three (in the unbiased case these would all be 25%).

Since the relative maximum in density at (0.4 0 0.4) in Fig. 2 is situated essentially along the line joining "t" sites, it is relevant to ask whether this site plays a role as a residence site. When the density is plotted from one "t" site to another, the location (0.4 0 0.4) is *a local minimum and not a maximum*; along this line the values of the density decrease from unity at ($\frac{1}{4}$ 0 $\frac{1}{2}$) to 0.63 at (0.31 0 0.44) and then to 0.37 at (0.375 0 0.375), point S. A concise manner of describing the situation is that the Ag ions reside at the "t" sites for about 3 psec, developing a thermal cloud of half-width 0.6 Å in the (100) plane and move between the "t" sites along rather narrow channels.

The work reported here makes it quite clear that using classic notions regarding the interparticle potentials in ionic materials it is possible to construct model systems which reproduce with fair precision the observed structural and dynamical behavior of α-AgI. We are therefore proceeding further on an analysis of the structure and properties of other forms of AgI as well as other materials like CuI. The analysis of density fluctuations in α-AgI is now in progress.

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Elementary Excitations in the Fermi Glass

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We evaluate the lifetime of quasiparticles in the Fermi glass. An anomalous contribution to the lifetime is found. It is shown that this effect leads to a $T^{1/4}$ law for the dc conductivity of a disordered system.

The importance of understanding the effects of electronic correlations in disordered systems has attracted some attention[1] in recent years —most notably, in interpreting various observations in the inversion layer at the semiconductor surface. In particular, in these systems where the number of carriers may be varied, density-dependent effective masses are reported[2] as well as an anomalous frequency dependence of the cyclotron resonance.[3] It is a rigorous result of the translationally invariant case, however, that for stationary processes, many-electron effects will make no contributions to the measurements for high enough fields. Thus it must be argued that either the nature of the disorder in these systems is such as to disturb the homogeneity of the samples or that many-electron effects behave somewhat differently in non–translationally-invariant systems.

In addition, the difficulties of incorporating various other anomalies, such as a free-electron–like Hall effect[4] and the presence of Shubnikov–de Haas oscillations[5] for densities well into the localized regime (as inferred from dc conductivity measurements) within the framework of the mobility-edge model, has tempted some workers[6] to abandon the independent-electron picture altogether. Nevertheless, it should be noted that there still exists no theory which treats the effect of a magnetic field on the Anderson localized

states except for some recent numerical work.[7] In addition, the many other successes of this model—for example, in providing a mechanism for the observed metal-insulator transition in many disordered systems and for explaining the observed minimum metallic conductivity, and our experience with the effects of interaction on metals—suggest a closer look at justifying the independent-particle model by establishing a corresponding quasiparticle picture.

In this Letter, we report and outline the procedure for deriving some fundamental properties of the Fermi glass, i.e., a system of interacting electrons localized in a random external potential. Up to now, attempts to generalize Landau's theory of the Fermi liquid to treat this system have been hampered by two essential complications. Firstly, the lack of translational invariance prohibits an algebraic reduction of Dyson's equation for the single-particle Green's function. This can be shown to lead to bare particles which are short-lived *near* the Fermi level. Secondly, unlike the case in Fermi-liquid theory where the unperturbed noninteracting Hamiltonian (Fermi gas) is exactly solved, the solutions for a single electron in a random potential are not known exactly. Nevertheless, the properties which are qualitatively understood about the noninteracting case (e.g., Anderson localization) are shown below to lead to considerable modification of the decay rates of elementary excitations in the glass.

We will assume that our system has certain properties. It is well known that degeneracy and symmetry are intimately related. Since a glass lacks the usual microscopic spatial symmetries, we can assume that the states of the noninteracting system are nondegenerate and described completely by their energy α. We assume, following Landau, that if we turn on the interaction at the proper rate that there is a one-to-one correspondence between the interacting and noninteracting states. Furthermore, since these states have the same symmetry, the states do not cross and the new states can also be labeled by their energy.

Because the system we consider is disordered, there is no symmetry which guarantees the existence of a basis in which the Green's function G is diagonal. Nevertheless, lifetime of an excitation can still be calculated by looking for the poles of the diagonal matrix elements of G. For the bare particles we use the α basis in which the noninteracting Green's function G^0 is diagonal. Thus we write

$$G_{\alpha\alpha}(\omega) = G_{\alpha\alpha}{}^{0}(\omega) + \int_{\alpha_1} G_{\alpha\alpha}{}^{0}(\omega)\Sigma_{\alpha\alpha_1}(\omega)G_{\alpha_1\alpha}(\omega), \tag{1}$$

where the symbols α label the exact eigenstates of the noninteracting random Hamiltonian H_0 and the sum is over all single-particle states. The states α diagonalize both H_0 and G^0. It can be shown that we may formally write the algebraic relation

$$G_{\alpha\alpha}(\omega) = G_{\alpha\alpha}{}^{0}(\omega) + G_{\alpha\alpha}{}^{0}(\omega)S_{\alpha}(\omega)G_{\alpha\alpha}(\omega),$$

where the quantity S_α may be expanded perturbatively in $\Sigma_{\alpha\alpha'}$ shown diagrammatically in Fig. 1 where the nth term has the structure

$$S_{\alpha}{}^{(n)}(\omega) = \int_{\alpha_1,\cdots,\alpha_{n-1}\neq\alpha} \Sigma_{\alpha\alpha_1}(\omega)G_{\alpha_1\alpha_1}{}^{0}(\omega)\cdots\Sigma_{\alpha_{n-1}\alpha}(\omega).$$

An expansion similar in structure has been developed in the noninteracting theory,[8] the convergence of which establishes the existence of localized single-particle states. In the Fermi-glass theory the existence of long-lived excitations of $G_{\alpha\alpha}$ requires $\mathrm{Im}S_\alpha$ to vanish near the Fermi level μ. The first term which just involves the diagonal contribution to the self-energy, $\Sigma_{\alpha\alpha}(\omega)$, behaves similarly to the proper self-energy which appears in the liquid theory, and $\mathrm{Im}\Sigma_{\alpha\alpha}(\omega)$ vanishes near μ since only processes which decay through many-particle states are allowed. Subsequent terms of S_α which contain off-diagonal matrix elements of $\Sigma(\omega)$ lead to finite contributions to the imaginary part since decay to *single-particle* states are permitted. These occur since there is no symmetry that excludes single-particle decay in the same way that momentum conservation does in the Fermi liquid.

In order to find a basis which avoids this diffi-

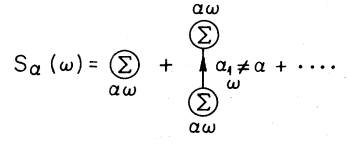

FIG. 1. Renormalization of the proper self-energy for the diagonal part of the interacting Green's function.

culty we define a *frequency*-dependent single-particle Hamiltonian by writing Dyson's equation in the form

$$[\omega - H_0 - \Sigma(\omega)]G(\omega) = 1. \quad (2)$$

In the basis in which $H_0 + \Sigma(\omega)$ is diagonal [the $m(\omega)$ basis], the eigenvalues determine the poles of G. However, $\Sigma(\omega)$ is not Hermitian and the eigenvalues of $H_0 + \Sigma(\omega)$ will, in general, be complex so that the corresponding states will decay. This is essentially a device for introducing an absorbing medium so that all single-particle states will decay through the many-particle system. The price is a non-Hermitian effective Hamiltonian which must be solved separately for each frequency ω. In general, both the eigenvalues and eigenvectors of $H_0 + \Sigma(\omega)$ will be frequency dependent.

By writing $G(\omega)$ in a representation which explicitly exhibits its frequency dependence (Lehman representation) it is readily seen that $G(\omega)$ is Hermitian for ω arbitrarily close to the chemical potential μ. Therefore, the eigenvalues of $H_0 + \Sigma(\omega)$ must also be real at $\omega = \mu$ and thus the excitations *at* the Fermi energy are stable.

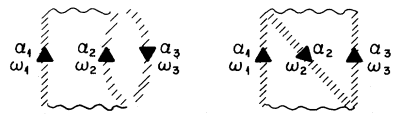

FIG. 2. Lowest contribution to the proper self-energy with nonzero imaginary part.

In particular, we evaluate the rate at which $\mathrm{Im}\Sigma(\omega)$ goes to zero near μ. The imaginary part of the eigenvalues of the effective Hamiltonian, and therefore the poles, will be dominated by this quantity. The Hartree-Fock terms give no imaginary parts; the lowest-order diagram which make a contribution are shown in Fig. 2. To emphasize the localized nature of the noninteracting states α we indicate the "propagators" as hatched lines. It is straightforward to show that the usual phase-space considerations lead to a $(\omega - \mu)^2$ contribution (for both diagrams) to $\mathrm{Im}\Sigma(\omega)$ which is, of course, the standard result. Modification of the lifetime occurs from consideration of the vertices. We may write the interaction as

$$U_{\alpha_1\alpha_2\alpha_3\alpha_4} = \int d\bar{x} \int d\bar{y}\, \varphi_{\alpha_1}(\bar{x})\varphi_{\alpha_3}(\bar{x})U(\bar{x} - \bar{y})\varphi_{\alpha_4}(\bar{y})\varphi_{\alpha_2}(\bar{y}), \quad (3)$$

where $U(x - y)$ is the two-particle interaction and φ_{α_i} are the single-particle localized states which behave at long distances from their centers as

$$\varphi_\alpha(\bar{x}) = \Phi_\alpha(\bar{x}) \exp(-\lambda_\alpha |\bar{x} - \bar{r}_\alpha|). \quad (4)$$

Here $\Phi_\alpha(x)$ is a well-behaved random-phased function which is needed for orthonormality. The state falls off exponentially with localization length λ_α^{-1}. Thus overlap with other states at long distances is exponential. Stability of the localized solutions requires the overlap matrix elements of any pair of such states to be less than their level spacing. In addition if the energies of a pair are E and E' where $|E - E'| < dE$, the two states must be a distance of order R apart, where R is given by

$$R = [\tfrac{4}{3}\pi N(E)|dE|]^{-1/3}, \quad (5)$$

with $N(E)$ the density of states per energy per volume. This correlation, inherent in the energy and distance scales of the unperturbed system, severely modifies the vertex contribution in Eq. (3), since for low energy or low-temperature processes only states near the Fermi energy are important. Specifically, evaluating Eq. (3) using the asymptotic form (4) we find

$$U_{\alpha_1\alpha_2\alpha_3\alpha_4} = \exp[-|\tilde{\lambda}/(E_1 - E_3)|^{1/3} - |\tilde{\lambda}/(E_2 - E_4)|^{1/3}]\, \tilde{U}_{\alpha_1\alpha_2\alpha_3\alpha_4}, \quad (6)$$

where $\tilde{U}_{\alpha_1\alpha_2\alpha_3\alpha_4}$, the reduced interaction, has a finite value at μ, and $\lambda = \lambda_\mu{}^3 3/4N(\mu)\pi$. As ω approaches μ, using (6), it is simple to show that $\mathrm{Im}\Sigma_{\alpha\alpha'}(\omega)$ approaches zero. Thus for ω near μ the major part of the effective Hamiltonian, $H_0 + \Sigma(\mu)$ has real eigenvalues. Using the standard techniques of Rayleigh-Schrödinger perturbation theory, and appealing to the general principles of adiabatic continuity and renormalization, we find the imaginary correction to the eigenvalues of the effective Hamiltonian to be

$$h_m \,\mathrm{sgn}(\mu - \omega)(\omega - \mu)^2 c^{-|\tilde{\lambda}/\omega - \mu|^{1/3}}, \quad (7)$$

where h_m is proportional to the square of the effective interaction.

It can also be shown[9] that further contributions have the same type of exponential behavior, although some may contain higher-order power prefactors. Thus, although $\tilde{\lambda}$ and h_m may be renor-

malized, the lifetime of the quasiparticle states is dominated by an exponential of the form (7). This result is in disagreement with the result obtained by Freedman and Hertz,[10] who find no modification from the liquid case. This is because of a failure to include the energy dependence of the matrix element. We would also like to point out that it has been implicitly assumed that interactions can generate continuum-like behavior. This matter will be discussed more fully elsewhere.[9]

Thus, we find proper quasiparticles whose lifetime is large near μ. In the absence of phonons, impurities, or any other scattering mechanism, we may use this result to evaluate the contribution to the conductivity. The dc conductivity σ, at finite temperature $1/\beta$, may be written as

$$\sigma = \int \sigma_E e^{-\beta E} dE, \tag{8}$$

where $e^{-\beta E}$ is the probability that a many-electron state of energy E is excited. In a simple Boltzmann theory, for hopping particles, σ_E may be taken to be proportional to the scattering rate for a quasiparticle excited to energy E above the Fermi level. Using the result for the lifetime in (7), the integral in (8) is easily evaluated and gives a temperature dependence, for low T, of

$$\sigma \propto \exp[-(\tilde{\lambda}/T)^{1/4}],$$

which is often observed in disordered systems. The important point is that this $T^{1/4}$ law (which is easily shown to be $T^{1/3}$ in two dimensions) is derived from a purely electronic theory without phonons. A similar temperature dependence has been derived by Mott from phonon considerations and is the well-known theory of variable-range hopping. The standard T^2 contribution, implied by the work of Freedman and Hertz should, if correct, show up at low temperatures. Experiments at temperatures in the millikelvin regime[11] on two-dimensional inversion layers find $T^{1/3}$ behavior.

As a final note, we wish to point out that in the extended regime although the bare states are still not long lived, there will be no exponential part in the lifetime. This is because there is no exponential contribution to the matrix elements due to the spatial distribution of the states.

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