lower amplitudes with the Pb beam. These findings seem to contradict the assumption that the conduction electrons play a major role in the damping.

The frequency term can be evaluated from a plot of the phase number of the oscillations versus the reciprocal ion velocity (Fig. 3). Approximately linear dependencies are found, the slope, which is proportional to $\langle Ea \rangle$, being steeper for the beam data, and for the additional oscillations observed in case of thin Pb films on Ni. The theoretical results seem to agree with the bulk results[5] or with the beam results.[6] The thin-layer results are intermediate between the theories. This difference of the theoretical values may be accidental in view of the simplifications and the arbitrary choice of some parameters entering into the calculations. Our results may help to define these parameters. Deviations from linearity (Fig. 3) are surprisingly small since this is only expected for $E \sin\vartheta = \mathrm{const}$ (E the energy, ϑ the scattering angle). Furthermore, it is an approximation for small ϑ only.[10,11] The data suggest a larger difference of the energy levels in case of the $5d$-$1s$ exchange for the beam than for the bulk. An even larger separation for the assumed $6s6p$-$2s2p$ exchange is indicated. Comparable data were obtained for Pb layers on Al, Si, and Cu substrates which will allow a more detailed discussion of the oscillatory fine structure.[9] Our main concern here is the direct comparison of the beam and the solid-state targets.

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Electron-Photon Coincidence Measurements in Electron Scattering from Atomic Hydrogen

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Angular correlations have been measured between electrons exciting the $2p$ state of hydrogen and the Lyman-α decay photons at incident energies of 70 and 100 eV and electron scattering angles of 5°, 10°, and 15°. The information obtained on the scattering amplitudes to the magnetic substates is compared with recent calculations, none of which are in good agreement with all of the data.

Electron-impact excitation of atoms in the low- to intermediate-energy range has been the subject of much theoretical work. Besides being of interest as a fundamental quantum mechanical scattering problem, electron-impact excitation has important applications in other fields of physics such as astrophysics and plasma physics. It is therefore essential to make adequate tests of the various scattering approximations. The more detailed the experimental data, the more stringent will be the test. Thus differential cross sections provide stricter tests of theoretical amplitudes than do total cross sections. Electron-photon coincidence measurements[1,2] provide an even more demanding test since they obtain detailed information on the excitation of the different magnetic sublevels of the final state.

The simplest inelastic scattering process is the electron-impact excitation of atomic hydrogen. Whereas other scattering problems are often complicated by the use of different bound-state wave functions and potentials, these are known exactly for atomic hydrogen, which therefore allows direct comparison of the scattering approximations. In this Letter we present the results of a study of electron-photon angular correlations in the electron-impact excitation of the $n=2$ state of atomic hydrogen at incident electron energies of 70 and 100 eV.

The atomic hydrogen source, a Pyrex dc discharge tube, is described by us in detail elsewhere.[3,4] The dissociation of the hydrogen effusing downwards to the collision region is typically $>75\%$. The apparatus is a modified version of the coplanar $(e, 2e)$ coincidence spectrometer described by McCarthy and Weigold[5] and by Fuss *et al.*[6] One of the two cylindrical-mirror electron-energy analyzers of the $(e, 2e)$ spectrometer is replaced by a photon detector. This consists of a collimator, grid, and LiF window placed in front of a channel electron multiplier. It is sensitive to Lyman-α photons (10.2 eV), but is blind to photons from more highly excited states of H and to most of the ultraviolet photons arising from the excitation of H_2.[3]

The angles subtended by the photon detector at the interaction region are 12° and 7° in the horizontal (θ) and vertical directions, respectively. The interaction region defined by the intersection of the atomic hydrogen beam and the incident and scattered electron beams was fully viewed by the photon detector at all angular settings. The entrance aperture of the electron-energy analyzer subtends a solid angle of 1×10^{-3} sr at the scattering center. This was reduced to 5×10^{-4} sr at the most forward angles studied ($\theta_e=5°$). The accuracy of the angular settings was better than $\pm0.3°$. The forward direction was obtained by optical alignment and confirmed by measuring differential cross sections on either side of $\theta=0$. Both the electron scattering angle and the photon angle in the scattering plane can be varied independently. The experiment was controlled by a PDP-11 computer.[5] The detection, control, and timing circuitry is essentially as described by McCarthy and Weigold.[5]

Care was taken to operate the neutral beam at sufficiently low density so that there was no significant trapping of resonant radiation. This has been investigated in a previous experiment[2] on He. Because of the lower resonance absorption cross section of the atomic hydrogen transition, the atomic H density can be somewhat greater than was the case for He. Also, the collision chamber was designed to take advantage of the fact that atomic hydrogen is effectively pumped by collisions with metal surfaces, so that the density of atomic hydrogen as seen by the photon detector dropped off far more rapidly with distance from interaction region than was the case for helium. As a check, angular correlations at 100 eV and 5° scattering angle were measured with atomic hydrogen densities differing by a factor of 2. They were not measurably different. The reported data are all obtained at the lower pressure. Coincidence energy-loss spectra, taken with the discharge both on and off,[3] show that the coincidence count rate is entirely due to excitation of the $n=2$ levels of H. With the discharge on, the coincidence energy-loss spectra are dominated by a peak at 10.2 eV (1.0 eV full width at half-maximum) well separated from the $n\geq3$ and undissociated H_2 contributions at higher energy losses.[3] Cascade contributions are therefore negligible. With the discharge off no coincidence counts were observed from H_2 at an energy loss of 10.2 eV. Contributions from vibrationally excited H_2 are also negligible (see Ref. 3 for details). The collision region was carefully shielded from stray electric fields to prevent $2s$ and $2p$ mixing. This was checked by measuring $H(e, 2e)H^+$ separation-energy spectra,[3,4] which are sensitive to stray fields. The timing resolution of 7×10^{-9} sec excludes any direct $2s$ contribution.

Macek and Jaecks[7] and Fano and Macek[8] developed general expressions for the angular correlation of photons detected in coincidence with a scattered electron after that electron has induced an atomic transition. For any s-p transition,

$$\frac{dN_c}{d\Omega_e d\Omega_\gamma}=\frac{\sigma}{\Sigma}K, \tag{1}$$

where $\sigma=\sigma_0+2\sigma_1$ is the differential cross section for the excitation, $\sigma_{|\mu|}$ being the cross section for exciting the $m=\mu$ sublevel, and Σ is the total integrated s-p excitation cross section, and in the case of atomic hydrogen[7] for $\varphi_e - \varphi_\gamma = \pi$

$$9K(k_i^2, \Omega_e, \Omega_\gamma) = [3\lambda + 4 + 3(1-2\lambda)\cos^2\theta_\gamma - 3\sqrt{2}R\sin 2\theta_\gamma], \quad (2)$$

where $\lambda = \sigma_0/\sigma$, $R = \mathrm{Re}\langle a_0 a_1\rangle/\sigma$, a_μ is the amplitude for exciting the $m=\mu$ sublevel, and $\langle a_\mu a_\mu\rangle = \sigma_\mu$. From symmetry $a_{-1} = a_1$. The quantities λ and R are related to the three alignment parameters of Fano and Macek through $A_0^{\mathrm{col}} = \frac{1}{2}(1-3\lambda)$, $A_{1+}^{\mathrm{col}} = \sqrt{2}R$, and $A_{2+}^{\mathrm{col}} = \frac{1}{2}(\lambda - 1)$. The experiment is, however, incapable of determining the orientation parameter O_{1-}^{col}. The magnitude of O_{1-}^{col} can be related to λ and R if and only if exchange scattering is negligible $\{|O_{1-}^{\mathrm{col}}| = [\lambda(1-\lambda) - 2R^2]^{1/2}\}$.

Figures 1 and 2 show the data obtained at incident energies of 70 and 100 eV and electron scattering angles of $-5°$, $-10°$, and $-15°$ (i.e., $\varphi_e = \pi$ and $\varphi_\gamma = 0$). The filled circles give the present data. At 100 eV and $\theta_e = -15°$ and $-10°$ some preliminary data have been reported by Williams.[9] These are indicated by crosses with the one-standard-deviation errors indicated by the dashed vertical lines. The experimental points have been normalized to the general expression given by Eq. (2), the solid line showing the best fit. The error loci for the quantities λ and R are also shown in the figures, the curves indicating the 95% confidence limits of the χ^2 tests, the solid circles and crosses indicating, respectively, the best-fit and Born-approximation values for λ and R. At 70 eV the values of λ and R obtained from the data are, respectively, 0.33 ± 0.05 and 0.28 ± 0.02 at 5°; 0.16 ± 0.06 and 0.28 ± 0.02 at 10°; and 0.21 ± 0.09 and 0.20 ± 0.04 at 15°. The 100-eV results are listed in Table I.

At 100 eV comparison can be made with several more sophisticated scattering approximations. The distorted-wave Born-approximation (DWBA) results of Calhoun, Madison, and Shelton[10] for λ and R (Table I) do not differ significantly from those of the simple Born approximation (BA), the respective angular correlations being indistinguishable. Roberts calculated the alignment and orientation parameters using a classical path approximation to the Coulomb T matrices (CPTM approximation).[11] Since exchange scattering is neglected in this calculation it is possible to derive values for R from his tabulations of λ and O_{1-}^{col}. They differ significantly from the experimental values and the Born results (Table I). The predicted CPTM angular correlations are shown by the dot-dashed curves in Fig. 2.

McDowell, Morgan, and Myerscough[12] and Morgan and McDowell[13] have used the distorted-wave polarized-orbital (DWPO) model to calculate λ and R. This approximation, which should be useful at energies above the ionization threshold and below the region of validity of the BA, is a single-channel model. It does, however, take into account distortion effects due to the short-range static local potential of the target ground state, and to the long-range polarization potential. It also includes off-diagonal s-p coupling via target polarization. The DWPO $1s \to (2p + 2s)$ differential cross sections are in excellent agree-

TABLE I. The parameters λ and R for the $1s$-$2p$ transition in H at 100 eV compared with the various approximations discussed in the text.

θ_e		Present results	BA	DWBA[a]	CPTM[b]	DWPO[c,d]	UGA[c,e]	CPB[e,f]
5°	λ	0.31 ± 0.03	0.31	0.31	0.52	0.32	0.28	0.07
	R	0.24 ± 0.02	0.33	0.33	0.29	0.33	0.32	~0.17
10°	λ	0.32 ± 0.07	0.14	0.13	0.19	0.20	0.11	0.01
	R	0.26 ± 0.03	0.25	0.24	0.15	0.26	0.22	~0.05
15°	λ	0.36 ± 0.09	0.11	0.10	0.09	0.17	0.06	0.01
	R	0.23 ± 0.04	0.22	0.20	0.02	0.25	0.17	~0.05

[a] Ref. 10.
[b] Ref. 11.
[c] Obtained from the published curves; may be in error by ± 0.01.
[d] Refs. 12 and 13.
[e] Ref. 14.
[f] Ref. 15.

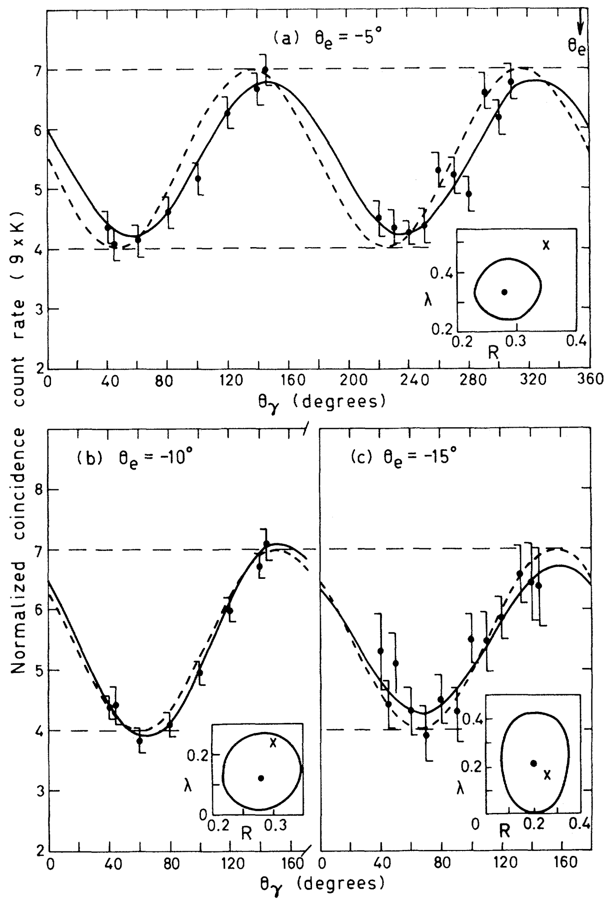

FIG. 1. The normalized coincidence count rate ($9K$) at an incident energy of 70 eV, plotted as a function of the photon detector angle, compared with the predictions of the Born approximation (dashed curve). The solid curve is the best fit of Eq. (2) to the data. The horizontal dashed lines indicate the theoretical maximum and minimum values of $9K$ [Eq. (2)]. The 95% confidence limits for the parameters λ and R are shown in the insets, the best-fit values are indicated by circles, and the BA values by crosses.

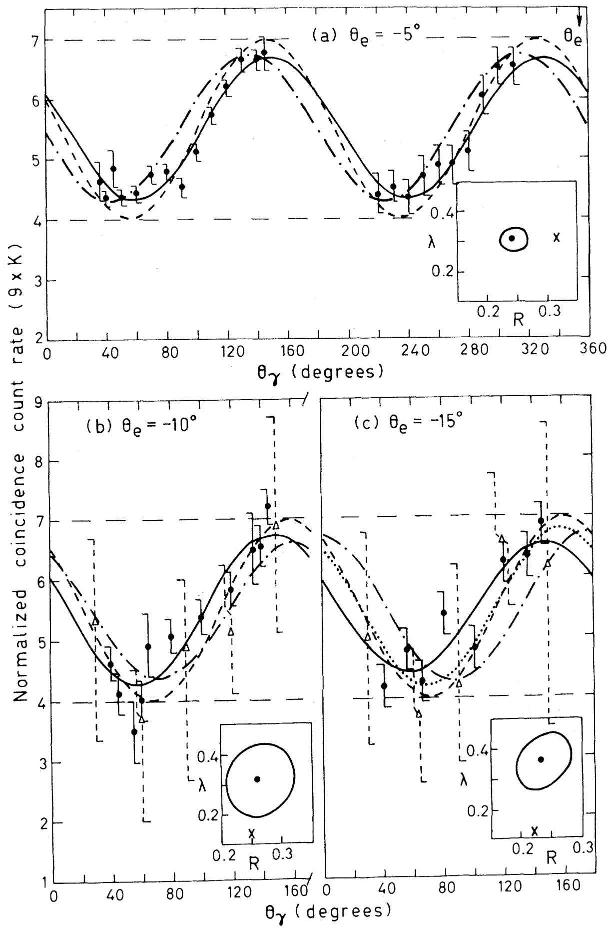

FIG. 2. As in Fig. 1 except for an incident energy of 100 eV. The triangles and dashed error bars indicate the preliminary experimental results of Williams (Ref. 9). The classical-path T-matrix calculation of Roberts (Ref. 11) is given by the dot-dashed curve, and the distorted-wave polarized-orbital (Refs. 12 and 13) calculation by the dotted curve.

ment with the intermediate-energy absolute measurements of Williams and Willis[16] at angles smaller than 60°. The DWPO values of λ and R are, however, in disagreement with experiment at 100 eV (Table I). At 5° and 10° the predicted angular correlations are nearly indistinguishable from those given by the BA. At 15° (dotted curve in Fig. 2) the DWPO angular correlation is intermediate between the predictions of the BA and the best fit to the data.

Some theoretical results in the unrestricted Glauber approximation (UGA) have also been reported by Gau and Macek.[14] The values for λ and R extracted from their figures are included in Table I. The values of λ and R are in poorer agreement with the data than even those of the BA. The predicted angular correlations are very similar to those of the BA at all angles and have not been included in the figure for the sake of clarity.

Morgan and Stauffer[15] have calculated the alignment and orientation parameters in the Coulomb-projected Born (CPB) approximation and its generalization at selected energies in the range 40–200 eV. Although the CPB approximation gives good agreement with the experimental differential cross sections for excitation of the $2s+2p$ levels in H, especially for scattering angles of less than 90°, its values for λ and R are in very poor agreement with the data. At 100 eV it predicts[15] an angular correlation with a minimum at 90° at all the measured electron scattering angles, in strong disagreement with the measurements. Again for clarity the angular correlations have not been included in Fig. 2.

Comparison of the 100-eV data with the various approximation therefore shows that although these approximations may describe the differential $2s+2p$ cross sections quite accurately, they do not give an adequate description of the coincidence measurements. The only approximation to improve noticeably on the BA description of the data is the DWPO model. The CPB, CPTM, and UGA models give much poorer results. The 70-eV data are in rather better agreement with the simple BA.

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Lie-Operator Approach to Mode Coupling in Nonuniform Plasma

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Hamiltonian perturbation theory based on recent Lie transform techniques is applied to the Hamiltonian of a single particle in nonuniform Vlasov plasma. A simple relation is derived between the field-plasma interaction energy and the transformed single-particle Hamiltonian. This relation implies as special cases a general formula for ponderomotive force in terms of the linear Vlasov susceptibility, and a symmetric Poisson-bracket formula for the general three-mode coupling coefficient.

Nonlinear interaction among waves and particles in plasma occurs in problems of parametric instability[1] and weak plasma turbulence[2]; these nonlinear processes have important applications to such areas as radio-frequency plasma heating, laser-plasma coupling, and the stabilization of linear instabilities. Several alternative theoretical approaches to these problems have evolved, including a direct perturbation expansion of the governing classical equations (e.g., the Vlasov-Maxwell system[3]), the temporary introduction of quantum mechanical ideas,[4] and the averaged-Lagrangian method.[5] Recently, the present authors have suggested a new approach[6-8] based upon a canonical transformation[9] of the single-particle Hamiltonian. This viewpoint, named the method of generalized ponderomotive forces, has been shown to provide a systematic and intuitive framework for the study of mode coupling in magnetized Vlasov plasma[8]; among its advantages