PHYSICAL REVIEW LETTERS

VOLUME 40 | 22 MAY 1978 | NUMBER 21

Precision Experimental Verification of Special Relativity

D. Newman, G. W. Ford, A. Rich, and E. Sweetman

Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan 48109

(Received 17 January 1978)

We compare the results of precision electron g-factor experiments at low energy and at 110 keV. The agreement between these measurements constitutes the most precise laboratory confirmation to date of the predictions of special relativity. Relativistic electromagnetic theory and Thomas precession are verified in this test. We also consider limits on possible effects of acceleration.

The theory of special relativity is assumed almost universally in physics, although it has been subjected to few high-precision tests, particularly for particles moving at high velocity. In this Letter we compare the results of a recent electron $g-2$ experiment,[1] done at $\beta \approx 5\times 10^{-5}$, with the results of an earlier $g-2$ experiment[2] carried out at $\beta \approx 0.5$. We conclude from the measured agreement of $g-2$ for free electrons at these different velocities that a major kinematic prediction of special relativity, the Thomas precession, has been verified to 5×10^{-9}. This is, by at least two orders of magnitude, the most accurate test of the Thomas precession to date. This conclusion is independent of the quantum-electrodynamic calculation of the g-factor anomaly. In addition, we remark that the agreement of these two results may be interpreted as verifying the assumed interaction of a relativistically moving magnetic moment with a magnetic field. As a final remark we argue that the absence of possible effects of acceleration on the g-factor is also verified.

Before discussing the $g-2$ work, we briefly review recent precision tests of special relativity in order to place our new comparison in context. These tests can be grouped into low-velocity and high-velocity experiments (see Table I). The low-velocity tests include measurements of the effect of Earth's velocity on laser frequency,[3] the temperature dependence of maser frequency,[4] the temperature dependence of the Mössbauer effect,[5] processes in rotating frames,[6,7] and the second-order Doppler effect.[8] Most recently, Brecher[9] in an article in this journal analyzed existing data on 70-keV x-ray pulses from the x-ray source Her X-1 ($\beta \approx 10^{-3}$), and concluded that the velocity of light is independent of the velocity of the source to an accuracy of 2×10^{-9}. We note that in these experiments β was never larger than 7×10^{-3}.

The high-velocity tests include the experiments of Grove and Fox[10] and of Zrelov, Tiapkin, and Farago,[4] who measured the masses of moving protons. Ayres *et al.*[12] measured both the lifetime and velocity of decaying pions in a beam at $\beta = 0.92$. They obtained γ from time dilation and β from time-of-flight measurements, thus verifying special relativity in a direct fashion to an accuracy of 4×10^{-3}. Alvager *et al.*[13] measured the velocity of γ rays from moving pions. Guiragossián *et al.*[14] compared the velocity of electrons and photons at 20 GeV to an accuracy of 2×10^{-7}. Bailey *et al.*[15] measured the time-dilated lifetime of muons decaying in the CERN 3-GeV storage

TABLE I. Tests of special-relativity predictions. In each test, the effect listed was measured at two velocities, β_1 and β_2. The resolution R is equal to the experimental uncertainty divided by the nominal magnitude of the effect. A quality factor F is also shown where applicable. F is defined as $R/\Delta\beta$ for velocity-of-light experiments and as $R/\Delta\gamma$ for the remaining experiments.

Ref	Effect Measured	Method	β_1	β_2	R
3	Ether Drift	Michelson-Morley Interference	0	10^{-4}	10^{-3}
4	Transverse Doppler	Temperature Dependence of Hydrogen Maser	9×10^{-6}	10×10^{-6}	3×10^{-2}
5	Transverse Doppler Effect	Temperature Dependence of Mössbauer Effect	2×10^{-4}	4×10^{-4}	10^{-1}
6	Transverse Doppler Effect	Rotating Absorber with Mössbauer Effect	0	7×10^{-7}	4×10^{-2}
7	Twin Paradox	Atomic Clocks	1×10^{-6}	2×10^{-6}	3×10^{-2}
8	Transverse Doppler Effect	Velocity Dependence of Atomic Line	0	7×10^{-3}	5×10^{-2}
9	Velocity of Light	Timing of Pulses From Binary Star	10^{-3}	-10^{-3}	2×10^{-9}
10	Relativistic Mass	Moving Protons	0	0.7	6×10^{-4}
11	Relativistic Mass	Moving Protons	0	0.81	10^{-3}
12	Pion Lifetime	Decaying Beam	0	0.92	4×10^{-3}
13	Velocity of Light	Decay of Moving Pions	0	0.99975	1.3×10^{-4}
14	Velocity of High Energy Electron	Comparison with Photon	1.(photon)	$1-5\times10^{-10}$	2×10^{-7}
15	Muon Lifetime	Storage Ring	0	0.9994	10^{-3}
21,22	Muon g-Factor	Precession in Storage Ring	0.38	0.9994	2.7×10^{-7}
1,2	Electron g-Factor	Precession in Electro-Magnetic Trap	5×10^{-5}	0.57	3.5×10^{-9}

ring. Using the lifetime at rest as determined by other workers,[16] they obtained the time-dilation factor γ. They compared this with the corresponding factor, which they called $\bar{\gamma}$, obtained from the cyclotron frequency. As shown in Table II, limits of order 10^{-3} in $(\gamma-\bar{\gamma})/\gamma$ at $\gamma=29.3$ were set.

The new evidence for special relativity which we point out here is that an existing measurement of the magnetic moment of the electron at an energy of 110 keV to an accuracy of 3×10^{-9} may now be compared directly with a new and even more precise (0.2×10^{-9}) magnetic-moment measurement performed on electrons with an energy of about 10^{-3} eV.

TABLE II. Summary of CERN results on muon decay (Ref. 15).

	μ^+	μ^-
Lifetime in flight	64.419(58)	64.368(29)
Lifetime at rest[a]	2.19711(8)	b
γ	29.320(26)	29.297(13)
$\bar{\gamma}$	29.327(4)	29.327(4)
$(\gamma-\bar{\gamma})/\gamma$	$(2\pm9)\times10^{-4}$	$(-10\pm5)\times10^{-4}$

[a]Ref. 16.

[b]Assuming the *CPT* theorem we use the measured μ^+ lifetime at rest for the μ^-. It is assumed that the observed effect of 2 standard deviations for μ^- is not experimentally significant.

In order to analyze the implications of these measurements, we will assume that the energy-momentum relation for a free electron, $E=E(p)$, differs from the usual relativistic form at high energies. Such a deviation could arise, for example, from a "band structure" due to a microscopic periodic structure of space felt by the electron. The electron's inertial rest mass (nonrelativistic mass) is

$$\frac{1}{m}=\lim_{p\to0}\frac{1}{p}\frac{dE}{dp}. \quad (1)$$

For the case of orbital motion perpendicular to a uniform magnetic field, the cyclotron rotation frequency will be

$$\omega_c=eB/\tilde{\gamma}mc, \quad (2)$$

where

$$\tilde{\gamma}=(p/m)dp/dE. \quad (3)$$

For this same case the spin-precession frequen-

TABLE III. Experiments testing the effect of acceleration on fundamental processes. The velocity and resolution are as defined in Table I.

Ref.	Method	Acceleration (cm/sec^2)	β	Resolution
5	Mössbauer effect	6×10^{16}	8×10^{-7}	10^{-1}
6	Rotating objects	1.7×10^{6}	7×10^{-7}	4×10^{-2}
7	Rotating objects	0.02	2×10^{-6}	3×10^{-2}
15	Muon lifetime	10^{21}	0.9994	10^{-3}
1,2	Electron g factor	10^{20}	0.57	3.5×10^{-9}

cy is given by

$$\omega_s = geB/2mc + (1-\gamma)\omega_c \tag{4}$$

where

$$\gamma = (1-\beta^2)^{-1/2}, \quad \beta = c^{-1}dE/dp. \tag{5}$$

The first term in (4) is the precession due to the interaction of the electron magnetic moment $\vec{\mu} = ge\vec{S}/2mc$ with the magnetic field. The second term is the well-known Thomas precession.[17] We emphasize that the Thomas precession is a result of the kinematics of special relativity as applied to accelerated systems. Hence the γ in (4) is given by the usual relativistic expression, while $\tilde{\gamma}$ in (2) arises from electron dynamics and need not be the same.

Wesley and Rich[2] at The University of Michigan trapped 110-keV electrons ($\beta = 0.57, \gamma = 1.2$) in a magnetic well at $B = 1.2$ kG. The quantity directly measured in this experiment is the difference frequency

$$\omega_D = \omega_s - \omega_c = \left(\frac{g}{2} - \frac{\gamma}{\tilde{\gamma}}\right)\frac{eB}{mc}. \tag{6}$$

Combining this with an NMR determination of eB/mc, they found $\frac{1}{2}g - \gamma/\tilde{\gamma} = 0.001\,159\,657\,70(350)$. This is to be compared with the result of Van Dyck, Schwinberg, and Dehmelt[1] at The University of Washington. In their experiment, a single electron of about 5×10^{-4} eV ($\beta = 5\times10^{-5}$, $\gamma - 1 = 10^{-9}$) in a Penning trap with $B = 20$ kG was excited with rf fields to measure the cyclotron and spin-cyclotron beat frequencies. Their result is $(\omega_s - \omega_c)/\omega_c = 0.001\,159\,652\,41(20)$. Combining these two measurements, we find

$$1 - \gamma/\tilde{\gamma} = (5.3 \pm 3.5)\times10^{-9}. \tag{7}$$

This verifies $\tilde{\gamma} = \gamma$ to this precision, in agreement with special relativity.

Note that this result does not depend upon quantum-electrodynamic calculations of $g-2$ since it is based upon a comparison of two experimental observations of ω_D, one at relativistic, the other at nonrelativistic, velocities.

Alternatively, the agreement between the $g-2$ experiments of Refs. 1 and 2 may be considered as a verification of the assumed theory of electron-spin motion for a relativistic Dirac particle. This theory can be based almost entirely upon the relativistic invariance of electromagnetism, as was done in the well-known paper of Bargmann, Michel, and Telegdi,[18] and even more explicitly in an unpublished report of Ford and Hirt.[19] Precision application of this theory to the spin motion in g-factor experiments has been given by Granger and Ford.[20]

An analogous result can be derived from measurements of the magnetic moment of the muon, which are less precise than the electron g-factor results and have a smaller variation of β, but reach higher values of γ. The current CERN results[21] at 3.0 GeV ($\beta = 0.9994$, $\gamma = 29$) and accurate to 10^{-8} in the magnetic moment are in agreement with their earlier measurements[22] of the muon magnetic moment at 1.27 GeV ($\beta = 0.92$, $\gamma = 12$) performed to an accuracy of $\pm 2.7\times10^{-7}$.

Finally, we note that possible effects of acceleration can also be considered.[23,24] As summarized in Table III, previous experiments have found no effects in fundamental processes from acceleration of rotation,[6,7] thermal vibrations,[5] or cyclotron motion.[15] The acceleration in the experiment of Wesley and Rich[2] was 1.3×10^{20} cm/sec^2, as compared with less than 10^{18} cm/sec^2 in the experiment of Van Dyck, Schwinberg, and Dehmelt.[1] No effects of such acceleration on the internal structure of electrons or on relativity which would affect spin precession in a magnetic field were observed at the previously considered accuracy of 5×10^{-9}.

We take pleasure in acknowledging stimulating conversations with H. R. Crane and L. Michel. This work was supported by the National Science Foundation under Grant No. PHY73-04587.

[1]R. Van Dyck, P. Schwinberg, and H. Dehmelt, Phys. Rev. Lett. 38, 310 (1977).
[2]J. Wesley and A. Rich, Phys. Rev. A 4, 1341 (1971).
[3]T. S. Jaseja, A. Javan, J. Murray, and C. H. Townes, Phys. Rev. 133, A1221 (1964).
[4]R. F. C. Vessot and M. W. Levine, Metrologia 6, 116 (1970).
[5]R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Lett. 4, 274 (1960).
[6]H. Hay, J. Schiffer, T. Cranshaw, and P. Egelstaff, Phys. Rev. Lett. 4, 165 (1960).
[7]J. C. Hafele and Richard E. Keating, Science 177, 166 (1972).
[8]H. I. Mandelberg and L. Witten, J. Opt. Soc. Am. 52, 529 (1962).
[9]K. Brecher, Phys. Rev. Lett. 39, 1051, 1236(E) (1977).
[10]D. J. Grove and J. G. Fox, Phys. Rev. 90, 378 (1953).
[11]V. P. Zrelov, A. A. Tyapkin, and P. S. Farago, Zh. Eksp. Teor. Fiz. 34, 555 (1958) [Sov. Phys. JETP 7, 384 (1958)].
[12]D. S. Ayres, A. M. Cormack, A. J. Greenberg, R. W. Kenney, D. O. Cladwell, V. B. Elings, W. P. Hesse, and R. J. Morrison, Phys. Rev. D 3, 1051 (1971).
[13]T. Alvager, F. J. M. Farley, J. Kjellman, and I. Wallin, Phys. Lett. 12, 260 (1964).
[14]Z. G. T. Guiragossián, G. B. Rothbart, M. R. Yearian, R. A. Gearhart, and J. J. Murray, Phys. Rev. Lett. 34, 355 (1975).
[15]J. Bailey, K. Borer, K. Combley, H. Drumm, F. Krienen, F. Lange, E. Picasso, W. von Rüden, F. J. Farley, J. H. Field, and P. M. Hattersley, Nature 268, 301 (1977).
[16]M. P. Balandin, V. M. Crebenyuk, V. G. Zindv, A. D. Konin, and A. N. Pondmarev, Zh. Eksp. Teor. Fiz. 67, 1631 (1974) [Sov. Phys. JETP 40, 811 (1974)].
[17]C. Møller, *The Theory of Relativity* (Oxford Univ. Press, London, 1952).
[18]V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Lett. 2, 435 (1959).
[19]G. W. Ford and C. W. Hirt, The University of Michigan Report, Contract No. Nonr 1224(15), 1961 (unpublished).
[20]S. Granger and G. W. Ford, Phys. Rev. D 13, 1897 (1976).
[21]J. Bailey, K. Borer, F. Combley, H. Drumm, F. Farley, J. Field, W. Flegel, P. Hattersley, F. Krienen, F. Lange, E. Picasso, and W. von Rüden, Phys. Lett. 68B, 191 (1977).
[22]J. Bailey, W. Bartl, B. vonBochmann, R. Brown, F. Farley, M. Giesch, H. Jöstlein, S. van der Meer, E. Picasso, and R. Williams, Nuovo Cimento A9, 369 (1972).
[23]J. E. Romain, Rev. Mod. Phys. 35, 376 (1963).
[24]C. W. Sherwin, Phys. Rev. 120, 17 (1960).

Instantons and the Hypothetical Light Boson

Varouzhan Baluni
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 10 March 1978)

The mass formula for the π, η, A (axion) system is analyzed taking into account explicit instanton effects. It is argued that small-size instantons serve as an independent source for the axion's mass which may render the axion more massive than the mixing of the bare axion with π and η.

There is a revived hope that the reality of Higgs particles can be finally established. Very recently Weinberg[1] and Wilczek[2] independently observed that the existence of an elementary pseudo Goldstone boson is an indispensable feature of a certain class of *CP-conserving* models which combine colored gauge interactions with unified electromagnetic and weak interactions. These models were defined earlier by Peccei and Quinn[3] by requiring an extended axial $U_{PQ}(1)$ symmetry for the entire Lagrangian. The symmetry is broken spontaneously by Higgs fields and explicitly by Adler-Bell-Jackiw anomalies. Hence the corresponding current A_μ^{PQ} is a source of pseudo Goldstone bosons—axions. In this note explicit effects of instantantons on properties of the axion will be exhibited.

The Lagrangian of the theory is given by

$$\mathcal{L} = \mathcal{L}_{G,W} + \mathcal{L}_{Y} + \mathcal{L}_{H,L}, \tag{1}$$

where $\mathcal{L}_{G,W}$ describes the gauge interaction of N-flavor quarks through colored gluons (G) and weak bosons (W) according to standard schemes SU(3) and $U_R(1)\otimes SU_L(2)$, respectively; $\mathcal{L}_{H,L}$ describes weak interactions of both leptons and *two* Higgs doublets $\varphi_i = (\varphi_i^+, \varphi_i^0)$, $i = 1, 2$, according

