]\mu^{1/4}\times\exp(-i\pi/4)$ which is obviously stabilizing. Adding the two contributions we obtain from (3) an analytical marginal condition for stability of the drift tearing mode in the weakly collisional regime:

$$\overline{\Delta}_0' = 2.12(4\mu)^{1/4}, \quad \alpha = 2.12/\pi\mu^{3/4}, \tag{5}$$

with $2.12 = 2\pi\Gamma(\frac{3}{4})/\Gamma(\frac{1}{4})$. One can verify, with the ordering in α and μ^{-1} implied by the result (5), that the overlapping of the two respective contributions is negligible in the computation of g —this justified our boundary-layer method; for α and μ^{-1} small, the analytical formula compares well with the numerical results. In the opposite case of large values of α and μ^{-1} we find with dimensionless arguments from Eqs. (3) and (4) that $\alpha \sim \mu^{-2/3}$ and $\overline{\Delta}_0' \sim \mu^{1/2}$, the numerical result being $\overline{\Delta}_0' = 1.13(2\mu)^{1/2}$. Let us finally mention that our condition for stability is within the range of parameters of TFR.[6] [The stability condition (5) gives a slow dependence, $\sim l^{1/2}$, of $\overline{\Delta}_0'$ with respect to the mode number ($l^{1/2}$ for Δ').]

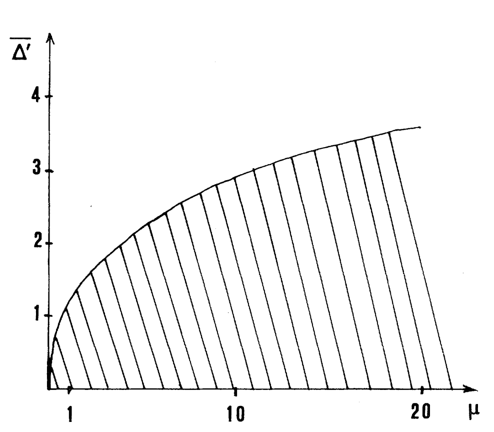

FIG. 1. Curve of marginal stability. $\overline{\Delta}' = \rho_i{}^2\Delta'\omega/\beta e Z(\omega - \omega_i{}^*)$; $\mu = (m_i/m_e)(\rho_i/\delta)^2(\omega_e{}^*/Z\nu_{ei})$.

Let us now discuss the main physical limitations of our model. First, Eq. (4) is no longer valid in the weakly collisional regime when the thickness δ of the electron current channel becomes smaller than the ion Larmor radius. The new ion contribution is easily computed by a kinetic treatment and we have obtained the following modifications of Eqs. (1) and (2): In Eq. (1) $Z(\omega - \omega_i{}^*)$ becomes $Z(\omega - \omega_i{}^*)\mathcal{L}(r)$ and in Eq. (2) $-(\omega - \omega_i{}^*)\Delta_\perp$ becomes $(\omega - \omega_i{}^*)2[1 - \mathcal{L}(r)]/\rho_i{}^2$. Here $\mathcal{L}(r)$ is the standard Larmor-radius operator defined by its Fourier transform upon the local variable $r - r_0$: $\mathcal{L}(r) \to I_0 e^{-b}$. Here I_0 is the modified Bessel function and $b \equiv 0.5\rho_i{}^2(k_r{}^2 + n^2/R^2)$. The results of a complete numerical treatment of the integrodifferential equation will be reported in a forthcoming paper. It is nevertheless clear that our analytical treatment has a much larger domain of validity than $\delta/\rho_i > 1$ in the weakly collisional limit for small α and μ^{-1}. Obviously, the electron current channel is not modified by the finite-ion-Larmor-radius effects. We have also found analytically, in agreement with numerical results that the ion contribution is important for large x ($x \lesssim \mu^{1/4}$) and consequently the validity of our treatment relies upon $\mu^{1/4}(\delta/\rho_i) \gtrsim 1$ instead of upon $\delta/\rho_i \gtrsim 1$. That mild condition, which is independent of ν_{ei}, reads $d\ln q/d\ln n^0 \lesssim q(R/r)Z^{-1/2}$. The last comment is based on the fluid approximation for the electron population: This is verified in the electron current channel as far as $\nu_{ei} > \omega_e^*$. Outside the electron current channel we have already shown that the electron contribution is negligible.

A kinetic approach[11] would be in all cases more precise but does not modify our conclusions: Without electron temperature gradient the drift frequency is independent of the individual particle velocity and the parallel conductivity has the same qualitative behavior in $k_\parallel$ and x. On the contrary, the conductivity is velocity dependent in the presence of a temperature gradient. As a consequence the drift tearing mode may be driven by $\Delta' > 0$ and/or by a temperature gradient.[13] In the weakly collisional regime, by solving Eq. (4) in the electron current channel ($x<1$) one finds that[11] $\omega \sim \omega_e^* + \frac{5}{4}\omega_{T_e}^* + \delta\omega$, with $\omega_T^* = (l/2eBr) \times (dT_e/dr)$. Outside the electron current channel ($x \gtrsim 1$), the temperature gradient disappears and Eq. (3) still holds. With the quoted frequency value, α is now large and the ion sound wave has a much smaller effect (of order $\alpha^{-1/2}$) in the expression for g. The stabilizing effect of the ion sound wave relies upon the expected nonlinear flattening of the electron temperature profile by island formation. However, the validity of the small-Larmor-radius expansion has not been verified and all the published results involving temperature gradients have to be considered as being approximate at best.

In conclusion, we have found a stabilizing mechanism for the drift tearing mode similar to the well known case of the electrostatic drift wave[14]; the condition for stability was in that case a consequence of the boundary condition: The electrostatic energy had to be outgoing. Now, the source of instability (in the absence of a temperature gradient) is outside the singular layer, and the boundary condition is to match the MHD solution for the plasma displacement ($\xi \to -1/x$). As is well known, that condition is equivalent to a vanishing electric component along $\vec{B}$. The radial ion displacement is coupled through the electrostatic potential to the ion motion along field lines: As a consequence of the magnetic shear, the contribution of the parallel ion inertia increases from the center of the singular layer and modifies the asymptotic behavior of the displacement. Since the parallel ion inertia drives the ion sound-wave propagation, we can conclude that the quasineutrality condition couples the tearing mode and the ion sound wave, resulting in the new stabilizing effect that we have found. As a consequence of our work, the drift tearing mode may be considered less harmful than previously thought in the confinement of hot plasmas.

[1]H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids 6, 459 (1963); G. Laval, R. Pellat, and M. Vuillemin, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1965), Vol. III, p. 259.

[2]B. Coppi, R. Galvao, R. Pellat, M. N. Rosenbluth, and P. H. Rutherford, Fiz. Plasmy 2, 961 (1976) [Sov. J. Plasma Phys. 2, 553 (1976)].

[3]M. N. Bussac, D. Edery, R. Pellat, and J. L. Soulé, in *Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976* (International Atomic Energy Agency, Vienna, 1977), Vol. 1, p. 607.

[4]B. B. Kadomtsec, Fiz. Plasmy 1, 710 (1975) [Sov. J. Plasma Phys. 1, 389 (1975)].

[5]R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, in *Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976* (International Atomic Energy Agency, Vienna, 1977), Vol. 1, p. 569.

[6]Equipe TFR, in *Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976* (International Atomic Energy Agency, Vienna, 1977), Vol. 1, p. 267.

[7]P. H. Rutherford, Phys. Fluids 16, 1903 (1973).

[8]H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids 16, 1054 (1973).

[9]A. H. Glasser, J. M. Greene, and J. L. Johnson, Phys. Fluids 18, 875 (1975).

[10]B. Coppi, Phys. Fluids 8, 2273 (1965).

[11]J. F. Drake and Y. C. Lee, Phys. Fluids 20, 1341 (1977).

[12]It is interesting to note that a Nyquist mapping from the α plane to the g plane also leads to a simple discussion of the results.

[13]R. D. Hazeltime, D. Dobrott, and T. S. Wang, Phys. Fluids 18, 1778 (1975).

[14]L. D. Pearlstein and H. L. Berk, Phys. Rev. Lett. 23, 220 (1969).