The experiment yields

$$\Gamma/K^2 = 4\times 10^{-8}\ \mathrm{cm^2/s}.$$

Using the following empirical values for the parameters in Eq. (9) $\mu = 10^{-2}$ cm/s K^4 and $\gamma_{sl} = 44$ mJ/m^2 we obtain the theoretical prediction $\Gamma/K^2 = 3.8\times 10^{-8}$ cm^2/s.

The agreement with the experimental value is gratifying and provides strong support for the proposed mechanism for the correlation time of the scattering fluctuations.

As to the amplitude A, all we can say is, that $(\bar{A})^2$ is proportional to the growth rate (Fig. 2) and hence to the rate at which the Gibbs free energy is released at the interface. Furthermore, since experimentally a growth rate threshold must be exceeded to initiate the fluctuations and also since these fluctuations can be maintained on reducing the growth rate below the threshold it seems likely that the fluctuations originate in an instability.

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Critical Dynamics of a Heisenberg Spin-Glass

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The spin-glass transition of a magnetic material with quenched impurities is studied with use of perturbative mode-mode coupling theory, without the use of the replica method. In mean-field theory we find that the transport coefficient $\tilde{\Gamma}$ diverges just above the spin-glass freezing temperature T_f like $(T-T_f)^{-1/2}$ for $d>4$ and $(T-T_f)^{2/d-1}$ for $2<d<4$.

Recently, there has been a considerable amount of interest in the spin-glass phase.[1-6] This phase, which occurs for certain amorphous magnetic systems, is characterized by a frozen magnetization with a vanishing spatial average. Most of the theoretical attempts to describe this phase have focused upon the static properties, and have employed the "replica" trick of letting the number of degrees of freedom, n, of the spins approach zero. Recently, however, Thouless, Anderson, and Palmer[4] and Ma and Rudnick[6] have performed mean-field calculations without using the replica method. Ma and Rudnick[6] have also examined dynamics using a time-dependent Ginzburg-Landau model. However, this model neglects the effects of the precession of a spin in the local field of neighboring spins which are known to play an important role in the dynamics of a Heisenberg magnet.[7] Since it is believed that in a real spin-glass, the spins interact via a Heisenberg interaction,[8] we believe that a proper theory should take account of the dynamical effects appropriate to that system. In this Letter, we do not use the replica ($n\to 0$) trick.

The dynamics of an isotropic Heisenberg spin system with quenched impurities may be defined by the equation of motion,[7,9]

$$\partial\vec{S}/\partial t = \lambda\vec{S}\times\vec{H} - \Gamma\nabla^2\vec{H} + \vec{\zeta}, \qquad (1)$$

where $\vec{S}(x,t)$ is a three-component vector spin field, $\vec{H}(x,t)$ is the local field, and $\vec{\zeta}(x,t)$ is a random noise simulating the effect of thermal fluctuations on the spins. We assume the local field is given by the derivative of the free energy $F\{\vec{S}\}$, which is assumed to be of the Ginzburg-Landau form for a given spin configuration,

$$F(\vec{S}) = \tfrac{1}{2}\int d^dx[(\nabla S)^2 + (r_0+\varphi)S^2 + \tfrac{1}{2}u(S^2)^2 - \vec{h}\cdot\vec{S}],$$
$$\vec{H}(x,t) = -\delta F/\delta\vec{S}(x,t), \tag{2}$$

where $\vec{h}$ is the external field, and $\varphi(x)$ is a random local variation in effective temperature. This model is similar to that discussed by Ma and Mazenko[7] for the isotropic Heisenberg ferromagnet, differing only by the inclusion of the random variable φ. This quantity is assumed to be a time-independent Gaussian random variable due to the particular configuration of the quenched impurities, and thus has a very different physical origin from that of $\vec{\zeta}(x,t)$, the Gaussina random noise. The average of each of these random variables is assumed to be zero, and the correlations in Fourier space are given by

$$\langle\varphi_k\varphi_{k'}\rangle = \Delta\delta_{k',-k},$$
$$\langle\zeta_k{}^i(t)\zeta_{k'}{}^j(t')\rangle = 2\Gamma k^2\delta_{k',-k}\delta_{ij}\delta(t-t'), \tag{3}$$

and higher cumulants are assumed to vanish. We note that Γ is the bare transport coefficient; Γr_0 is a diffusion constant. If $\lambda=0$, this model is a time-dependent Ginzburg-Landau model with a conserved spin. (Ma and Rudnick considered the case of a nonconserved spin.) The λ term is of the form $-\lambda\vec{S}\times\nabla^2\vec{S}$, which is closely related to the usual Heisenberg equation of motion. We shall take the viewpoint that the spin-glass freezing temperature T_f is higher than the Curie temperature T_C. We are interested here only in determining the behavior near T_f; we shall not discuss the competition between spin-glass and ferromagnetic order.[3]

Our new result is that the transport coefficient $\tilde{\Gamma}$ diverges at the spin-glass freezing temperature T_f like $\tilde{\Gamma}\propto(T-T_f)^{-1/2}$. The freezing temperature T_f is the temperature at which the order parameter q becomes nonvanishing. This order parameter, q, for the spin-glass phase is given by

$$q\delta_{ij} = [\langle S^i\rangle\langle S^j\rangle]_{\rm av}, \tag{4}$$

where $\langle\;\rangle$ stands for a thermal average, and $[\;]_{\rm av}$ is a configuration average.

We shall now briefly discuss the derivation of this result, beginning with the statics. To leading order in the external static field $\vec{h}$, for a fixed φ, the thermal average $\langle S_k{}^i\rangle$ is given by

$$\langle S_k{}^i\rangle \equiv \int d^dx\, e^{-ik\cdot x}\langle S^i(x,t)\rangle = G_0(k,0)h_k{}^i + G_0(k,0)\int d^dk'(2\pi)^{-d}\varphi_{k-k'}G_0(k',0)h_{k'}{}^i + \dots, \tag{5}$$

where $G_0(k,\omega) = (r_0+k^2-i\omega/\Gamma k^2)^{-1}$ is the zeroth-order spin response function and r_0 includes the one-loop self-energy correction arising from the u term in F. Equation (5) is shown diagrammatically in Fig. 1(a). We now square Eq. (5) and average over φ. Keeping only terms shown in Fig. 1(b) and 1(c) for $[\langle S_k{}^i\rangle]_{\rm av}$ and $[\langle S_k{}^i\rangle\langle S_{-k}{}^i\rangle]_{\rm av}$, the single-loop and "rainbow" diagrams indicated in Fig. 1(b) merely shift r_0 to a new value r, and we have

$$q = \Pi_0(0,0)h^2/[1-\Delta\Pi_0(0,0)], \tag{6}$$

where

$$\Pi_0(0,\omega) = \int d^dk(2\pi)^{-d}G_0(k,\omega)G_0(-k,-\omega) \tag{7}$$

with r_0 replaced by r in G_0. Thus, q can be nonvanishing for $h\to 0$ if the denominator of the right-hand side of Eq. (6) vanishes. This defines the spin-glass freezing temperature T_f, which we take to be greater than T_C, the temperature for ferromagnetic ordering.

Below T_f, the mean-field self-energy acquires

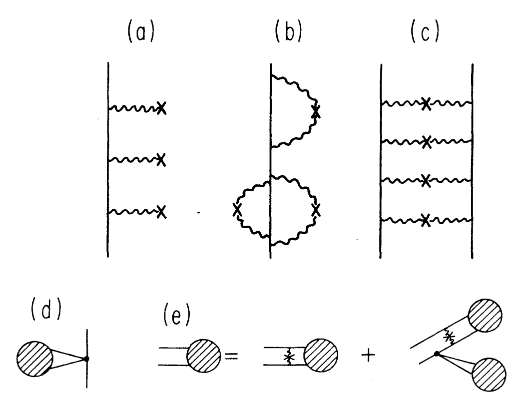

FIG. 1. (a) Graph of Eq. (5) in the text. The straight lines are zeroth-order spin response functions, and the wavy lines represent the random part, $\varphi(x)$, of the quadratic coefficient $r(x)$ in the Landau-Ginzberg function (2). (b) Graph for the configuration average [over $\varphi(x)$] of the spin correlation function. (c) Graph obtained by averaging over configurations the product of two graphs such as shown in (a). (d) New self-energy diagram below T_f. (e) Diagrammatic equation for q.

an extra term proportional to the spin-glass order parameter q [Fig. 1(d)]. This gives a q dependence to the parameter r in G, and thus to Π; we write $\Pi_q(0,\omega)$ where we previously had $\Pi_0(0,\omega)$. Then the equation for the self-consistent value of the order parameter [Fig. 1(e)] can be given by

$$q = \Delta \Pi_q(0,0) q. \tag{8}$$

Thus the ladder sum $(1-\Delta\Pi_q)^{-1}$ remains divergent at $\omega = 0$ everywhere below T_f. [Ma and Rudnick[6] have performed this calculation for a scalar spin, but the form of Eq. (8) does not depend upon the number of components of the spin.]

We now turn to the dynamics. To lowest order in λ, the shift in the transport coefficient $\tilde{\Gamma}$ $(=\Gamma + \Delta\Gamma)$ is given by a set of graphs, one of which is

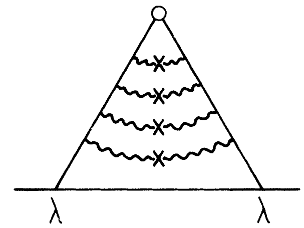

FIG. 2. Typical graph contributing to the shift in the transport coefficient $\Delta\Gamma$. A line with a small circles represents a C_0 function.

pictured in Fig. 2. Corrections to the λ vertices due to the fluctuations in φ vanish. Corrections to the "noise vertex" do not vanish, because $[|G^2|]_{\text{av}} \neq |[G]_{\text{av}}|^2$. In the ladder approximation which we used for the statics (Fig. 2) we have

$$(r+k^2)\frac{\Delta\Gamma}{\Gamma}$$

$$= -\frac{-2\lambda^2}{\Gamma^2 k^2}\int d^dq \int d^dq' \int d\nu (2\pi)^{-(2d+1)}\left[\frac{(k^2-q^2)[q^2-(q+k)^2]}{(q+k)^2}\,\frac{\Delta C_0(q',\nu)G_0(q,\nu)G_0(-q,-\nu)G_0(k+q,\nu)}{1-\Delta\Pi_0(0,\nu)}\right], \tag{9}$$

where $C_0(k,\omega) \equiv (2/\omega)\,\mathrm{Im}G_0(k,\omega)$. In Eq. (9), the momentum factors are due to the λ vertices[7,9] since the Fourier transform of $\lambda\vec{S}\times\nabla^2\vec{S}$ has an explicit k^2 factor. Equation (9) differs qualitatively from the corresponding equation for a pure ferromagnet because of the factor $[1-\Delta\Pi_0(0,\nu)]^{-1}$, which is large near T_f for small ν. We may expand $\Pi_0(0,\nu)$ for small ν to obtain

$$\begin{aligned} &\Pi_0(0,\nu) = \Pi_0(0,0) - (\nu/\Gamma^*)^2 + O(\nu^4) \quad (4<d),\\ &\Pi_0(0,\nu) = \Pi_0(0,0) - (\nu/\Gamma^*)^{d/2} + O(\nu^2) \quad (2<d<4),\end{aligned} \tag{10}$$

where

$$\begin{aligned} &\Gamma^2/\Gamma^{*2} = \tfrac{1}{3} r^{(d-12)/2} 2^{-(d+1)} \pi^{-d/2} \Gamma(\tfrac{1}{2}d - 2)\Gamma(6-\tfrac{1}{2}d)/\Gamma(\tfrac{1}{2}d) \quad (4<d<12),\\ &(\Gamma/\Gamma^*)^{d/2} = r^{-(d+4)/2} 2^{-(d+1)} \pi^{-d/2}\Gamma(\tfrac{1}{4}d)\Gamma(1-\tfrac{1}{4}d) \quad (2<d<4).\end{aligned} \tag{11}$$

Here $\Gamma(x)$ is the gamma function. We now define $a^2 = 1-\Delta\Pi_0(0,0) \propto T - T_f$. Since the integrand of the ν integral in (9) is peaked sharply at $\nu = 0$, we may obtain the dominant behavior by setting $\nu = 0$ in the integrand except in $\Pi_0(0,\nu)$:

$$\begin{aligned} &\frac{\Delta\Gamma}{\Gamma} = \frac{4\lambda^2\Gamma^* AB\Delta^{1/2}}{ra\Gamma^3} \quad (d>4),\\ &\frac{\Delta\Gamma}{\Gamma} = \frac{4\lambda^2\Gamma^* AB\Delta^{1-2/d}}{ra^{2-4/d}\Gamma^3}\,\frac{4\Gamma(2/d)\Gamma(1-2/d)}{d\pi} \quad (2<d<4),\end{aligned} \tag{12}$$

where

$$A = \int d^dk (2\pi)^{-d} k^{-2}(r+k^2)^{-2}, \quad B = \int d^dk (2\pi)^{-d}(\hat{q}\cdot\hat{k})^2(2r+k^2)(r+k^2)^{-4}.$$

Thus, recalling the definition of a, we have $\Delta\Gamma \propto (T-T_f)^{-1/2}$ for $d>4$ and $\Delta\Gamma \propto (T-T_f)^{(2/d)-1}$ for $d<4$. A proper analysis at this level of the spin-glass phase below T_f would involve interactions between Halperin-Saslow spin-wave modes.[5] This more complex problem will be discussed in another paper.

One worry about a model such as Eq. (2) is that it looks superficially like a random ferromagnet,[10] and contains no obvious spin-glass physics, despite the fact that a spin-glass instability falls out naturally in ladder approximation. We have therefore also considered a different model which explicitly contains random exchange interactions between spins on a lattice. It is a Wilson-Fisher "soft-spin"[11] version of the classical random Heisenberg model considered by Edwards and Anderson[1]:

$$\beta H_{\rm eff}[\vec{S}] = \tfrac{1}{2}\beta\sum_{ij} J_{ij}\vec{S}_i\cdot\vec{S}_j + \tfrac{1}{2}\sum_i r\vec{S}_i^{\,2} + \tfrac{1}{4}u\sum_i(\vec{S}_i^{\,2})^2, \tag{13}$$

where J_{ij} is a Gaussian random variable. The dynamics of the model are specified by an equation of motion like (1), with a finite-difference expression replacing the $\nabla^2\vec{H}_i$ at site i given by

$$\beta\vec{H}_i = \beta\sum_j J_{ij}\vec{S}_j + r\vec{S}_i + u\vec{S}_i(\vec{S}_i^{\,2}). \tag{14}$$

This model can be solved to leading order in the reciprocal of the number of nearest neighbors and gives the same singular behavior in $\Delta\Gamma$ that we found above in the Ma-Rudnick model. The reason is that the equations for the order parameter, the spin-spin correlation function, and $\Delta\Gamma$ have very similar structures in the two models. This calculation will be published in a later paper.

Since the mean-field results are valid for $d > 6$,[3] we expect that this approach will only be valid in that dimensionality regime. For $d<6$, a renormalization-group expansion in $\epsilon = 6 - d$ should be performed. That calculation is in progress.

We note the similarity between the result for the transport coefficient for a Heisenberg spin-glass, and that for a Heisenberg antiferromagnet. That is, long-wavelength spin fluctuations exhibit "critical speeding-up," although the critical exponents are different in the two cases. (The near equality of the exponents for $d=3$ is just a coincidence.) This behavior should be observable in inelastic neutron scattering experiments.

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