Critical Surface Density of the Superfluid Component in ⁴He Films

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It is shown that published third-sound measurements are in very good agreement with the theoretical result by Nelson and Kosterlitz that in the limit as $T \to T_c^-$ the ratio of the density per unit area of superfluid component to the critical temperature, T_c, is the universal constant $(2/\pi)(m/\hbar)^2 k_B$.

In a recent Letter Nelson and Kosterlitz[1] discuss the theoretical result that the ratio of the density per unit area of the superfluid component of a helium film to the temperature at the critical point (superfluid onset point) as $T \to T_c$ from below T_c, the critical temperature, is a universal constant given by

$$(\bar{\rho}_{sf} d/T)_{T_c} = (2/\pi)(m/\hbar)^2 k_B$$
$$= 3.49 \times 10^{-9} \text{ g cm}^{-2} \text{ T}^{-1}. \quad (1)$$

$\bar{\rho}_{sf}$ is the average volume density of superfluid component in the film and d is the thickness of the film, m is the mass of the helium atom, and T, $\hbar$, and k_B have their customary meanings.

There are many theories which have been advanced and not all of them lead to Eq. (1). Nelson and Kosterlitz suggest that agreement of experimental results with Eq. (1) could be used as a criterion for choosing which of the theories, if any, are correct. They comment that "measurements of third-sound propagation in films are not inconsistent with the prediction" [Eq. (1)]. The purpose of the present Letter is to point out that a much stronger statement can be made, namely,

that third-sound measurements provide values which are in *very good agreement* with Eq. (1).

Table I lists $\bar{\rho}_{sf}$, d, and $\bar{\rho}_{sf}d$ at superfluid onset obtained from Ref. 2. $\bar{\rho}_{sf}$ and d were determined by the relations

$$\frac{\bar{\rho}_{sf}}{\rho} = C_3^2 \left(1 + \frac{TS}{L}\right)^{-1} \left(\frac{3k_B T}{m} \ln \frac{p_0}{p}\right)^{-1}, \quad (2)$$

$$\frac{\alpha}{d^3} = \frac{k_B T}{m} \ln \frac{p_0}{p}, \quad (3)$$

where α/d^3 is the van der Waals potential, p_0 is the saturated vapor pressure, p is the vapor pressure in equilibrium with the film, C_3 is the velocity of third sound, ρ is the density of He II, S is its entropy, and L its latent heat. Therefore

$$\frac{\bar{\rho}_{sf}d}{T} = \frac{\rho C_3^2 \alpha^{1/3}}{3T} \left(\frac{k_B T}{m} \ln \frac{p_0}{p}\right)^{-4/3} \left(1 + \frac{TS}{L}\right)^{-1}. \quad (4)$$

In obtaining the values listed in the table the density ρ was taken to be that of bulk helium at vapor pressure. It is, however, greater in the film and allowance for this would increase the experimental value of $\bar{\rho}_{sf}d$ and decrease the discrepancy with the theoretical value.

There are additional data by Scholtz, McLean, and Rudnick[3] and by Scholtz[4] which are collected in Table II. Two different substrates—glass and

TABLE I. Critical values of thin film superfluid helium (on glass substrate) parameters obtained from Ref. 2.

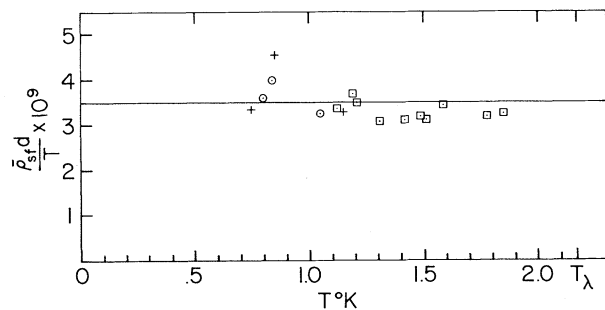| T (°K)	$\bar{\rho}_{sf} \times 10^2$ (g cm⁻³)	d (Atomic layers; 1 layer = 3.6 Å)	$(\bar{\rho}_{sf}d/T) \times 10^{+9}$ (g cm⁻² T⁻¹)
1.125	4.04	2.600	3.361
1.192	4.21	2.904	3.692
1.205	4.02	2.938	3.530
1.310	3.74	2.986	3.070
1.415	3.80	3.189	3.083
1.495	3.84	3.453	3.193
1.512	3.73	3.507	3.114
1.586	3.94	3.879	3.469
1.778	3.18	4.976	3.204
1.852	2.98	5.585	3.235
Mean			(3.30 ± 0.21)

FIG. 1. Plot of values of $\bar{\rho}_{sf}d/T$ from Tables I and II. Squares, values from Table I; plusses, values from Table II, glass substrate; circles, values from Table II, CaF₂ substrate. The line is Eq. (1).

TABLE II. Critical values of thin-film superfluid helium parameters obtained from Refs. 3 and 4.

Substrate	T (°K)	$\bar{\rho}_{sf} \times 10^2$ (g cm^{-3})	d (Atomic layers; 1 layer = 3.6 Å)	$(\bar{\rho}_{sf}\, d/T) \times 10^9$ (g cm^{-2} T^{-1})
Glass	0.75	3.32	2.11	3.362
	0.85	4.53	2.39	4.580
	1.15	3.96	2.64	3.280
CaF$_2$	0.80	3.77	2.12	3.597
	0.84	4.15	2.25	4.006
	1.05	3.87	2.45	3.254
Mean (excluding value at 0.85°K)				(3.36 ± 0.26)

single-crystal CaF$_2$—were used. I have no explanation for the occurrence of the rather large value of $\bar{\rho}_{sf}/T$ at 0.85°K on the glass substrate. It is interesting to note that a decrease of only 7% in d would reduce the value to 3.49×10^{-9}. The results in Table II suggest that $\bar{\rho}_{sf}d/T$ remains constant down to low temperatures and its value is the same, or not very different, for the two substrates. The mean (excluding the value at 0.85°K) is seen to agree with the value given in Table I and the value in Eq. (1).

In appraising the results it helps to see them graphically as shown in Fig. 1. A least-squares fit (excluding the point at 0.85 °K) gives

$$\bar{\rho}_{sf}d/T = (3.83 - 0.37T) \times 10^{-9}.$$

Thus the change in $\bar{\rho}_{sf}d/T$ in the temperature range covered (0.75°K → 1.85°K) does not exceed the range of the standard deviations given in Tables I and II.

It is important to emphasize that the values of C_3 determined in the experiments, and consequently the values of the average superfluid density, were experimentally determined to be frequency independent and are the values in the low-frequency limit.

Note added.—A copy of new results by D. J. Bishop and J. D. Reppy of Cornell University, obtained with an "Andronikashvili pendulum"—also showing very good agreement with Eq. (1)—has just been received.

[1] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. 39, 1201 (1977).

[2] I. Rudnick and J. C. Fraser, J. Low Temp. Phys. 3, 225 (1970). See also I. Rudnick, in Proceedings of the Twelfth International Conference on Low Temperature Physics, Kyoto, 1970, edited by E. Kanda (Keigaku Publishing Co., Tokyo, Japan, 1971), p. 29. In both articles the quantity $\langle \rho_{sf} \rangle$ is listed in their respective tables. The quantity listed in the present article is $\bar{\rho}_{sf} = \langle \bar{\rho}_{sf} \rangle (1 - 1/d)$, where d in atomic layers is the quantity listed in the 1970 articles. The film thicknesses shown in the present article are 0.677 times those given in the 1970 articles because an incorrect van der Waals potential was in use then. See Ref. 10 of J. H. Scholtz, E. O. McLean, and I. Rudnick, Phys. Rev. Lett. 32, 147, 569(E) (1974), for a discussion of this. An earlier report of this work is R. S. Kagiwada, J. C. Fraser, I. Rudnick, and D. Bergman, Phys. Rev. Lett. 22, 338 (1969).

[3] Scholtz, McLean, and Rudnick, Ref. 2.

[4] J. H. Scholtz, Ph.D. dissertation, University of California, Los Angles, Calif., 1973 (unpublished).