[10]V. V. Alikaev *et al.*, Fiz. Plasm. 3, 230 (1977) [Sov. J. Plasma Phys. 3, 127 (1977)]; A. G. Litvak *et al.*, Nucl. Fusion 17, 659 (1977).

[11]T. H. Stix, *Theory of Plasma Waves* (McGraw-Hill, New York, 1962), Chap. 3.

[12]V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon, New York, 1964), Chap. V.

[13]J. Preinhaelter and V. Kopecky, J. Plasma Phys. 10, 1 (1973).

[14]S. Ichimaru, *Basic Principles of Plasma Physics* (Benjamin, Reading, Mass., 1973), Chap. 5.

[15]S. Tanaka *et al.*, Phys. Lett. 59A, 290 (1976).

[16]J. J. Schuss and J. C. Hosea, Phys. Fluids 18, 727 (1975).

[17]J. C. Hosea, V. Arunasalam, and R. Cano, Phys. Rev. Lett. 39, 408 (1977); A. E. Costley and TFR Group, Phys. Rev. Lett. 38, 1477 (1977).

# Three-Dimensional Velocity Diffusion in Two-Stream Turbulence

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Thr turbulent state initiated by the electron-beam–ion two-stream instability is investigated. The spectrum of fluctuations is concentrated on a Cherenkov cone with a single modulus of phase velocity, azimuthally symmetric around the beam axis. The associatedd diffusion in velocity space occurs mainly on the surface of a double cone, complementary to the Cherenkov emission cone. This model is supported by an experiment in which the three-dimensional diffusion coeffient is measured with use of space-time correlation functions.

Little is known about the properties of the turbulence generated by the intense-electron-beam–ion two-stream (EBITS) instability.[1] In numerical simulations, two- and three-dimensional spectra of turbulent fluctuations have been observed.[2] In laboratory experiments, however, only one-dimensional standing eigenmodes of the two-stream instability have been observed in a bounded geometry.[3] The only reported observations of three-dimensional electrostatic turbulence of which we are aware have been of unstable ion-acoustic modes.[4] In this Letter, we present details of the three-dimensional structure of two-stream turbulence. Since charged-particle transport properties[5] are determined by the velocity-space diffusion coefficient, we have developed a method to measure this coefficient using space-time correlation functions.

In experiments on the EBITS instability, an interesting effect occurs when a stationary electron population is present, as is often the case when the electron beam is injected and no external electric field is applied. The instability-growth rate can remain large, but maximizes for a beam speed of about twice the electron thermal speed[6] $[=(KT_e/m_e)^{1/2}]$. Thus for a beam speed $v_b \gtrsim 2v_{Te}$, oblique Cherenkov emission can be expected,[7] at an angle $\varphi$ relative to the beam axis, given by

$$\varphi = \arccos[2(KT_e/m_e)^{1/2}/v_b]. \tag{1}$$

In velocity space, the modes of the turbulent spectrum will therefore be concentrated on the surface of a Cherenkov cone (Fig. 1) lying along the beam axis. The modes all have a phase speed $c$ (the phase speed of the most unstable modes) and each mode lies at an angle $\varphi$ with respect to the beam axis. Because of wave-particle resonance, each mode will diffuse particles whose velocity component along the direction of propagation $\varphi$ is equal to the phase speed $c$. Thus, a mode will diffuse all particles whose velocity vectors terminate on a plane normal to the phase-velocity vector at its extremity. The ensemble of such planes, each perpendicular to a mode on the Cherenkov cone, generates the diffusion domain $D$ (Fig. 1). $D$ is the volume exterior to a double cone aligned along the beam axis with its apex lying at $c/\cos\varphi$ relative to the origin, and with an open angle $2(\pi/2-\varphi)$. No diffusion occurs outside $D$.

We can describe the diffusion in velocity space due to a random, homogeneous, stationary, electrostatic electric field, in terms of the particle-velocity variances. These are obtained by integrating the particle equation of motion along un-

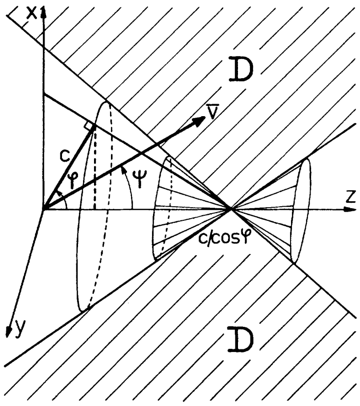


FIG. 1. Cherenkov turbulence in phase space. The phase-velocity vectors all lie on a Cherenkov cone, at an angle $\varphi$ with respect to the beam axis, with a constant modulus $c$. The diffusion planes perpendicular to each mode fill the diffusion domain $D$ (cross-hatched region), bounded by the double-cone complementary to the Cherenkov cone.

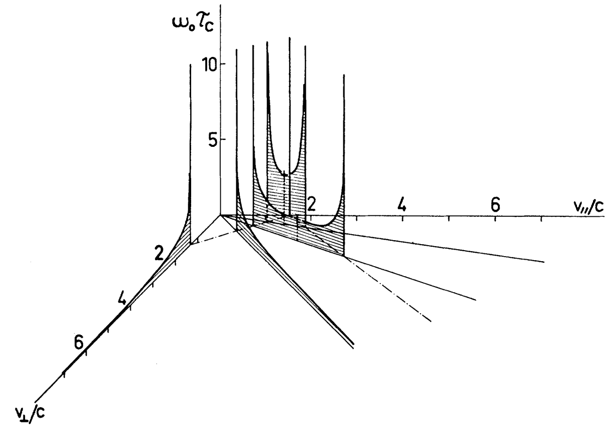


FIG. 2. Correlation time $\tau_c$ (or diffusion coefficient), Eq. (6), for Cherenkov turbulence. Here $\varphi = 50°$ and $\vec{v}_\parallel$ is parallel to the beam axis. $\tau_c(\vec{v})$ is plotted vs polar coordinates, for different values of the angle $\psi$ relative to the $\vec{v}_\parallel$ (beam) axis: $\psi = 0°$, 14°, 26°, 45°, and 90°. The correlation time diverges on the boundaries of the diffusion domain $D$ [dashed line in the $(v_\perp, v_\parallel)$ plane].

perturbed particle trajectories.[8] The variance increases linearly with time at a rate given by the diffusion-coefficient tensor,

$$\vec{\vec{D}}(\vec{v}) = \frac{\langle \Delta\vec{v}\Delta\vec{v}\rangle}{2t} = \left(\frac{q}{m}\right)^2 \langle E^2\rangle \vec{\vec{\tau}}(\vec{v}), \tag{2}$$

where $\vec{\vec{\tau}}_c(\vec{v})$ is the correlation-time tensor. This correlation time can be expressed in terms of the electric-field space-time correlation function,

$$\tau_{ij}(\vec{v}) = \langle E^2\rangle^{-1} \int_0^\infty dt \langle E_i(0,0) E_j(\vec{r} = t\vec{v}, t)\rangle. \tag{3}$$

According to Eqs. (2) and (3), the correlation time $\vec{\vec{\tau}}(\vec{v})$ and the diffusion coefficient are both related to the form of the noise spectrum. However, $\vec{\vec{\tau}}(\vec{v})$ is the more convenient quantity to use experimentally, since it does not depend on the particular species of particle, nor on the spectrum amplitude. Equation (3) will be used to find $\vec{\vec{\tau}}(\vec{v})$ from measured space-time correlation functions.

In order to find theoretical values for $\vec{\vec{\tau}}(\vec{v})$, we rewrite Eq. (3) in terms of the spectral density $S(\omega, \vec{K})$ of the potential fluctuations,

$$\tau_{ij}(\vec{v}) = \frac{\pi \int d\omega\, d^3k\, k_i k_j S(\omega, \vec{k}) \delta(\omega - \vec{k}\cdot\vec{v})}{\int d\omega d^3k\, k^2 S(\omega, \vec{k})}. \tag{4}$$

To calculate the diffusion created by the Cherenkov turbulence of Fig. 1, the relevant spectral density is of the form

$$S(\omega, \vec{k}) = F(\omega)\delta(\omega - kc)\delta(\vec{n}\cdot\vec{v}_b{}^0 - \cos\varphi), \tag{5}$$

where $\vec{v}_b{}^0$ is the unit vector along the beam axis. If the total stochastic velocity diffusion is of interest, then only the trace of the diffusion tensor is retained. Inserting Eq. (5) into Eq. (4) we can obtain the corresponding trace $\tau_c$ of the correlation-time tensor:

$$\tau_c = \omega_0^{-1}\{[(v/c)\cos(\psi - \varphi) - 1][1 - (v/c)\cos(\psi + \varphi)]\}^{-1/2} \tag{6}$$

for velocities $\vec{v}$ lying inside the diffusion domain $D$ (Fig. 1), and $\tau_c = 0$ outside $D$. Here $\omega_0$ is a weighted mean frequency of the spectrum.[9]

Figure 2 shows this correlation time, normalized to $\omega_0^{-1}$, as a function of $\vec{v}$ for five values of the angle $\psi$ relative to the $\vec{v}_\parallel$ (beam) axis ($\psi = 0°$, 14°, 26°, 45°, and 90°), when $\varphi = 50°$. Note that $\tau_c$ diverges on the boundaries of the diffusion domain $D$. This result can be understood in the following way: For given $\vec{v}$, Eq. (4) retains those values of $S(\omega, \vec{k})$ on the $(\omega, \vec{k})$ resonant surface for which $\omega - \vec{k}\cdot\vec{v} = 0$; that is, Eq. (4) retains those modes whose phase velocity $\omega/k$ is equal to the component of $\vec{v}$ along $\vec{k}$. Thus instead of the four-dimensional $(\omega, \vec{k})$ space, one can use a three-dimensional $(\omega/k, \vec{k}^0)$ phase-velocity space to describe $S$. In this space, resonant modes lie on the surface of diameter $v$. For Cherenkov turbulence [Eq. (5)], $S$ is nonzero on the Cherenkov circle $C$ of Fig. 1 (modulus $c$, angle $\varphi$ relative to the beam axis). The intersection of this circle $C$ with the diffusing sphere determines the resonant modes. When $\vec{v}$ lies within the diffusion domain $D$, the sphere intersects $C$ at two points. When $\vec{v}$ lies on the boundary of $D$, the sphere is tangent to $C$, which leads to a divergence. When $\vec{v}$ lies at the apex of $D$, the sphere includes all of the circle $C$, leading to the $\delta$ function in the diffusion coefficient.

For given $\psi$, the divergence occurs once for $\psi=0$, twice when $0<\psi<\frac{1}{2}\pi-\varphi$, and then once when $\psi>\frac{1}{2}\pi-\varphi$. The strongest diffusion occurs not for particle velocities terminating on the Cherenkov circle ($\psi=\varphi$, $v=c$), but at the apex of the complementary cone ($\psi=0$, $v=c/\cos\varphi$). Because of the azimuthal symmetry of the turbulent spectrum, the figure should be extended symmetrically about the $(\tau_c, \vec{v}_\parallel)$ plane.

To check these predictions, experiments were carried out in a triple-plasma device, which is an extension of the multipole double-plasma arrangement.[10] Three plasmas are located end to end along a common axis and are separated by fine mesh grids (30 cm diam). The two end plasmas are used as electron-beam and ion-beam sources. These two counterstreaming beams enter the central interaction region (15 cm long). In order to obtain an intense ($I=1$ A) plane beam of electrons close to thermal speed ($v_{be}/v_{Te}\gtrsim 1$, $V_{be}=5-10$ eV), an extra set of hot tungsten filaments in the electron-beam source is connected directly to the chamber wall (anode) to replace the extracted electrons. Ions stream slowly from the ion source at the opposite side into the central region. In the following experiment, the condition were an argon pressure of $4\times10^{-4}$ Torr, $T_e\sim 3$ eV, $T_i\sim 0.2$ eV, $V_{be}=6$ V, and $n_{be}/n_i\gtrsim 0.7$ in the central chamber.

As predicted for the electron-beam–ion two-stream instability,[1] a spectrum of turbulent modes is observed to grow exponentially away from the electron-beam injection grid, with $k_i/k_r=0.6$. The spectrum peaks in frequency at $f_0=300$ kHz with $\Delta f/f_0=\frac{1}{3}$. The spectrum noise power saturates in about 14 mm with $(\delta n/n)_{\rm rms}=0.11$, and then decays. Two-probe spatial correlation measurements along the beam axis yield a damped oscillation with $k_\parallel=5.1$ cm$^{-1}$. Perpendicular to the beam axis, we find the oscillating behavior of a Bessel function, $J_0(k_\perp x)$, with $k_\perp=5.7$ cm$^{-1}$. This behavior is characteristic of a Cherenkov $k$ spectrum, peaking off-axis with $k_0\simeq 8$ cm$^{-1}$ at an angle $\varphi=\arctan(k_\perp/k_\parallel)\simeq 48°$.

Two small, positively biased, spherical probes (diam $\approx\lambda_D\approx 0.5$ mm) are used to measure the correlation function. One probe is fixed at a point on the beam axis defined as the origin ($x=0$, $z=0$) about 5 mm in front of the electron-beam injection point. The position of the second probe is stepped along $x$ (perpendicular axis) and $z$ (parallel axis) in small increments, chosen so that the probe is displaced along a straight line at an angle $\psi$ relative to the beam axis and passing through the origin (Fig. 1). At each incremental step along a given line $\psi$, the two-probe normalized correlation function for the electric-current fluctuation

$$C(x,z,\tau)=\frac{\langle\tilde{J}_1(0,0,0)\tilde{J}_2(x=r\sin\psi;z=r\cos\psi;\tau)\rangle}{[\langle\tilde{J}_1^{\,2}\rangle\langle\tilde{J}_2^{\,2}\rangle]^{1/2}} \tag{7}$$

is taken as a function of the time delay $\tau$, with use of a 5-MHz correlator. A family of about twenty such correlation curves (twenty steps along $\psi$) is recorded for each value of $\psi$, and five values of $\psi$ (0°, 14°, 26°, 45°, and 90°, same as in Fig. 2) are used. Each such family of space-time correlation curves can be processed in the same way as for previously reported results[8] for one-dimensional plane turbulence.

To obtain the correlation time $\tau_c(\vec{v})$ [Eq. (3)] for each value of $\vec{v}$ (expressed in polar coordinates $|\vec{v}|, \psi$, we graphically integrate $C(x,z,\tau)$ [Eq. (7)] along the unperturbed orbits $\vec{r}=\tau\vec{v}$,

$$\tau_c(v,\psi)=\int_0^\infty d\tau\, C(x=v\tau\sin\psi,\ z=v\tau\cos\psi,\ \tau). \tag{8}$$

Thus for each of the five fixed values of $\psi$, one obtains the variation of $\tau_c$ as a function of the modulus of $\vec{v}$.[11] These curves, shown in Fig. 3, resemble the theoretical results of Fig. 2. There is a single peak for $\psi=0°$, two peaks for $\psi=14°$ and 26°, and then a single peak for $\psi=45°$ and 90°. In addition, plotting the loci of the maxima of $\tau_c$ in the $(v_\perp, v_\parallel)$ plane shows that they are aligned along two straight lines symmetric about the $\vec{v}_\parallel$ axis. These lines are just the boundaries of the diffusion domain. The angle between these lines gives the opening of the diffusion domain, from which we can find the Cherenkov angle $\varphi$. The two lines intersect on the beam axis at the apex of the complementary cone. The velocity $c_A$ at the apex is related to the mean phase speed $c$ of the turbulent modes by $c=c_A\cos\varphi$ (Fig. 1). We thus find $\varphi=49.3°$ and $c=3.16\times10^5$ cm/sec for the turbulent spectrum used in these measure-

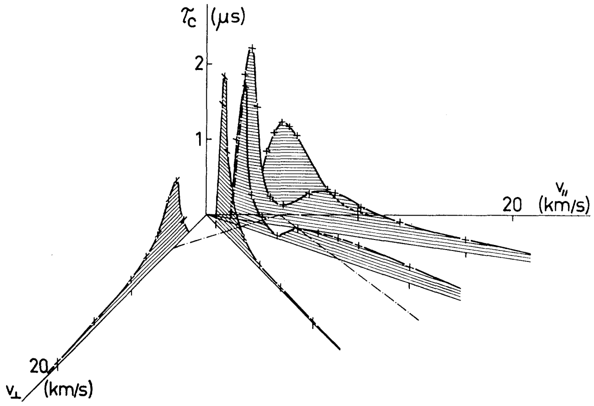


FIG. 3. Experimental correlation times in electron-beam–ion two-stream turbulence, obtained from correlation-function measurements, Eq. (9). Same polar coordinates as in Fig. 2. These curves show experimentally that the turbulence has a Cherenkov structure, with $\varphi = 50°$ and $c = 3.14$ km/sec.

ments.

The experimental curves are strongly broadened in comparison to the theoretical results of Fig. 2, particularly the $\psi = 0°$ curve and the second maxima for the $\psi = 14°$ and $\psi = 26°$ curves. The reason for this broadening is that the turbulent modes do not all terminate on an infinitely thin Cherenkov circle. The boundaries and apex of the diffusion domain are therefore increasingly broadened as one gets farther from the Cherenkov circle.

We can conclude from these results that the diffusion coefficient is far from isotropic in velocity space. This conclusion should hold as well for other types of three-dimensional turbulence, even if the Cherenkov character is not pronounced. One consequence of this is that the frequently used isotropic Lorentz pitch-angle collision model[5] is not accurate enough to describe two-stream or ion-acoustic turbulent processes. Furthermore, the quasilinear analysis used here shows which regions of three-dimensional velocity space are most affected by the turbulence. Quasilinear theory does not strictly apply in the region of the peaks,[12] however; the particle trapping times $\tau_b = (e\langle k\rangle E_{\rm rms}/m)^{1/2}$ are calculated to be about 1 $\mu$m sec for the ions and 3.7 nsec for the electrons, and these times are smaller than the spectrum correlation time at the peaks.

[1]O. Buneman, Phys. Rev. Lett. 1, 8 (1958), and Phys. Rev. 115, 503 (1959).

[2]M. Lampe, I. Haber, J. H. Orens, and J. P. Boris, Phys. Fluids 17, 428 (1974); D. Biskamp, K. U. Von Hagenow, and H. Welter, Phys. Lett. 39A, 351 (1972).

[3]K. Saeki, S. Dizuka, N. Sato, and Y. Hatta, in *Proceedings of the Fourteenth International Conference on Physics of Ionized Gases, East Berlin, German Democratic Republic* (The Physical Society of the German Democratic Republic, East Berlin, 1977).

[4]R. E. Slusher, C. M. Surko, D. R. Moler, and M. Porkolab, Phys. Rev. Lett. 36, 674 (1976); D. B. Ilic, Phys. Fluids 20, 1717 (1977); R. L. Stenzel, Phys. Fluids 21, 99 (1978).

[5]L. I. Rudakov and L. V. Korablev, Zh. Eksp. Teor. Fiz. 50, 220 (1966) [Sov. Phys. JETP 23, 145 (1966)]; W. Horton, Jr., D. I. Choi, and R. A. Koch, Phys. Rev. A 14, 424 (1976); C. T. Dum, Phys. Rev. Lett. 35, 947 (1975).

[6]T. D. Mantei, F. Doveil, and D. Gresillon, Plasma Phys. 18, 705 (1976).

[7]K. Estabrook and I. Alexeff, Phys. Lett. 36A, 95 (1971). However, in our work the phase velocity of the emitted modes is much smaller than the beam speed, since it is close to the ion acoustic speed $[c = 1.9(kT_e/m_i)^{1/2} \ll v_b]$.

[8]D. Gresillon, T. D. Mantei, and F. Doveil, Phys. Rev. Lett. 36, 1138 (1976).

[9]Specifically, $\omega_0 = \int d\omega\, \omega^4 F(\omega)/\int d\omega\, \omega^3 F(\omega)$.

[10]R. Limpaecher and K. R. MacKenzie, Rev. Sci. Instrum. 44, 726 (1973); R. J. Taylor, K. R. MacKenzie, and H. Ikezi, Rev. Sci. Instrum. 43, 1675 (1972).

[11]The current correlation function used in Eq. (8) is equal to the potential fluctuation correlation. Equation (8) is thus equivalent to Eq. (3), although we should have used the $E$-field correlation. The relation between the measured $\tau$, $\tau_{\rm meas}$ [Eq. (8)], and the theoretical $\tau$, $\tau_{\rm theor}$ [Eq. (3)], is $\tau_{\rm meas}/\tau_{\rm theor} = \langle k^2\rangle/\langle k^2(\vec{v})\rangle$, where $\langle k^2\rangle$ is the mean $k^2$ value over the spectrum, and $\langle k^2(\vec{v})\rangle$, is the mean $k^2$ value over the spectrum of modes in resonance with a particle of velocity $\vec{v}$.

[12]D. Pesme, G. Laval, and R. Pellat, J. Phys. (Paris), Colloq. 38, C3-213 (1977).