[d]Permanent address: University of Tokyo, Tokyo, Japan.

[e]Now at Physikalishes Institut, Technische Hochschule Aachen, Aachen, Germany.

[1]For $\pi^0$ correlations, see K. Eggert *et al.*, Nucl. Phys. B98, 49 (1975); F. W. Büsser *et al.*, Phys. Lett. 51B, 311 (1974). For charged-particle correlations, see M. Della Negra *et al.*, CERN Report No. CERN/EP/PHYS77-10 (to be published); see also recent reviews by G. C. Fox and H. J. Frisch, Brookhaven National Laboratory Report No. BNL-50598, edited by H. Gordon and R. F. Peierls, Proceedings of the American Physical Society Meeting, Division of Particles and Fields, Upton, New York, 1976 (unpublished); by H. Bøggild, in Proceedings of the Seventh International Symposium on Multiparticle Dynamics, Kayserberg, June 1977 (unpublished); by M. Della Negra, in Proceedings of the European Conference on Particle Physics at Budapest, July 1977 (unpublished).

[2]C. Kourkoumelis, CERN Report No. CERN 77-06 (unpublished); J. H. Cobb *et al.*, Phys. Lett. 68B, 101 (1977).

[3]Although we have taken data by separate trigger with acceptance at smaller angles, their analysis is not yet complete.

[4]A more certain, but biased $\pi^0$ definition requires two reconstructed photons with $\pi^0$ effective mass. However, this definition yields poor efficiency at high $P_T$. When two reconstructed showers coexist in the same octant, the $\pi^0$ energy was taken as that of the highest-energy shower. The inclusion of the lower-energy shower or additional nearby energy in the $\pi^0$ definition has no significant effect on our results.

[5]The spectrum has been weighted for geometrical and energy efficiency as determined by calibration runs at the CERN proton synchrotron so that the two $\pi^0$'s, when given random angles would have a flat distribution, independent of transverse momentum above $P_T = 1.2$ GeV/$c$. Trigger corrections due to the ISR intersection angle have been made. The weight of an event varies from 0.3 to 3, with an average weight of 1.

[6]We have also examined the product $\Pi = P_{T_1} \cdot P_{T_2} \cdot P_{TG}$. We find distributions similar to those of the $E_T$ selection.

[7]C. Michael and L. Vanryckegham, University of Liverpool Report No. LTH 31, May 1977 (to be published), and private communication. We compared our spectrum in $\Delta\varphi$ with that predicted by the independent-emission model for events with $9 < E_T < 12$ GeV and $P_{T_1} > 1$ GeV; $P_{T_2} > 3$ GeV. The 412 events in our sample have a distribution essentially the same as that of Fig. 2(b). The model predicts a nearly uniform distribution with variations of $< 20\%$ over our azimuthal coverage.

[8]The diplot of Fig. 3 is not corrected for efficiency which varies by less than 30% in the populated regions.

[9]Recent measurements of $\Upsilon(9.5) \to e^+e^-$ in the same apparatus [J. H. Cobb *et al.*, Phys. Lett. 72B, 273 (1977)], combined with an assumed 3% branching ratio to electrons, would indicate that there are several hundred decays of $\Upsilon(9.5)$ to hadrons in the data sample from which the present events are drawn.

# Atomic Electron Correlation in Nuclear Electron Capture

Mau Hsiung Chen and Bernd Crasemann
*Department of Physics, University of Oregon, Eugene, Oregon 97403*
(Received 21 February 1978)

The effect of electron-electron Coulomb correlation on orbital-electron capture by the nucleus has been treated by the multiconfigurational Hartree-Fock approach. The theoretical $^7$Be $L/K$ capture ratio is found to be 0.086, and the $^{37}$Ar $M/L$ ratio, 0.102. Both ratios are smaller than the independent-particle predictions. Measurements exist for the Ar $M/L$ ratio, and agreement between theory and experiment is excellent.

Benoist-Gueutal's insight[1] that atomic electrons must be included in a complete description of orbital-electron capture by the nucleus[2] led to the introduction of atomic exchange and imperfect-overlap factors in the theoretical capture probability.[3-6] All existing work on electron capture has been carried out in the independent-particle approximation; effects due to electron-electron Coulomb correlation have been neglected. Here we report on a first effort to take correlation into account, by using the multiconfigurational Hartree-Fock (MCHF) approach.[7] We calculate the $^7$Be $L/K$ and $^{37}$Ar $M/L$ capture ratios.

The nuclear-electron-capture rate is[2]

$$\lambda_i = \lambda_i^0 B_i, \quad i = K, L, M, \ldots, \tag{1}$$

where $\lambda_i^0$ is the rate obtained when atomic matrix elements are neglected,[8] and $B_i$ is the exchange-

 

overlap correction factor. For example, if the initial and final states are represented by a single Slater determinant, then

$$B_K = K\{\langle 2s'|2s\rangle\langle 3s'|3s\rangle - \langle 2s'|1s\rangle\langle 3s'|3s\rangle[R_{2s}(0)/R_{1s}(0)] - \langle 2s'|2s\rangle\langle 3s'|1s\rangle[R_{3s}(0)/R_{1s}(0)]\}^2, \quad (2)$$

where

$$K = \langle 1s'|1s\rangle^2\langle 2s'|2s\rangle^2\langle 2p'|2p\rangle^{2q(2p)}\langle 3s'|3s\rangle^{2[q(3s)-1]}\langle 3p'|3p\rangle^{2q(3p)}. \quad (3)$$

Here, $q(nl)$ is the occupation number of the $nl$ shell, and primes denote the daughter atom. Bahcall[2-6] set $K=1$, while Vatai[2,9] retained the factor. Similar expressions exist for $B_L$ and $B_M$.

The capture ratio for shells $i$ and $j$, in allowed transitions, is[2]

$$(\lambda_i/\lambda_j) = (\lambda_i/\lambda_j)^0(B_i/B_j), \quad (4)$$

where

$$(\lambda_i/\lambda_j)^0 = [R_i^2(0)/R_j^2(0)](q_i^2/q_j^2), \quad i,j = K, L_1, M_1. \quad (5)$$

The $R$'s are electron radial wave functions, evaluated at the origin, and the $q$'s are neutrino energies. The contributions from $L_2$ and $M_2$ electrons are neglected here.

In our MCHF calculation, the ground state is

$$\Psi_g(\gamma LS) = \sum_i C_i\,\Phi(\gamma_i LS), \quad (6)$$

and the final-state wave function, describing the hole state after capture, is

$$\Psi_j'(\gamma LS) = \sum_i C_{ji}'\,\Phi_i'(\gamma_i LS). \quad (7)$$

The atomic matrix elements become

$$\langle \Psi_j|O|\Psi_g\rangle = \sum_{i,k} C_{jk}'C_i\langle \Phi_k'|O|\Phi_i\rangle, \quad (8)$$

where we have $O = \sum_b a_b R_b(0)$, and $a_b$ is the destruction operator.[4] The exchange-overlap correction factor is

$$B_i = \sum_j |\langle \Psi_j|O|\Psi_g\rangle/R_i(0)|^2, \quad (9)$$

where the summation extends over the states included in the multiconfigurational expansion.

For the $^7$Be $L/K$-capture-ratio calculation, the ground state is represented by

$$\Psi_g = C_1\Phi_1(1s^22s^2) + C_2\Phi_2(1s^22p^2). \quad (10)$$

The $1s$-hole state after $K$ capture is

$$\Psi_j = C_{j1}'\Phi_1'(1s2s^2) + C_{j2}'\Phi_2'(1s2p^2). \quad (11)$$

The $2s$-hole state after $L_1$ capture is represented by the single configuration

$$\Psi_j = \Phi'(1s^22s). \quad (12)$$

For the $^{37}$Ar $M/L$-capture-ratio calculation, we take the ground-state MCHF wave function to be

$$\Psi_g = C_1\Phi_1(1s^22s^22p^63s^23p^6) + C_2\Phi_2(1s^22s^22p^63p^63d^2(^1S)) + C_3\Phi_3(1s^22s^22p^63s^23p^4(^1S)3d^2(^1S)) + C_4\Phi_4(1s^22s^22p^63s^23p^4(^3P)3d^2(^3P)) + C_5\Phi_5(1s^22s^22p^63s^23p^4(^1D)3d^2(^1D)). \quad (13)$$

The $2s$-hole state is

$$\Psi_j = C_{j1}\Phi_1'(1s^22s2p^63s^23p^6) + C_{j2}\Phi_2^1(1s^22s2p^63s^23p^4(^1S)3d^2(^1S)) + C_{j3}\Phi_3'(1s^22s2p^63s^23p^4(^3P)^4P3d^2(^3P)) + C_{j4}\Phi_4'(1s^22s2p^63s^23p^4(^3P)^2P3d^2(^3P)) + C_{j5}\Phi_5'(1s^22s2p^63s^23p^4(^1D)3d^2(^1D)). \quad (14)$$

The $3s$-hole state after $M_1$ capture is

$$\Psi_j = C_{j1}\Phi_1'(1s^22s^22p^63s3p^6) + C_{j2}\Phi_2'(1s^22s^22p^63s^23p^4(^1D)3d) + C_{j3}\Phi_3'(1s^22s^22p^63s3p^4(^1S)3d^2(^1S)) + C_{j4}\Phi_4'(1s^22s^22p^63s3p^4(^3P)^4P3d^2(^3P)) + C_{j5}\Phi_5'(1s^22s^22p^63s3p^4(^3P)^2P3d^2(^3P)). \quad (15)$$

TABLE I. MCHF $\langle nl'|nl\rangle$ overlap integrals for $_4$Be electron capture.

| | $K$ capture | | | $L_1$ capture | |
|---|---|---|---|---|---|
| | $\|1s\rangle$ | $\|2s\rangle$ | $\|2p\rangle$ | $\|1s\rangle$ | $\|2s\rangle$ |
| $\langle 1s'\|$ | 0.972 09 | −0.190 99 | | 0.962 47 | −0.155 91 |
| $\langle 2s'\|$ | 0.171 93 | 0.967 85 | | 0.082 71 | 0.882 83 |
| $\langle 2p'\|$ | | | 0.992 60 | | |

TABLE II. MCHF $\langle nl'|nl\rangle$ overlap integrals for $_{18}$Ar electron capture.

| | $\|1s\rangle$ | $\|2s\rangle$ | $\|2p\rangle$ | $\|3s\rangle$ | $\|3p\rangle$ | $\|3d\rangle$ |
|---|---|---|---|---|---|---|
| | | | $L_1$ capture | | | |
| $\langle 1s'\|$ | 0.998 73 | −0.029 77 | | −0.006 30 | | |
| $\langle 2s'\|$ | 0.027 05 | 0.992 50 | | −0.104 96 | | |
| $\langle 2p'\|$ | | | 0.998 58 | | −0.022 79 | |
| $\langle 3s'\|$ | 0.007 98 | 0.101 77 | | 0.992 74 | | |
| $\langle 3p'\|$ | | | 0.021 42 | | 0.999 27 | |
| $\langle 3d'\|$ | | | | | | 0.999 54 |
| | | | $M_1$ capture | | | |
| $\langle 1s'\|$ | 0.998 75 | −0.029 21 | | −0.006 28 | | |
| $\langle 2s'\|$ | 0.026 23 | 0.992 28 | | −0.097 36 | | |
| $\langle 2p'\|$ | | | 0.994 45 | | −0.081 77 | |
| $\langle 3s'\|$ | 0.00702 | 0.090 20 | | 0.989 13 | | |
| $\langle 3p'\|$ | | | 0.075 52 | | 0.990 47 | |
| $\langle 3d'\|$ | | | | | | 0.932 00 |

TABLE III. Electron radial wave-function ratios $R_{ns}^2(0)/R_{n's}^2(0)$, exchange-overlap correction factors $B_i$, and capture ratios $\lambda_i/\lambda_j$.

| Element | Quantity | | Result |
|---|---|---|---|
| $^7_4$Be | $R_{2s}^2(0)/R_{1s}^2(0)$, | HF | 0.0332 |
| | | MCHF [c] | 0.0300 |
| | $B_K$, | HF(V) [a] | 0.816 |
| | | HF(B) [b] | 0.900 |
| | | MCHF [c] | 0.792 |
| | $B_L$, | HF(V) [a] | 2.222 |
| | | HF(B) [b] | 3.045 |
| | | MCHF [c] | 2.259 |
| | $\lambda_L/\lambda_K$, | HF(V) [a] | 0.090 |
| | | HF(B) [b] | 0.112 |
| | | MCHF [c] | 0.086 |
| $^{37}_{18}$Ar | $R_{3s}^2(0)/R_{2s}^2(0)$, | HF | 0.0977 |
| | | MCHF [c] | 0.0669 |
| | $B_L$, | HF(V) [a] | 1.121 |
| | | HF(B) [b] | 1.171 |
| | | MCHF [c] | 1.098 |
| | $B_M$, | HF(V) [a] | 1.322 |
| | | HF(B) [b] | 1.549 |
| | | MCHF [c] | 1.674 |
| | $\lambda_M/\lambda_L$, | HF(V) [a] | 0.115 |
| | | HF(B) [b] | 0.129 |
| | | MCHF [c] | 0.102 |
| | | Experiment [d] | $0.104^{+0.007}_{-0.003}$ |

[a]Hartree-Fock, Vatai's approach (Refs. 2 and 9).
[b]Hartree-Fock, Bahcall's approach (Refs. 2 and 3–6).
[c]Present multiconfigurational HF calculation.
[d]Ref. 11.

The MCHF wave functions, including the amplitudes $C$, were computed with the Froese Fischer program.[7] The electrostatic interaction matrix elements were calculated with Hibbert's program.[10] The one-electron overlap integrals are listed in Tables I and II. The electron radial-wave-function ratios at the origin and the overlap-exchange correction factors $B_i$ as well as the electron-capture ratios are listed in Table III. For comparison, theoretical single-configuration HF capture ratios[2] and the experimental result[11] for $^{37}$Ar are also listed; there is no measurement of the $^7$Be $L/K$ ratio.

Electron correlation is seen to have a substantial effect on nuclear capture ratios when outer electrons are involved. Compared with single-configuration HF results according to Vatai's approach,[2] the MCHF $L/K$ capture ratio of $^7$Be is reduced by 4.4%; the $^{37}$Ar $M/L$ ratio is reduced by 11% and brought into excellent agreement with experiment.[2,11]

This work was supported in part by the U. S. Army Research Office under Grant No. DAAG29-78-G-0010 and by the National Aeronautics and Space Administration under Grant No. NGR 38-003-036.

[1]P. Benoist-Gueutal, C. R. Acad. Sci. 230, 624 (1950).

[2]For a recent review, see W. Bambynek, H. Behrens, M. H. Chen, B. Crasemann, M. L. Fitzpatrick, K. W. D. Ledingham, H. Genz, M. Mutterer, and R. L. Intemann, Rev. Mod. Phys. 49, 78 (1977).

[3]J. N. Bahcall, Phys. Rev. Lett. 9, 500 (1962).
[4]J. N. Bahcall, Phys. Rev. 129, 2683 (1963).
[5]J. N. Bahcall, Phys. Rev. 131, 1756 (1963).
[6]J. N. Bahcall, Nucl. Phys. 71, 267 (1965).
[7]C. Froese Fischer, Comput. Phys. Commun. 4, 107 (1972).
[8]H. Brysk and M. E. Rose, Rev. Mod. Phys. 30, 1169 (1958).
[9]E. Vatai, Nucl. Phys. A156, 541 (1970).
[10]A. Hibbert, Comput. Phys. Commun. 2, 180 (1971).
[11]J. P. Renier, H. Genz, K. W. D. Ledingham, and R. W. Fink, Phys. Rev. 166, 935 (1968).

# Population of Resonant $^{12}C+^{12}C$ States via the Reaction $^{12}C(^{16}O, \alpha)^{24}Mg$

A. J. Lazzarini, E. R. Cosman, A. Sperduto, S. G. Steadman, W. Thoms, and G. R. Young
*Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
(Received 10 November 1977)

Excitation functions for the reaction $^{12}C(^{16}O,\alpha)^{24}Mg$ have been measured from $E_{lab}(^{16}O) = 62$ to 110 MeV using a counter telescope at $\theta_{lab} = 7.5°$. Selective population of relatively few states at very high excitation energies in $^{24}Mg$ [$E_x(^{24}Mg) > 20$ MeV] is seen. A possible correspondence is found between these states and the narrow resonances reported in $^{12}C + ^{12}C$ reactions. In addition, a possible correspondence between averaged yields in $^{12}C(^{16}O,\alpha)^{24}Mg$ and gross structure seen in several $^{12}C + ^{12}C$ reaction channels is cited.

The reaction $^{12}C(^{16}O, \alpha)^{24}Mg$ has been studied extensively because of its striking final-state selectivity. It is interesting to consider whether this behavior reflects special structures in the initial, compound, and final systems. In this work we have significantly extended the $^{12}C(^{16}O, \alpha)^{24}Mg$ excitation functions. An apparent correlation has been found between individual pronounced transitions in that reaction and resonant states which have been seen in $^{12}C + ^{12}C$ reactions. Furthermore, the envelope of the $^{12}C(^{16}O, \alpha)^{24}Mg$ transitions shows a weak correlation with gross structure variations in the $^{12}C$ strength function as indicated by several $^{12}C + ^{12}C$ reaction-channel excitation functions.

High-resolution spectra from the reaction $^{12}C(^{16}O, \alpha)^{24}Mg$ were measured from $E_{lab}(^{16}O) = 62.0$ to 100.0 MeV in 1-MeV steps. Two measurements at $E_{lab}(^{16}O) = 105.0$ and 110.0 MeV were also taken. The experiment was performed at the Brookhaven National Laboratory Tandem Van de Graaff facility and employed a surface-barrier counter telescope placed at $\theta_{lab} = 7.5° \pm 0.25°$. Targets were nominally 45-$\mu g/cm^2$ natural carbon. The experimental resolution was typically 90 keV.

The primary objective of the study was to determine if the $^{12}C(^{16}O, \alpha)^{24}Mg$ spectra at higher energies show any gross- or fine-structure correlations to previously known $^{24}Mg$ states which have been observed via $^{12}C + ^{12}C$ resonance reactions. The $\alpha$ spectra contain numerous previously unseen transitions in the range of $E_x(^{24}Mg) = 20$ to 35 MeV—a region in which most $^{12}C + ^{12}C$ resonances have been recorded. The $\alpha$-transition yields exhibit compound-nuclear fluctuations; to average out this effect and to enhance the persistently strong transitions, the spectra were averaged over the incident $^{16}O$ energy. This was performed by linearizing the 39 individual $\alpha$ spectra to a common $^{24}Mg$ excitation-energy scale. Kinematic corrections were performed so the energy-summed spectra would reflect center-of-mass cross sections. A smooth evaporative background was subtracted by hand from each spectrum to further enhance strong discrete transitions in the summed spectra. Figure 1 shows three typical linearized spectra and background curves. It is understood that the magnitude of the underlying

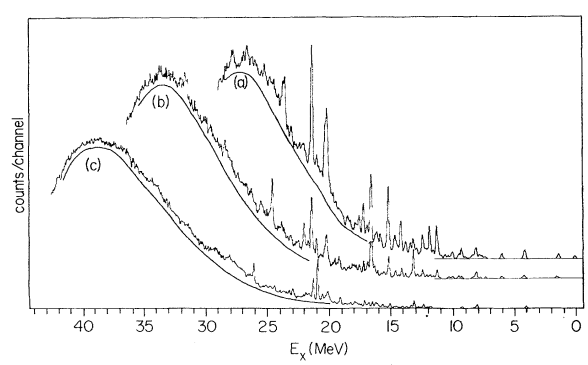


FIG. 1. Typical $^{12}C(^{16}O,\alpha)^{24}Mg$ spectra at $E_{lab} = 63$ (curve *a*), 77 (curve *b*), and 91 MeV (curve *c*). They have been linearized in $E_x(^{24}Mg)$ and the smooth curves are hand drawn to represent the background that is to be subtracted.